Continuity of care for a home health care provider: how much is too much?

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Data Analytics in Healthcare

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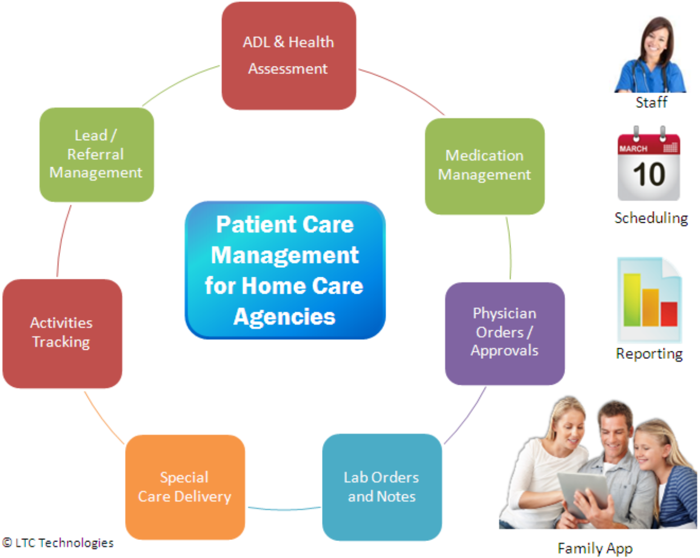
Home health care services

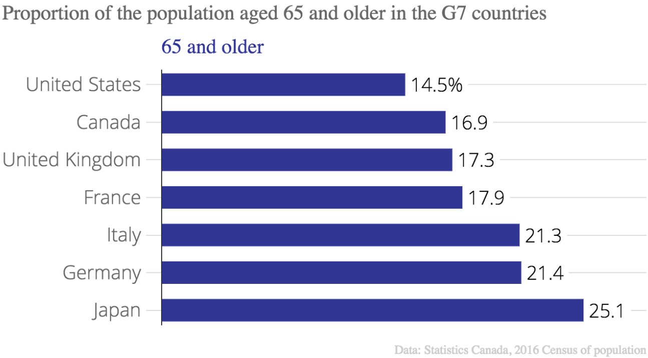
Home health care services

- Home Health Care (HHC) is an alternative to traditional hospitals.
- HHC is currently regarded as an essential service in patient-centric health systems.
- HHC is delivered via authorized HHC providers through licensed health practitioners, such as registered nurses, physical therapists, and/or personal support workers.

HHC agency responsibilities



Significance: Aging population in G7

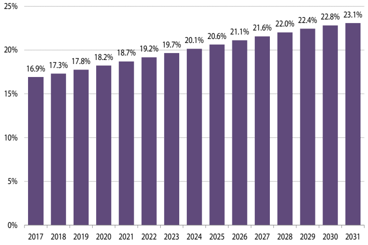


Significance: U.S.

- HHC is one of the world's most rapidly growing industries.
- In 2014, HHC was the fastest-growing U.S. industry with a projected growth of almost 5% per year through 2024.
- The National Association for Home Care and Hospice reports
    - 12 million patients received services from 33,000 agencies in North America in 2010.
    - 78.7% of these agencies are for-profit organizations.

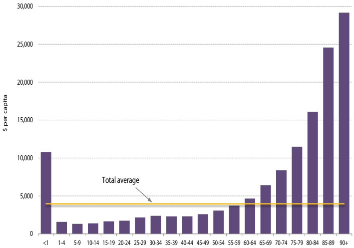
Significance: Canada

Figure 7: Proportion of the Population, 65 Years and Over, 2017-2031



Sources: Statistics Canada, 2014a and 2016a; calculation by authors.

Figure 8: Health Care Expenditure per Capita by Age Group, Canada, 2014



Source: CIHI, 2016.

Significance: Ontario

- Over 150,000 patients in Ontario rely on HHC services.
  - 34,500 patients patients in Toronto receive HHC services.
- Over 2.5 Billion was spent in Ontario for HHC services (5% of Ontario's total health budget).
- 92% of HHC patients in Ontario are satisfied with the services they have received.
- Provisioning care to terminally ill patients in an acute-care hospital is 10 times more expensive than at-home care.

