Physician Rostering with Downstream Capacity Constraints

Arik Senderovich
School of Information Technology, York University
Rotman School of Management, University of Toronto

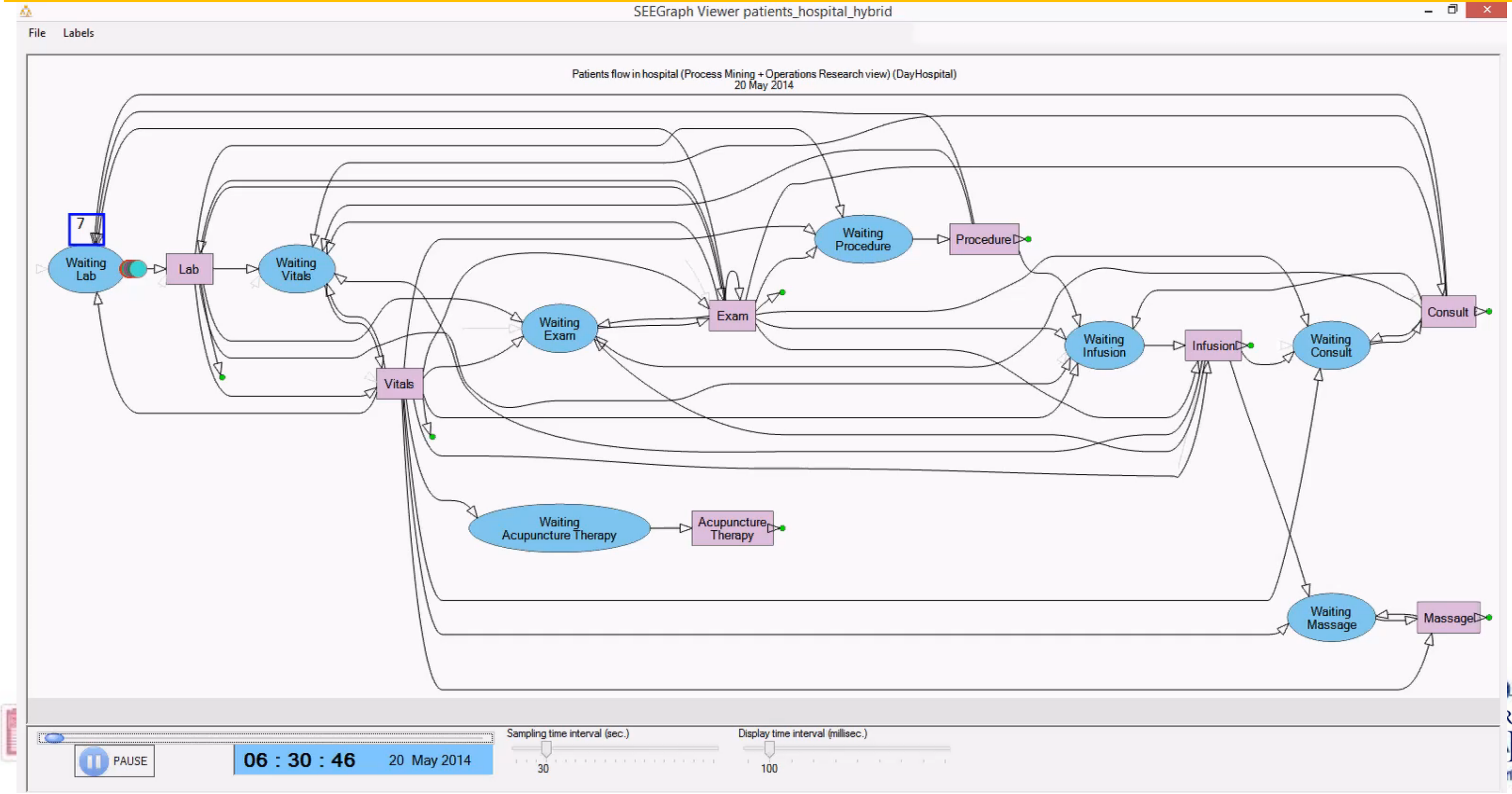
Yashi Huang, Yaron Shaposhnik
Simon Business School, University of Rochester

May 8, 2024

Motivation

- Collaboration with Dana-Farber Cancer Institute (DFCI)
- Large: ~1000 outpatients per day, two centers
- Diversity in physicians' specializations
  e.g., kidney cancer, liver cancer
- Different types of appointments (consultations, exams, chemotherapy)
- RTLS sensors at both sites; 10 years of data
- Providers are **rostered** to long-term running "exam sessions" (slots of time; usually cyclic)

