The Composition of Market Participants and Asset Dynamics*

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Abstract

We develop a dynamic equilibrium model where heterogeneous investors endogenously choose to enter/exit the stock market. We characterize the equilibrium and present a conditional consumption-CAPM. The model implies small changes in the composition of stockholders, which generate a strongly countercyclical stockholders' amount of consumption risk. The model provides a new perspective on the main drivers of asset dynamics. It is the procyclical consumption risk-sharing implied by changes in stockholders' composition that contributes to the dynamics of risk premium, excess volatility, and price-dividend ratio. We provide empirical evidence on market participation, amount of risk, and price of risk, supporting our theory.

JEL Classification: *G11, G12, G17*

Keywords: *Time-varying composition of stockholders, Heterogeneous investors, Conditional asset pricing, labor income risk, recursive utility*

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1 Introduction

Considerable research in financial economics has established the low level of stock market participation rate and its wide-ranging effects on asset prices. While it is well-accepted that this rate exhibits very mild time-series variation, recent studies have documented a simultaneous and active stock market entry/exit of investors.[1] These findings suggest that while the level of the stock market participation rate is quasi-stable, the composition of market participants at each time may be different. This article exploits this changing composition of stock market participants and offers a new perspective on some asset pricing puzzles.

This article shows that, once the participation decision is endogenized, a mild change in the composition of investors in terms of their risk aversions can be generated. The model-implied variation in investors' composition generates different dynamics for stockholder's consumption risk versus aggregate consumption risk. On the one hand, our model implies weakly procyclical to countercyclical aggregate consumption risk. This dynamics is well known in the literature and constitutes the Achilles heel of consumption-based asset pricing models when tested empirically.[2] On the other hand, our model-implied stockholders' consumption risk is strongly countercyclical. We demonstrate that these different dynamics are a strong feature of the data, robust to different state variables, methodologies, and sample periods. The model distinction of aggregate versus stockholders' consumption risk dynamics (i) sheds light on why previous research testing the conditional CCAPM have

[1] Mankiw and Zeldes (1991) show evidence of limited participation. Wolff (2017) shows a low variation in the participation rate. The mild variation is largely due to inactive trading activities in retirement income (e.g., Samuelson and Zeckhauser, 1988; Agnew, Balduzzi, and Sundén, 2003; Mitchell, Mottola, Utkus, and Yamaguch, 2006; Benartzi and Thaler, 2007). Vissing-Jørgensen (2002a), Brunnermeier and Nagel (2008), and Bonaparte, Korniotis, and Kumar (2018) present evidence on active turnover of market entry and exit in non-retirement accounts. For example, Brunnermeier and Nagel (2008) show that from the PSID, among the current stockholders (non-stockholders) about 19% (34%) on average exit (enter) the market in two years from 1999-2003. Including more recent periods, Bonaparte, Korniotis, and Kumar (2018) document 28.8% for stockholders' exit and 23.8% for non-stockholders' entry from 1999-2011.

[2] See Duffee (2005), Nagel and Singleton (2011), Roussanov (2014), and Xu (2021).

found implausibly high and negative values of the implied price of consumption risk, and (ii) most importantly, provides a new understanding of the dynamics of the equity premium, excess volatilities, and price dividend ratio.

To support our setup, we first document empirical evidence on the characteristics of investors entering and exiting the stock market as well as the timing of these decisions. We find that (i) heterogeneous risk aversion is a significant driver of investors' entry and exit decisions after controlling for age, changes in wealth, labor income, and other household controls, and (ii) risk-averse investors tend to enter (exit) the market in good (bad) times.[3]

Motivated by our empirical evidence, we propose a general equilibrium model where investors endogenously choose to enter or exit from the stock market. More precisely, we consider an overlapping generations economy populated by heterogeneous risk-averse investors with recursive preferences, and non-financial income. Our overlapping generations setup is to obtain a nondegenerate stationary equilibrium. Non-financial income shocks are positively correlated with dividend growth, giving rise to a short-selling demand for investors with a high risk aversion.[4] In the presence of short-selling constraints, unconstrained investors are stockholders who trade in both a riskless bond and a risky asset, whereas constrained investors are non-stockholders who trade only in a riskless bond. Due to time-varying investment opportunities, short-selling constraints bind intermittently for investors, and therefore, the optimal decision to enter and exit the market is state-dependent. As a result, in line with our empirical findings, our model predicts that investors with high risk aversions are more likely to stay in the stock market in good times than bad times.

This economy produces a novel conditional CCAPM. It highlights the importance of stockholders' consumption and risk aversions. Both have first-order effects on risk pre-

[3]This result is based on the assumption that household risk aversion is proportional to the probability of a household reporting no tolerance for investment risk available on the Survey of Consumer Finances (e.g., Haliassos and Bertaut, 1995; Bucciol and Miniaci, 2011).

[4]A higher short-selling demand for risk-averse investors is consistent with Veronesi (2019).

mium level and dynamics.[5] In our conditional CCAPM, the equilibrium equity premium is given by the product of two components: the stockholders' price of risk, which is the consumption-weighted harmonic mean of stockholders' risk aversions, and the stockholders' amount of risk, which is the covariance between the stockholders' consumption growth and stock returns.