Province-wide healthcare overhaul measure

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Sudbury

Ontario bill would overhaul $50 billion health care system, close CCACs

Health Minister Eric Hoskins isn't prepared to put a number on it, but says he expects "significant savings"

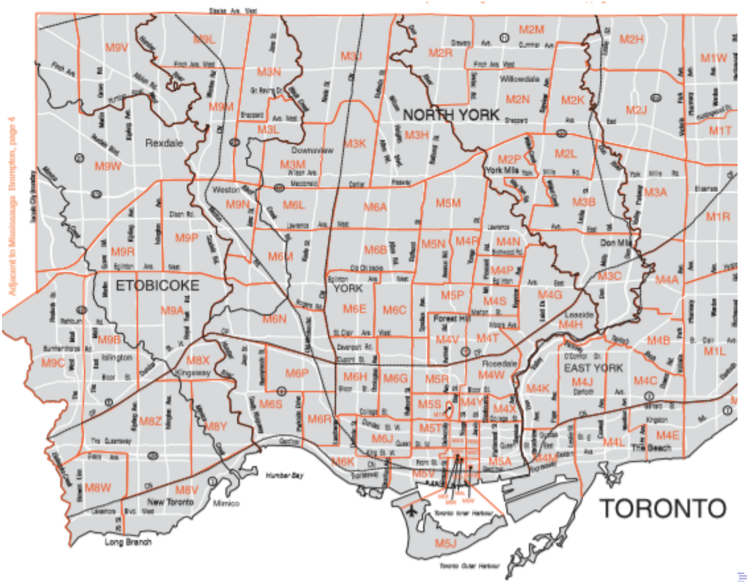
The Canadian Press · Posted: Jun 02, 2016 2:16 PM ET | Last Updated: June 2, 2016

Ontario Health Minister Eric Hoskins announced that all CCACs, created by the previous provincial government, will be shut down. (Darryl Dyck/Canadian Press)

Province-wide healthcare overhaul measure

- Government will shut down CCACs and integrate them into one of the 14 LHINs
  - Government needs to locate new HHC facilities
  - Home aides will be government employees.
  - Hiring/firing of aides will the government responsibility.

Toronto map: 96 FSAs



Locating HHC facilities in Toronto

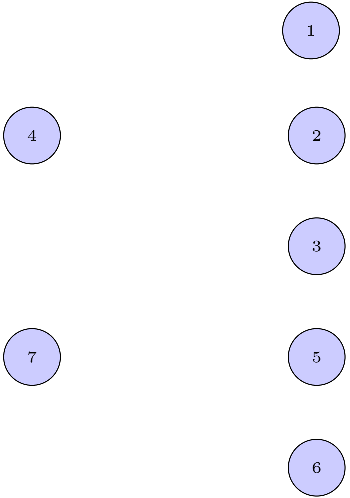
- 96 potential HHC demand locations
- 96 potential HHC facility sites
- Amount of each demand type from each demand node
  - Proportion of residential population
  - Proportion of commercial population
- 5 nursing demand types from each demand node
  - Proportion of each demand type
- 20 different time periods: Each equal to three months

Practical considerations

- Continuity of care
  - Full: permanent demand node to facility allocation
  - Partial: period-based demand node to facility allocation
- Nurse flexibility
- Nurse pooling
- Uncertainty in demand

Static location-allocation problem: baseline model

Figure: Potential facilities



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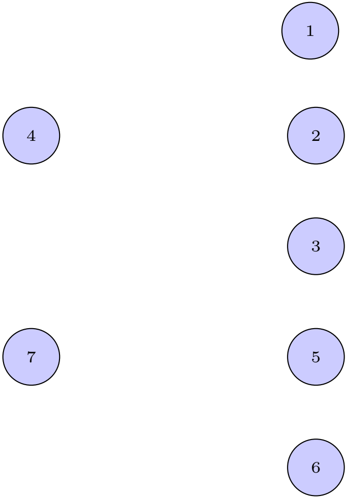
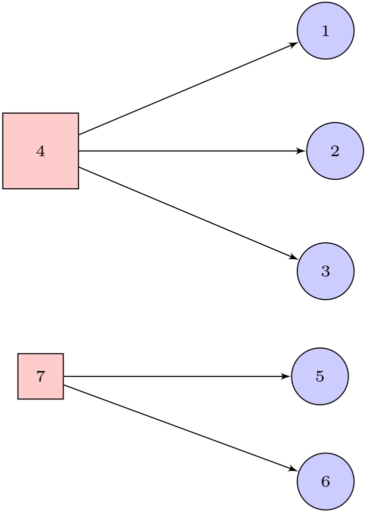