SEENimation of the DFCI process



The Patient Flow

Activities

Arrival (check in) → Examination → Chemotherapy Treatment → Departure

Arrival (check in) → Blood Test → Examination

Examination → Departure

Resources

- Blood Test: Nurses/ Phlebotomists, Lab rooms
- Examination: Providers, Exam rooms
- Chemotherapy Treatment: Nurses, Infusion chairs

Providers: Session

- Hours long
- Multiple patients
- Variability in sessions' frequencies (weekly or biweekly)

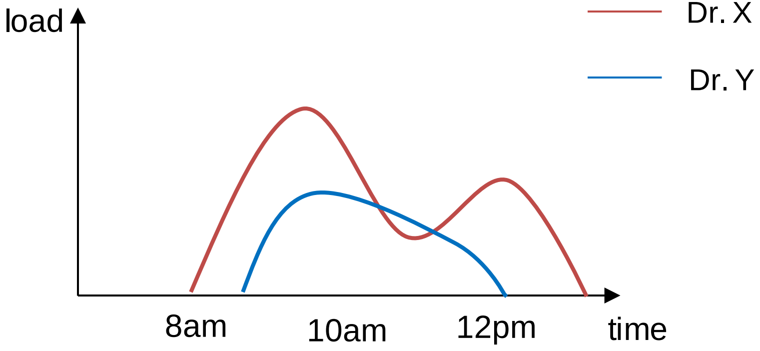
Impact of Exam on Infusion

- Sessions have impact on downstream infusion load (number of chairs)
- Sessions held by different providers impact differently

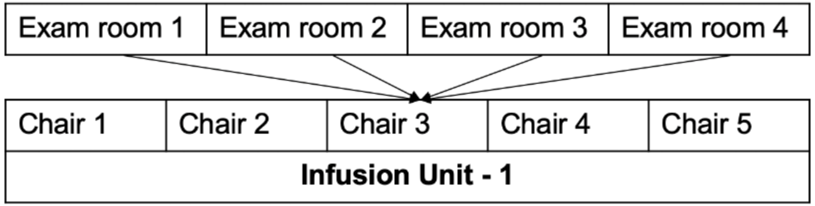
Dr. X and Dr. Y's session time

8:00am-9:00am	9:00am-10:00am	11:00am-12:00pm
Dr. X		-
Dr. Y		-

Infusion load for Dr. X and Dr. Y's sessions



Problem Setting



- Multiple sessions at the same time
- Total infusion load can vary for different combinations of sessions

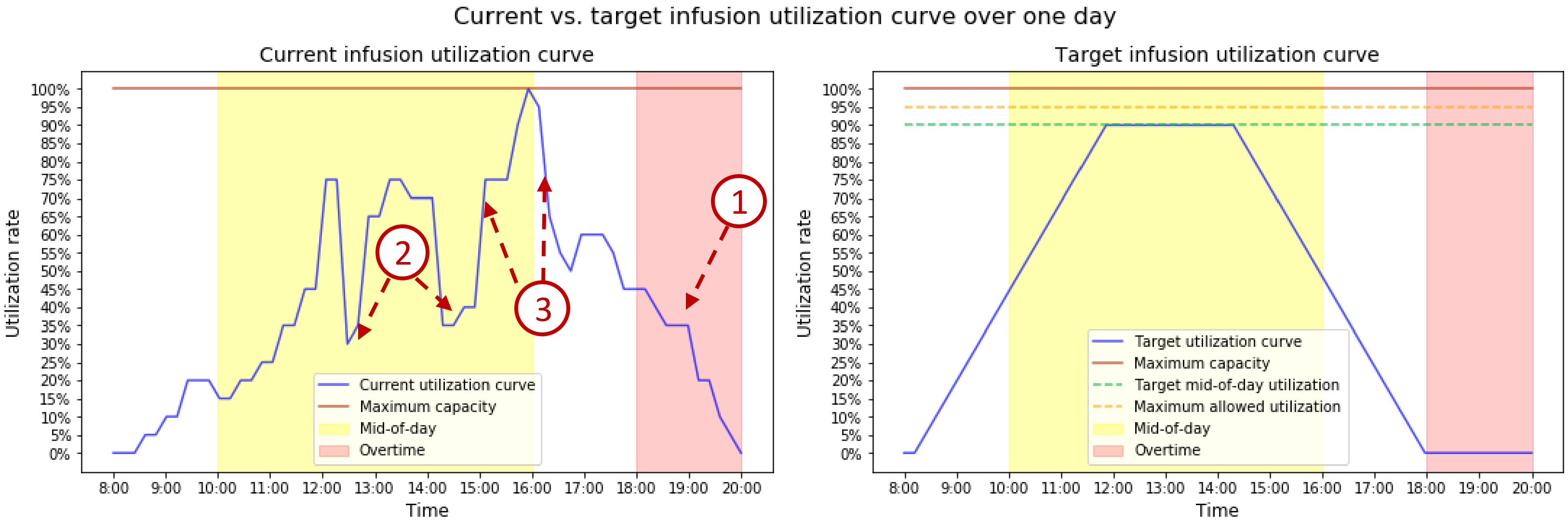
- **Research question**: How to roster provider sessions?
  - balance resource utilization (infusion chairs)
  - subject to downstream capacity constraints
  - make **as few changes** to existing roster as possible