Figure: Established facilities



Decision variables

- **Location decisions**
  - where to establish home care facilities

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  - which region/demand type to serve by each open facility
- **Capacity allocation decisions**
  - how many nurses of each type to allocate to open facilities
- **Provisional capacity allocation decisions**
  - what should be the size of each open facility

Deterministic mixed-integer programming model

$$\underset{x,y,z,z_0,w^+,w^-}{\text{maximize}} \overbrace{\sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}}\Big(\eta_k - (R_{ij}+S_k)\theta_k - R_{ij}\Omega\Big)\bar{D}_{jtk}x_{ijk}}^{\text{Service provisioning revenue-cost}} - \Bigg(\overbrace{\sum_{i\in\mathcal{I}} CF_i y_i}^{\text{Facility set-up cost}} +$$

$$\overbrace{\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}} C_k^{\text{Provisional}} z_{i0k}}^{\text{Facility provisional capacity cost}} + \overbrace{\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}} C_k^{\text{First}} z_{i1k}}^{\text{First-period hiring cost}} + \overbrace{\sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}\setminus\{1\}}\sum_{k\in\mathcal{K}}\Big(C_k^+ w_{itk}^+ + C_k^- w_{itk}^-\Big)}^{\text{Periods hiring/firing cost}}$$

subject to

$$z_{i0k} \le L_k y_i \qquad \forall i\in\mathcal{I}, k\in\mathcal{K} \qquad (1)$$

$$z_{itk} \le z_{i0k} \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}, k\in\mathcal{K} \qquad (2)$$

$$\sum_{i\in\mathcal{I}} x_{ijk} \le 1 \qquad \forall j\in\mathcal{J}, k\in\mathcal{K} \qquad (3)$$

$$\sum_{j\in\mathcal{J}}\Big(\Big(R_{ij}+S_k\Big)\bar{D}_{jtk}\Big)x_{ijk} \le z_{itk} \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}, k\in\mathcal{K} \qquad (4)$$

$$w_{itk}^+ \ge z_{itk} - z_{i,t-1,k} \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}\setminus\{1\}, k\in\mathcal{K} \qquad (5)$$

$$w_{itk}^- \ge z_{i,t-1,k} - z_{itk} \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}\setminus\{1\}, k\in\mathcal{K} \qquad (6)$$

Objective function

- **Maximize service revenue:** η_k

Objective function

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- **Minimize service provisioning costs**:
  - service cost: transit time+transportation cost+service time
  - fixed cost of opening facilities
  - variable cost of acquiring provisional capacity
  - hiring/firing costs of nurses

Constraints

- **Unique Assignment.** Allocate each demand type from each demand node <u>to at most one</u> of the open facilities

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- **Maximum facility size.** Set maximum possible provisional capacity

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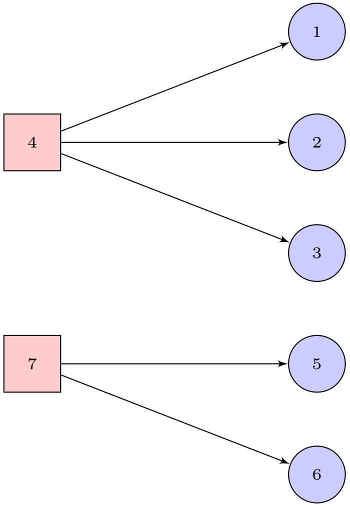
- **Period-based capacities.** Allocate required capacity to each demand type in each open facility in each period
- **Provisional capacities.** Determine provisional nursing capacity for each open facility
- **Maximum facility size.** Set maximum possible provisional capacity
- **Hiring/firing.** Compute hiring/firing of each nursing type
- **Budget limit.** Ensure the total cost of provisional capacity+facility opening does not exceed the considered budget

Static allocation: Full continuity of care

Figure: Location-allocation: $t = 1$



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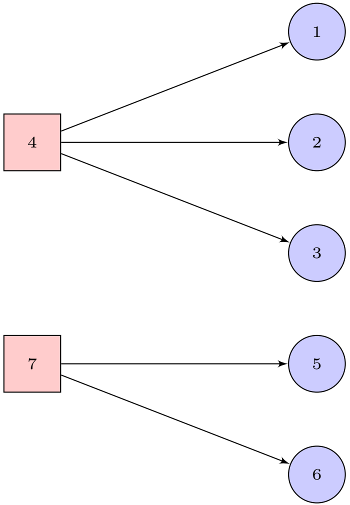
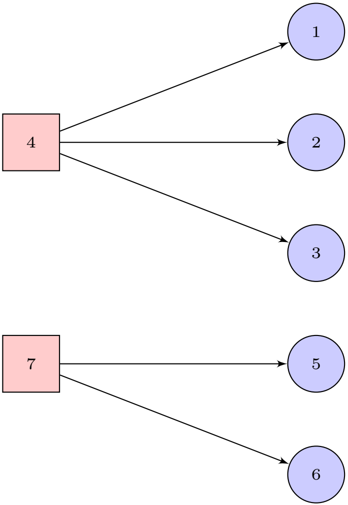


Figure: Location-allocation: $t = 2$



Dynamic allocation: Partial continuity of care

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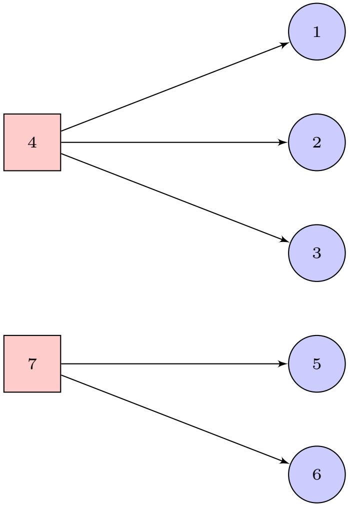

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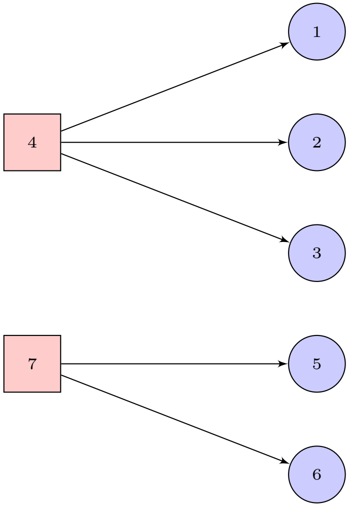

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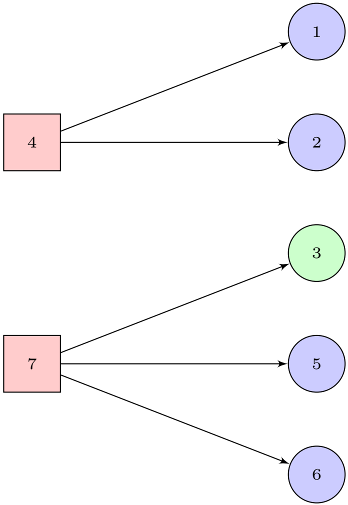
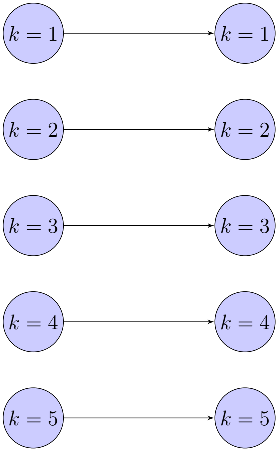


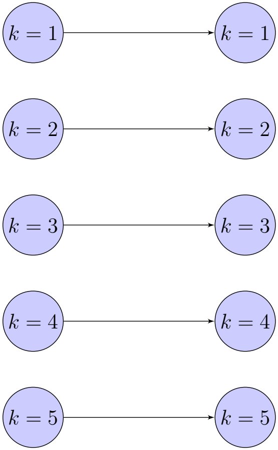
Nurse flexibility

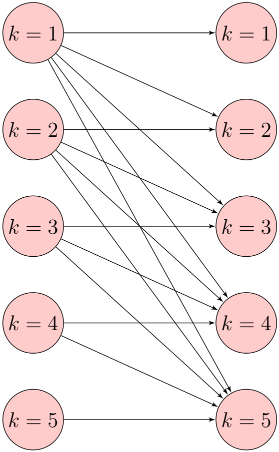
Each nurse performs exclusively the task that s/he specializes in.