Outline

- ~~Introduction~~
- **Problem Description**
- Numerical Experiment
- User Interface
- Takeaways and future steps

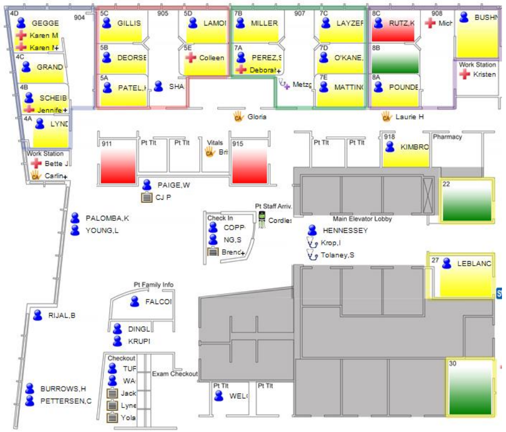
Objectives

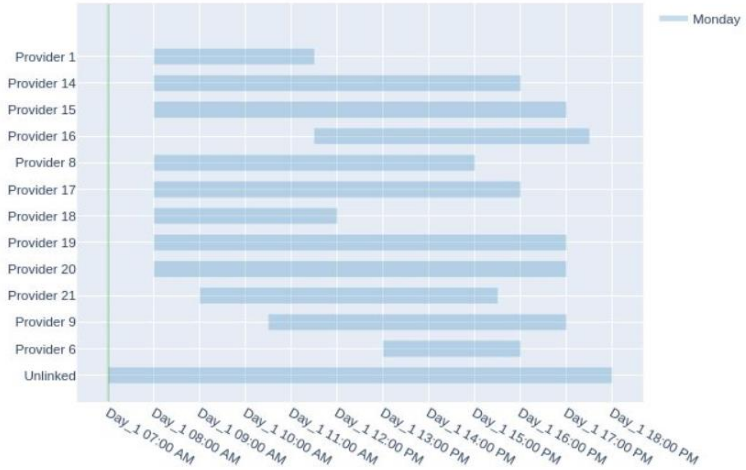
1. Total overtime;
2. Distance from an ideal mid-of-day target infusion utilization curve;
3. Fluctuations of the infusion curve.



Constraints

- Downstream capacity constraint - number of infusion chairs
- Exam room capacity constraint
- Provider availability
- Number of changes allowed (in terms of provider)





Physician Rostering with Downstream Capacity Constraints (PRDCC)

$$\min\ pc^{ot} \cdot \sum_{t \in \mathcal{T} \setminus \mathcal{T}^{r}} (t - LastRegSlot(t)) \sum_{\rho \in [R]} Q_{t,\rho} + pc^{tar} \cdot \sum_{t \in \mathcal{T}^{mid}} \sum_{\rho \in [R]} U_{t,\rho} + \sum_{t \in \mathcal{T}} \sum_{\rho \in [R]} D_{t,\rho} \tag{1}$$

$s.t.$

$$A_{\sigma,t} = A^{0}_{\sigma,t}, \forall \sigma \in \mathcal{S} : p_\sigma \in \mathcal{P}^{F} \tag{2}$$

$$B_{\sigma,t} = B^{0}_{\sigma,t}, \forall \sigma \in \mathcal{S} : p_\sigma \in \mathcal{P}^{F} \tag{3}$$

$$\sum_{t \in \mathcal{T}} B_{\sigma,t} = 1, \forall \sigma \in \mathcal{S} \tag{4}$$

$$B_{\sigma,t} = 0, \forall t \in \mathcal{T} \setminus \mathcal{T}^{\text{start}}, \sigma \in \mathcal{S} \tag{5}$$