Nurse flexibility

Higher-skilled nurses can perform the tasks of lower-skilled nurses.





Nurse pooling

- We only consider the network hiring/firing cost.
- We only penalize the surplus or shortage of the network with respect to the previous period.

Uncertainty in demand: Scenario-based approach

- We consider stochasticity in demand using scenarios:
  - $\bar{D}_{jtk} \longrightarrow D_{jtk}^{(s)}$

Almost Robust Mixed-Integer Optimization (ARMIO)

- **ARDO**[1] is a soft-constrained approach to robust optimization that
  - models robust optimization problems with binary variables,
  - trades off infeasibility versus objective function value, and
  - incorporates exogenous risk tolerance.

[1] Baron, O., Berman, O., Fazel-Zarandi, M. M., and Roshanaei, V., (2019). Almost Robust Discrete Optimization (ARDO), European Journal of Operational Research, In press.

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- **ARMIO** generalizes the concept of ARDO to
  - solve robust mixed-integer optimization problems,
  - trades off suboptimality versus objective function value, and
  - incorporates endogenous risk tolerance.

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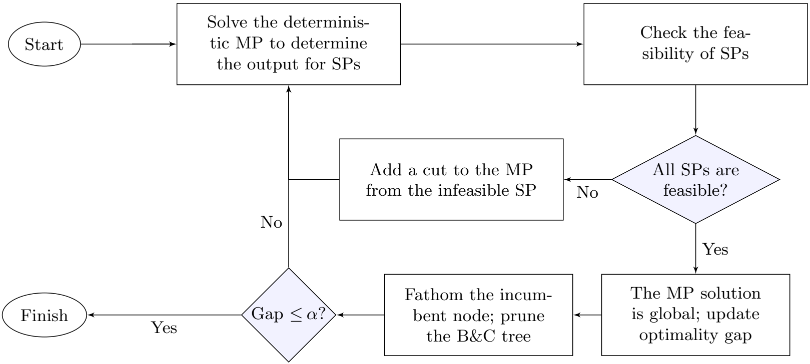
Size of the ARMIO model

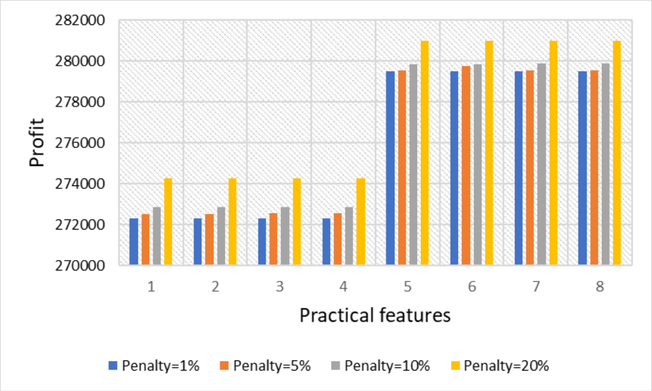
- Static variant: $\mathcal{O}(|\mathcal{I}| \times |\mathcal{J}| \times |\mathcal{K}|) \approx$ 50,000 variables
- Dynamic variant: $\mathcal{O}(|\mathcal{I}| \times |\mathcal{J}| \times |\mathcal{T}| \times |\mathcal{K}|) \approx$ 1,000,000 variables

Branch-and-Benders-Cut (B&BC) for ARMIO



Uncapacitated variants with varying risk tolerances

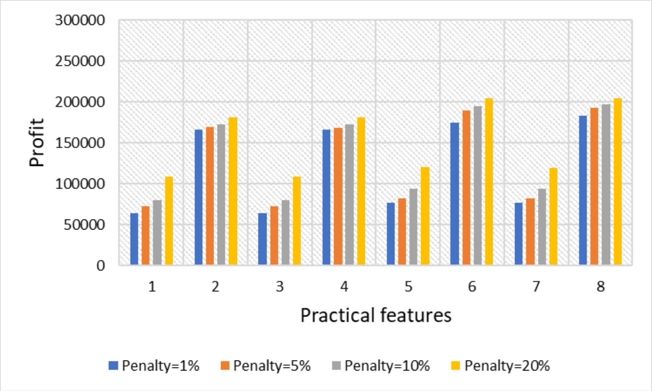
- Features 1 to 4 are static variants and 5 to 8 are dynamic variants.