$$\sum_{t \in \mathcal{T}} A_{\sigma,t} = l_\sigma, \forall \sigma \in \mathcal{S} \tag{6}$$

$$B_{\sigma,t} \le A_{\sigma,t+\tau}, \forall \sigma \in \mathcal{S}, t \in \mathcal{T}, \tau \in \{0 \le \tau \le l_\sigma - 1\} \tag{7}$$

$$A_{\sigma,t} = 0, \forall \sigma \in \mathcal{S}, t \in \mathcal{T}^{day\_ends} \tag{8}$$

$$\sum_{\sigma : p_\sigma = p} A_{\sigma,t} \le 1, \forall p \in \mathcal{P}, t \in \mathcal{T} \tag{9}$$

$$A_{\sigma,t} - \sum_{\sigma' : p_{\sigma'} = p_j} A_{\sigma',t} \le 1 - I^{attn}_{p_i,p_j}, \forall \sigma \in \mathcal{S} : p_\sigma = p_i,\ p_i, p_j \in \mathcal{P}, t \in \mathcal{T} \tag{10}$$

$$M_p \ge B_{\sigma,t} - B^{0}_{\sigma,t}, \forall \sigma \in \mathcal{S} : p_\sigma \in \mathcal{P}^{A}, t \in \mathcal{T} \tag{11}$$

$$\sum_{p \in \mathcal{P}^{A}} M_p \le M \tag{12}$$

$$\sum_{\sigma \in \mathcal{S}} A_{\sigma,t} \le C^{\text{exam}}_{t}, \forall t \in \mathcal{T} \tag{13}$$

$$Q_{t,\rho} = \sum_{\sigma \in \mathcal{S}} \sum_{n \in Z : Day(t-n) = Day(t)} B_{\sigma,t-n} \cdot W_{\sigma,n \cdot R + \rho}, \forall \rho \in [R], t \in \mathcal{T} \tag{14}$$

$$Q_{t,\rho} \le u^{max} \cdot C^{infusion}_{t}, \forall \rho \in [R], t \in \mathcal{T} \tag{15}$$

$$D_{t,\rho} \ge Q_{t,\rho} - Q_{t,\rho-1}, \forall \rho \ge 2, \rho \in [R], t \in \mathcal{T} \tag{16}$$

$$D_{t,\rho} \ge -Q_{t,\rho} + Q_{t,\rho-1}, \forall \rho \ge 2, \rho \in [R], t \in \mathcal{T} \tag{17}$$

$$D_{t,1} \ge Q_{t,1} - Q_{t-1,R}, \forall t \in \mathcal{T} : Day(t) = Day(t-1) \tag{18}$$

$$D_{t,1} \ge -Q_{t,1} + Q_{t-1,R}, \forall t \in \mathcal{T} : Day(t) = Day(t-1) \tag{19}$$

$$D_{t,1} = 0, \forall t \in \mathcal{T} : Day(t) \ne Day(t-1) \tag{20}$$

$$U_{t,\rho} \ge Q_{t,\rho} - u^{tar} \cdot C^{infusion}_{t}, \forall \rho \in [R], t \in \mathcal{T} \tag{21}$$

$$U_{t,\rho} \ge -Q_{t,\rho} + u^{tar} \cdot C^{infusion}_{t}, \forall \rho \in [R], t \in \mathcal{T} \tag{22}$$

$$A_{\sigma,t}, B_{\sigma,t}, M_p \in \{0,1\} \tag{20}$$

Details in the paper

PRDCC - Complexity

Lemma 1 *The PRDCC problem is NP-hard.*

Proof: We begin by defining NPP. Subsequently, we construct an instance of the PRDCC problem, and then we demonstrate its reduction from NPP.

Definition 1 (Number Partition Problem) *(Mertens 2006) Given a list of positive integer numbers* $w_1^{NPP}, w_2^{NPP}, ..., w_N^{NPP}$*, find a partition, i.e., a subset* $\mathcal{S}_1^{NPP} \subset [N^{NPP}]$ *such that the discrepancy*

$$D^{NPP}(\mathcal{S}_1^{NPP}) = |\sum_{i \in \mathcal{S}_1^{NPP}} w_i^{NPP} - \sum_{i \notin \mathcal{S}_1^{NPP}} w_i^{NPP}|,$$