- Largest contribution to profit (2.6%) due to dynamic allocation (feature 5)

Capacitated variants with varying risk tolerances

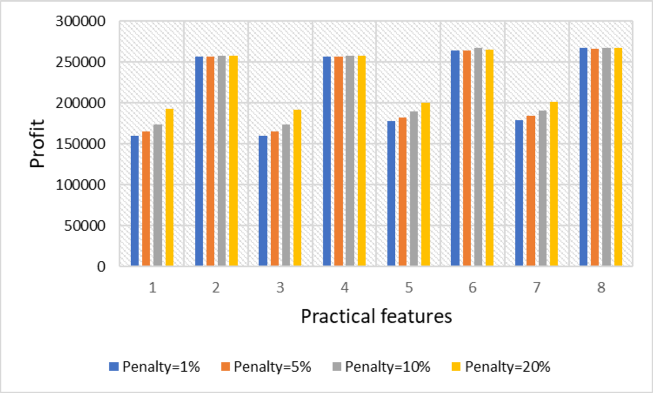
- Capacity of maximum **10 nurses** of each demand type



- Largest contribution to profit (2.5 times) due to nurse flexibility (feature 2)

Capacitated variants with varying risk tolerances

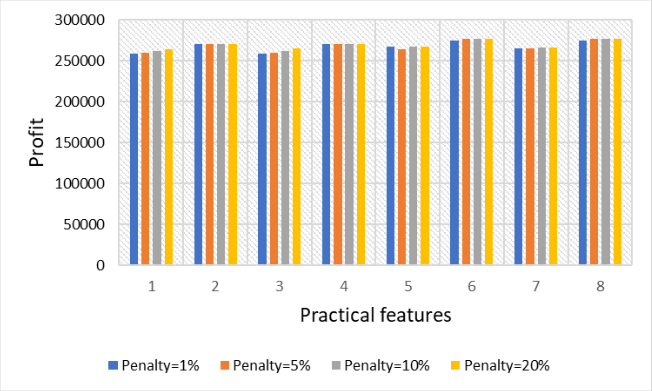
- Capacity of maximum **20 nurses** of each demand type



- Largest contribution to profit due to nurse flexibility (feature 2)

Capacitated variants with varying risk tolerances

- Capacity of maximum **50 nurses** of each demand type



- Diminishing the impact of practical features

Conclusion

- We developed new models and methods for locating HHC facilities in Toronto
  - Continuity of care, nurse flexibility, nurse pooling, stochasticity in demand

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- We developed new models and methods for locating HHC facilities in Toronto
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- We developed new models and methods for locating HHC facilities in Toronto
  - Continuity of care, nurse flexibility, nurse pooling, stochasticity in demand
- **Nurse flexibility** is most useful under capacity restriction. It can increase profit by 2.5 times (250%).
- **Dynamic allocation** of demand nodes to facilities has the largest contribution on profit (2.6%) when facilities can acquire unlimited capacities.
- Static allocation plus nurse flexibility is a reasonable trade-off among tractability, profitability, and continuity of care in the presence of unlimited capacity.

Thanks for your attention.

- Nursing capacity allocation in the absence of flexibility

$$\sum_{j \in \mathcal{J}} \Big(R_{ij} + S_k\Big) \bar{D}_{jtk} x_{ijk} \leq z_{itk} \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}$$

- Nursing capacity allocation in the presence of flexibility

$$\sum_{j \in \mathcal{J}} \sum_{k' \leq k} \Big(R_{ij} + S_{k'}\Big) \bar{D}_{jtk'} x_{ijk'} \leq \sum_{k' \leq k} z_{itk'} \qquad \forall i \in \mathcal{I}, k \in \mathcal{K},$$