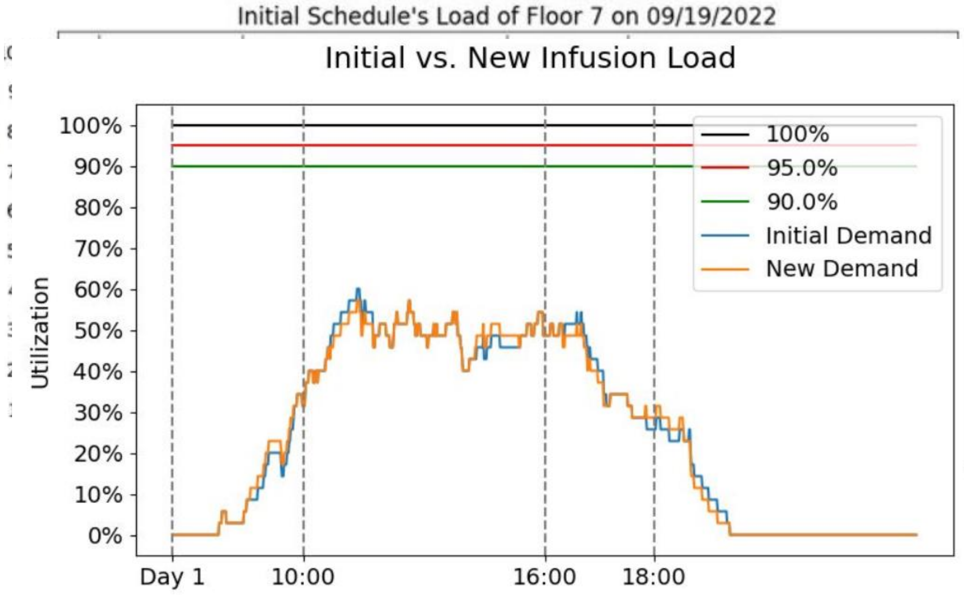
is minimized. A partition with $D^{NPP} = 0$ $(D^{NPP} = 1)$ *for* $\sum w_j^{NPP}$ *even (odd) is called* ***perfect partition****.*

- Complexity: total number of sessions & number of time slots
- In practice, MILP provides fast solutions to DFCI-size problems with one-month horizon

Predictive model for infusion load

Problem Description

- Optimization model - input output

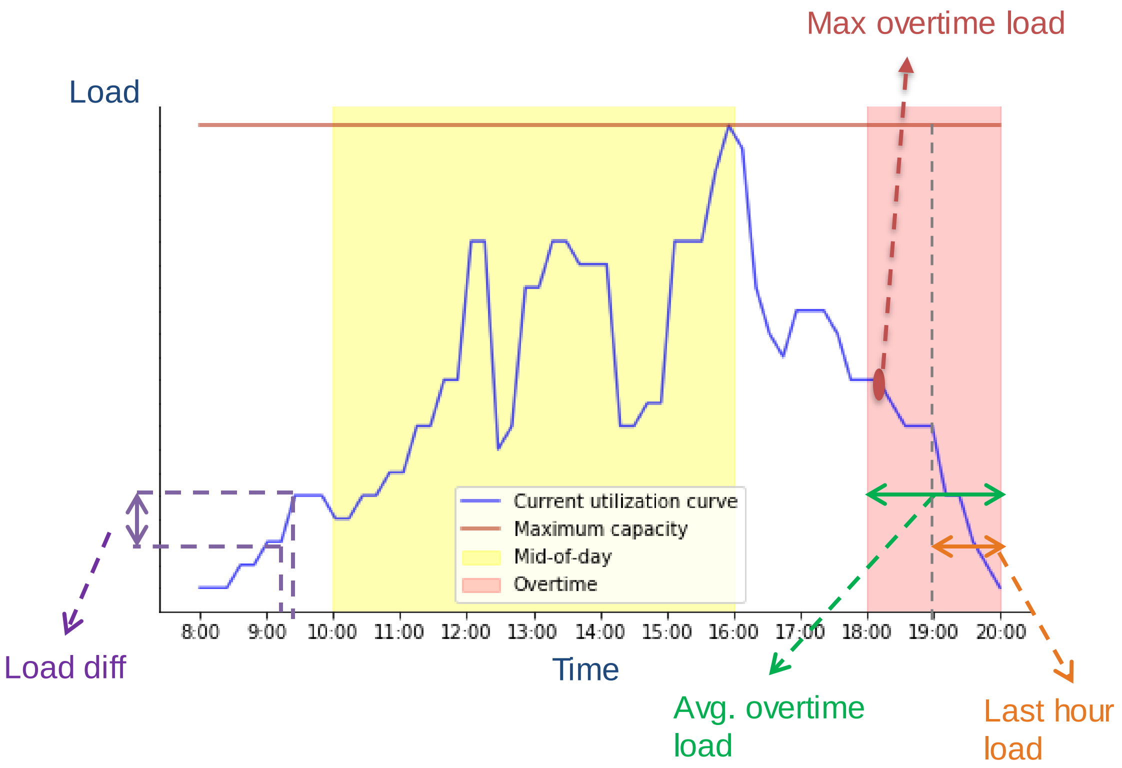


Outline

- ~~Introduction~~
- ~~Problem Description~~
- **Numerical Experiment**
- User Interface
- Takeaways and future steps