- Extensions can be developed for
  - $x_{ijk} \geq 0$ and $x_{ijk} \in \{0, 1\}$
  - $x_{ijtk} \geq 0$ and $x_{ijtk} \in \{0, 1\}$

Inter-facility nurse pooling

- **No inter-facility nurse pooling:**

$$w^{+}_{itk} \geq z_{itk} - z_{i,t-1,k} \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \setminus \{1\}, k \in \mathcal{K}$$
$$w^{-}_{itk} \geq z_{i,t-1,k} - z_{itk} \qquad \forall i \in \mathcal{I}, t \in \mathcal{T} \setminus \{1\}, k \in \mathcal{K}.$$

- **Inter-facility nurse pooling:** Fired nurses of type k from each facility can work in other facilities with deficit in the same nursing category.

$$w^{+}_{tk} \geq \sum_{i \in \mathcal{I}} z_{itk} - \sum_{i \in \mathcal{I}} z_{i,t-1,k} \qquad \forall t \in \mathcal{T} \setminus \{1\}, k \in \mathcal{K},$$
$$w^{-}_{tk} \geq \sum_{i \in \mathcal{I}} z_{i,t-1,k} - \sum_{i \in \mathcal{I}} z_{i,t,k} \qquad \forall t \in \mathcal{T} \setminus \{1\}, k \in \mathcal{K}.$$

Uncertainty in demand

$$\sum_{j \in \mathcal{J}} \left(\left(R_{ij} + S_k \right) D_{jtk}^{(s)} \right) x_{ijk} \leq z_{itk} \qquad \forall i \in \mathcal{I}, t \in \mathcal{T}, k \in \mathcal{K}, s \in \mathcal{S},$$

B&C master problem with deterministic demand

$$\underset{x,y,z,z_0,w^+,w^-}{\text{maximize}} \quad \tau$$

$$\text{s.t. } \tau \leq \sum_{i\in\mathcal{I}}\sum_{j\in\mathcal{J}}\sum_{t\in\mathcal{T}}\sum_{k\in\mathcal{K}}\Big(\eta_k - \big(R_{ij}+S_k\big)\theta_k - R_{ij}\Omega\Big)\bar{D}_{jtk}x_{ijk} - \Bigg(\sum_{i\in\mathcal{I}}K_i y_i +$$

$$\sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}C_k^{\text{Provisional}}z_{i0k} + \sum_{i\in\mathcal{I}}\sum_{k\in\mathcal{K}}C_k^{\text{First}}z_{i1k} + \sum_{i\in\mathcal{I}}\sum_{t\in\mathcal{T}\setminus\{1\}}\sum_{k\in\mathcal{K}}\Big(C_k^+ w_{itk}^+ + C_k^- w_{itk}^-\Big)\Bigg)$$

$$\sum_{i\in\mathcal{I}} x_{ijk} \leq 1 \qquad \forall j\in\mathcal{J}, k\in\mathcal{K}$$

$$\sum_{j\in\mathcal{J}}\Big(R_{ij}+S_k\Big)\bar{D}_{jtk}x_{ijk} - z_{itk} \leq \ell_k \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}, k\in\mathcal{K}$$

$$z_{i0k} \leq L_k y_i \qquad \forall i\in\mathcal{I}, k\in\mathcal{K}$$

$$z_{itk} \leq z_{i0k} \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}, k\in\mathcal{K}$$

$$w_{itk}^+ \geq z_{itk} - z_{i,t-1,k} \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}\setminus\{1\}, k\in\mathcal{K}$$

$$w_{itk}^- \geq z_{i,t-1,k} - z_{itk} \qquad \forall i\in\mathcal{I}, t\in\mathcal{T}\setminus\{1\}, k\in\mathcal{K}$$

Master problem output for subproblems at incumbent h

- $\hat{\mathcal{I}}^{(h)}$: set of open facilities
- $\hat{\mathcal{J}}_i^{(h)}$: set of demand nodes allocated to open facility i
- $\hat{\mathcal{K}}_i^{(h)}$: set of nursing types served by open facility i
- $\hat{Z}_{itk}^{(h)}$: capacity of nursing type k at period t in open facility i
- $\hat{Z}_{i0k}^{(h)}$: provisional capacity of nursing type k for open facility i