Performance Measures

- Overtimes
  - Average overtime load (average load during 18:00 - 20:00)
  - Max overtime load (peak load during 18:00 - 20:00)
  - Last hour load (average load during 19:00 - 20:00)
- Fluctuations
  - Average of 10-min load difference



Counterfactual Experiment

- DFCI Session data + "Action log"
  - September-December 2022 (18 weeks sample)
  - One-week planning horizon, per floor
- Infusion load (as function of exams) remains the same (no prediction)- only start-time shifts
- Change in rostering due to PRDCC OPT

Table 1 A statistical table of the current infusion load measures by infusion unit.

Unit	Num_sessions	Max Overtime Load				Average Overtime Load				Last Hour Load				Daily Fluctuation (10 min)			
	mean	mean	min	50%	max	mean	min	50%	max	mean	min	50%	max	mean	min	50%	max
6	60.76	9.06	5.0	9.0	13.0	2.67	1.27	2.57	4.27	1.11	0.45	1.01	2.33	0.58	0.50	0.59	0.65
7	65.47	17.00	11.0	18.0	25.0	5.74	3.28	5.08	12.38	2.52	0.94	2.26	6.52	0.98	0.87	0.98	1.13
8	87.81	17.12	12.0	17.0	22.0	6.97	5.00	7.17	9.98	3.68	1.98	3.49	5.94	0.94	0.88	0.94	1.05
9	78.78	14.78	12.0	14.0	20.0	5.84	3.73	5.91	11.38	2.74	1.68	2.44	7.34	0.90	0.80	0.89	1.04
10	66.56	17.94	6.0	18.0	24.0	6.31	2.88	6.04	9.46	2.79	1.55	2.24	5.00	0.90	0.62	0.92	1.01
11	43.53	7.12	3.0	7.0	11.0	2.05	0.63	1.87	4.20	0.78	0.12	0.71	1.95	0.50	0.45	0.48	0.61

Results

- Performance by number of changes allowed

Table 2 A table of statistics of load improvements (%) by number of changes.