Subproblem: Penalty function for each scenario

The penalty function for each scenario of hth MP solution:

$$Q_{ikt}^{(s)} = \left(\sum_{j \in \hat{\mathcal{J}}_i^{(h)}} \left(\left(R_{ij} + S_k \right) D_{jtk}^{(s)} \right) - \hat{Z}_{itk}^{(h)} \right)^{+} \quad \forall i \in \hat{\mathcal{I}}^{(h)}, t \in \mathcal{T}, k \in \hat{\mathcal{K}}_i^{(h)}, s \in \mathcal{S}$$

$\hat{Z}_{itk}^{(h)}$: capacity of nursing type k in facility i at period t obtained via deterministic demand: $\bar{D}_{itk}$

Expected penalty over all scenarios

$$\bar{Q}_{ikt} = \sum_{s \in \hat{\mathcal{S}}_{itk}^{(h)}} p_s Q_{ikt}^{(s)}.$$

Violations and Benders cuts

Upon observing any violation, develop a Benders cut that

1. Increases capacity z_{itk};
2. Removes at least one demand node from $\hat{\mathcal{J}}_i^{(h)}$; and/or
3. Implements both strategies.

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$$\tilde{Z}_{itk}^{(h)} \left(1 - \left(\sum_{j \in \hat{\mathcal{J}}_i^{(h)}} \left(1 - x_{ijk} \right) \right) \right) - z_{itk} \leq \ell_{itk} \ \forall i \in \hat{\mathcal{I}}^{(h)}, t \in \mathcal{T}, k \in \hat{\mathcal{K}}_i^{(h)},$$

where $\tilde{Z}_{itk}^{(h)} = \hat{Z}_{itk}^{(h)} + \bar{Q}_{ikt}^{(h)}$.

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where $\tilde{Z}_{itk}^{(h)} = \hat{Z}_{itk}^{(h)} + \bar{Q}_{ikt}^{(h)}$.

Theorem

The above inequality is a valid Benders cut and does not remove any globally integer feasible solution.

Subproblem with nurse flexibility

$$
\begin{aligned}
\bar{Q}_{it}^{(h)} := \min \quad & \sum_{k \in \mathcal{K}} \sum_{s \in \mathcal{S}} p_s Q_{itk}^{(s)} && \text{(LP model)} \\
\text{subject to} \quad & \sum_{k' \geq k} e_{itkk's} \leq \hat{Z}_{itk} && \forall k \in \mathcal{K}, s \in \mathcal{S}, \\
& Q_{itk}^{(s)} \geq \sum_{j \in \hat{\mathcal{J}}_i} \Big(R_{ij} + S_k \Big) D_{jtk}^{(s)} - \ell_k - \sum_{k' \leq k} e_{itk'ks} && \forall k \in \mathcal{K}, s \in \mathcal{S}, \\
& e_{itkk's} \geq 0 && (k, k') \in \mathcal{K} \, | k' \geq k, s \in \mathcal{S} \\
& Q_{itk}^{(s)} \geq 0 && k \in \mathcal{K}, s \in \mathcal{S},
\end{aligned}
$$

Toronto data

- 96 demand nodes (centroid of each region)
- 150,000 HHC patients served in Ontario
- 34,500 HHC patients service in Toronto (23% of Ontario population)
  - residential population of each demand node is known.
- Fraction of each nursing demand type: [5.2%, 0.7%, 31.5%, 56.9%, and 5.7%]
- Nursing cost: [40, 35, 30, 25, 20]
- Revenue per visit: [60, 50, 40, 35, 25]
- Transportation cost: 41 cents per km
- Service time: 50 minutes
- Budget: 50,0000,000
- Fixed cost of facilities ≈ U[800,000,1,700,000]
- Scenarios: 100

Future work

- Robustness Index (RI)

$$\text{RI} = \frac{\text{improvement in objective function value}}{\text{increase in total penalty}} = \frac{c^T x_\ell^* - c^T x_0^*}{\bar{Q}(x_\ell^*)^T I_{1\times J}}$$

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- **Location decisions**

$$y_i = \begin{cases} 1, & \text{if facility } i \text{ is established} \\ 0, & \text{otherwise} \end{cases}$$

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- **Provisional capacity allocation decisions**
  $z_{i0k} \geq 0$: provisional capacity allocation to type k nursing demand in open facility i