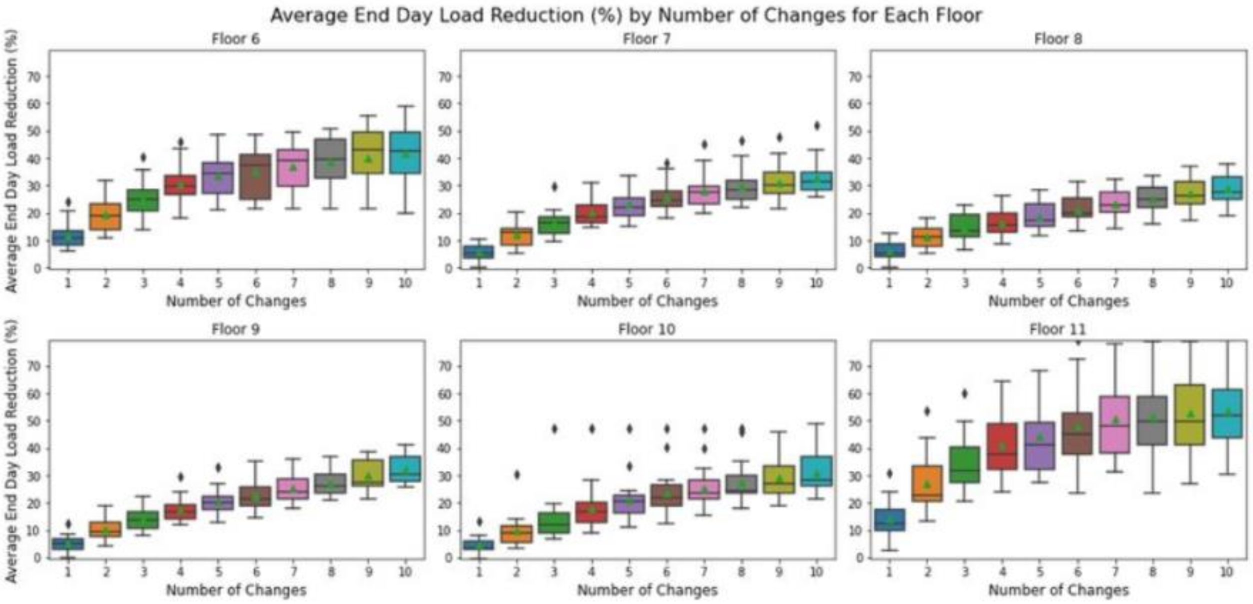
Changes	Max Overtime Load mean	min	50%	max	Average Overtime Load mean	min	50%	max	Last Hour Load mean	min	50%	max	Daily Fluctuation (10 min) mean	min	50%	max
1	1.86	-20.00	0.00	28.57	7.93	-0.43	7.05	30.98	17.70	0.00	14.26	100.0	0.78	-2.81	0.49	6.17
2	4.68	-30.00	4.17	42.86	14.92	3.15	13.11	53.42	31.33	4.12	25.70	100.0	1.66	-4.03	1.19	8.59
3	7.46	-25.00	7.14	42.86	19.87	6.48	17.21	59.83	40.52	11.24	34.18	100.0	2.45	-4.06	1.88	11.09
4	10.44	-33.33	11.11	45.45	24.04	8.73	20.15	64.28	46.58	16.31	40.29	100.0	3.13	-3.19	2.63	14.86
5	10.40	-40.00	11.56	50.00	26.91	10.96	23.62	68.06	51.42	19.76	46.60	100.0	3.76	-1.57	3.13	16.00
6	11.49	-25.00	10.00	57.14	29.37	12.31	25.40	79.29	55.33	24.08	51.12	100.0	4.35	-2.39	3.52	15.89
7	15.14	-20.00	14.29	71.43	31.59	14.21	27.93	78.16	58.42	25.10	53.60	100.0	4.99	-3.72	4.20	17.71
8	15.10	-33.33	14.29	71.43	33.26	16.03	29.22	78.79	60.53	25.22	57.05	100.0	5.82	-2.62	4.62	17.01
9	15.79	-33.33	14.29	71.43	34.96	17.06	31.81	78.79	62.19	29.02	59.17	100.0	6.39	-2.26	5.51	20.80
10	17.64	-21.43	15.38	71.43	36.73	18.96	33.62	79.33	63.46	32.78	60.16	100.0	7.15	-6.34	6.55	21.14

Objective (future work to test sensitivity to params):

$$10 * \sum_{t \in T} |D_t - D_{t-1}| + 20 * \sum_{t \in T^{mid}} |D_t - 0.9 * Capacity| + 50 * \sum_{t \in T^{end}} t * D_t$$

More (Graphical) Results

Figure 13 Average Overtime Load reduction % versus number of changes for each infusion unit.

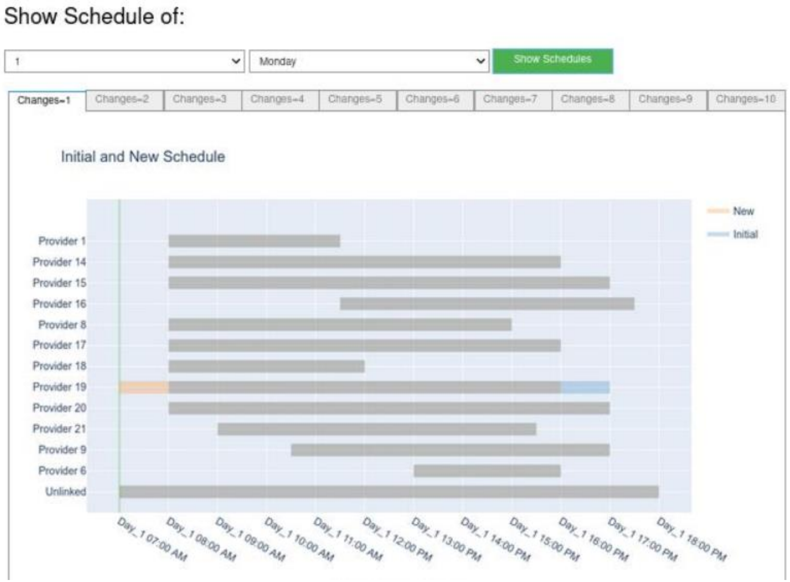


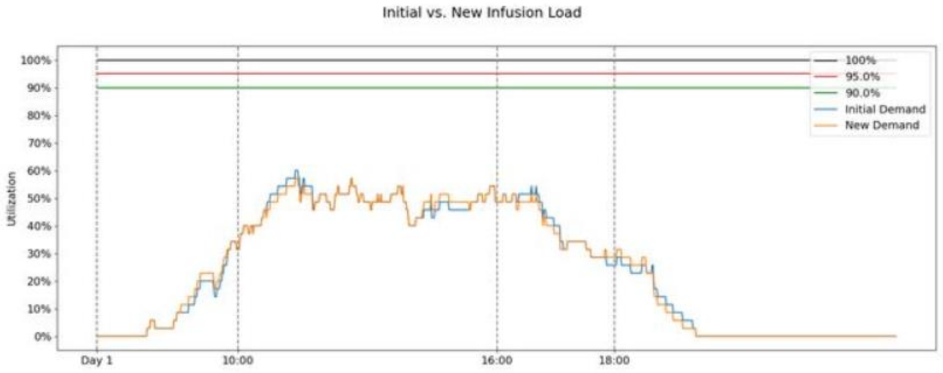
Outline

- ~~Introduction~~
- ~~Problem Description~~
- ~~Numerical Experiment~~
- **User Interface**
- Takeaways and future steps

User Interface

- A user interface (UI) to review and analyze the optimized rosters.
  - roster visualization





User Interface

- A user interface (UI) to review and analyze the optimized rosters.
  - performance metrics display
    - statistics table

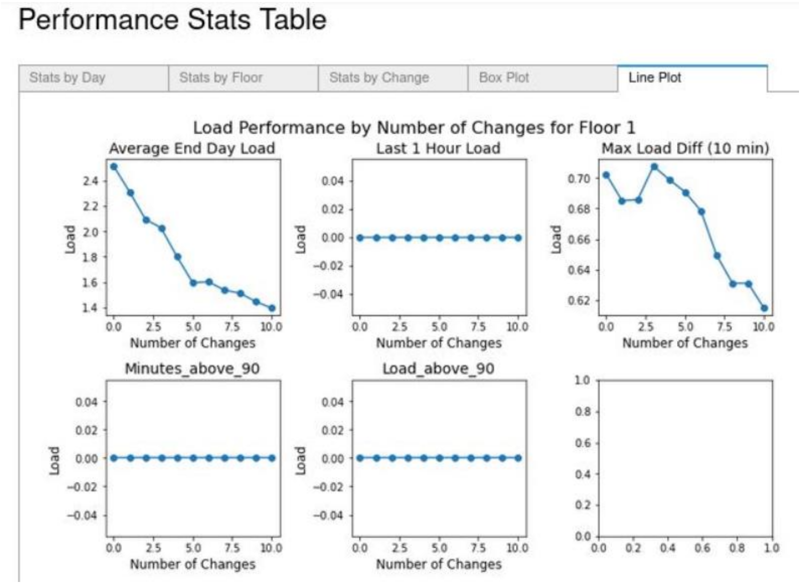
Performance Stats Table

Stats by Day | Stats by Floor | Stats by Change | Box Plot | Line Plot

	Average End Day Load Reduction				Last 1 Hour Load Reduction				Max Load Diff (10 min) Reduction				Minutes_above_90 Reduction				Load_above_90 Reduction			
	mean	min	50%	max	mean	min	50%	max	mean	min	50%	max	mean	min	50%	max	mean	min	50%	max
Date																				
Monday	0.78	0.00	0.83	1.12	0.0	0.0	0.0	0.0	0.02	-0.04	0.03	0.09	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
Tuesday	0.79	0.25	0.93	1.16	0.0	0.0	0.0	0.0	0.05	0.00	0.03	0.11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

User Interface

- A user interface (UI) to review and analyze the optimized rosters.
  - performance metrics display
    - plots



Outline

- ~~Introduction~~
- ~~Problem Description~~
- ~~Numerical Experiment~~
- ~~User Interface~~
- Takeaways and future steps

Takeaways

- Formulated an optimization model to solve rostering problem with downstream capacity constraints and proved its complexity
- Developing a predictive model to estimate the infusion load.
- Designed a user interface to present outcomes.

Future Steps

- Predictive experiment (out-of-sample) using real DFCI data for training and testing
- Providing a heuristic that would quickly solve PRDCC (for large instances and longer time horizons)
- Integrate predictions into the UI and run a pilot at DFCI

Thank you!

sariks@yorku.ca