The model-implied stockholders' amount of risk is strongly countercyclical. In bad times, more risk-averse investors leave the market. As a result, aggregate dividends are shared only by the relatively risk-tolerant investors and hence account for a larger fraction of stockholders' consumption. Therefore, a change in consumption of the stock market participants is highly sensitive to dividend shocks and equity returns. Simultaneously, the model-implied share of dividends in aggregate consumption is procyclical because aggregate dividends account for a smaller fraction of aggregate consumption in bad times. This procyclical dynamics of the dividend share in aggregate consumption leads to a weakly procyclical to countercyclical aggregate amount of risk, consistent with the well-known empirical evidence. We present empirical evidence consistent with our theoretical finding in the empirical section.

Endogenizing market participation uncovers interesting and intuitive implications for the price of risk. We show that the optimal behavior of investors' entry and exit of the stock market leads to a procyclical price of risk. In our model, the price of risk is driven by two counterbalancing effects: (i) time-varying entry/exit and (ii) a time-varying cross-sectional consumption re-distribution effect, which is discussed in Chan and Kogan (2002). On the one hand, we find that, in bad times, the exit of some risk-averse investors drives down the consumption-weighted mean of stockholders' risk aversion. On the other hand, at the same time, consumption of the relatively risk-tolerant market participants declines the most in response to a negative shock because they heavily invest in the stock. This decrease in the consumption of risk-tolerant investors drives up the consumption-weighted mean

[5] Note that non-participating investors also have an indirect effect on the equity premium through their influence on equilibrium parameters via the markets clearing conditions.

of stockholders' risk aversion, holding market participation unchanged. Taken together, we show that even a small change in the composition of investors in terms of their risk aversion due to market entry and exit can dominate the cross-sectional consumption redistribution effect, resulting in a procyclical price of risk with a reasonable average of 5.9.[6] Using the Consumer Expenditure data, we find empirical evidence on both entry/exit and consumption-redistribution effects on the dynamics of the price of risk, consistent with our theory.

In sum, this article shows that the countercyclical stockholders' amount of risk, due to relatively ineffective risk-sharing in bad times, essentially explains the countercyclical equity premium even with the procyclical price of risk. This insight is in stark contrast to the previous understanding that given the weakly procyclical aggregate amount of risk, consumption-based asset pricing models require a strongly countercyclical price of risk to explain the observed equity premium. From the lens of our theory, we also illustrate why models that impose full participation of investors can generate extreme levels of the price of risk.[7]

We examine the level and dynamics of stock volatility. In our model, stock volatility is tightly linked to the ratio of (i) the aggregate dividend share in stockholders' consumption, and (ii) the stockholders' consumption-weighted mean of risky asset share in their total wealth. Excess volatility is generated when the first term is greater than the second term. We find that our model-implied stock volatility is countercyclical as the dispersion between the two terms is higher in bad times. As discussed before, the aggregate dividend share

[6]To illustrate the effects of entry/exit and consumption redistribution, consider an example of three investors with the risk aversion of 3, 6, 9 and the consumption share of each investor with 50%, 30%, 20%, respectively, resulting in the price of risk of 5.1. In a bad state, the consumption share is re-distributed to 40%, 30%, and 30%, which leads to the price of risk of 5.7, higher than before. However, if the most risk-averse investor leaves the market, the consumption share is 57% and 43% for the first two investors, resulting in the price of risk of 4.3, lower than before.

[7]Our implied price of risk based on aggregate consumption also reproduces the negative values as in Duffee (2005), Nagel and Singleton (2011), and Roussanov (2014).

in stockholders' consumption is countercyclical as in bad states, the share of dividends in stockholders' consumption is higher than in good states due to ineffective risk-sharing. With regards to the second term, the stockholders' consumption-weighted mean of risky asset share in total wealth is generally acyclical. Therefore, state-dependent stockholders' risk-sharing helps generate countercyclical equity excess volatility. In contrast, we show that our nested economies of (i) a full participation economy and (ii) a representative-agent economy generate counterfactually procyclical excess volatility.

Finally, we examine our model-implied price-dividend ratio. In the literature, it is challenging to produce a procyclical variation in the price-dividend ratio with the elasticity of intertemporal substitution less than one (e.g., Ju and Miao, 2012; Chabakauri, 2015b) with few exceptions (e.g., Guvenen, 2009). We show that our model produces the empirically observed procyclical price-dividend ratio due to both a procyclical risky asset holding and the countercyclical excess volatility. We also find that our model generates long-horizon predictability of the equity premium with a quantitatively similar R^2 as in the data.

The rest of the paper unfolds as follows: Section 2 reviews the literature. Section 3 discusses the economic setup. Section 4 solves the equilibrium. Section 5 simulates the model and discusses our results. Section 6 presents empirical evidence supporting our model setup and theoretical results. Section 7 concludes. The online appendix presents additional analysis, details, and extensions. However, readers can understand the paper without having to rely on this online appendix.

2 Literature review

This article belongs to the consumption-based asset pricing literature. Leading representative-agent dynamic asset pricing theories (e.g., Campbell and Cochrane, 1999; Bansal and Yaron, 2004; Barro, 2009) and subsequent studies have been successful in explaining the salient features of financial markets. To the best of our knowledge, this article is the first to empirically identify and theoretically generate countercyclical stockholders' amount of consumption risk simultaneously with a procyclical aggregate amount of consumption risk. Our model illustrates that it is the countercyclical amount of risk rather than the countercyclical price of risk that drives the dynamics of risk premium, volatility, and price-dividend ratio.

Our work is directly related to the studies that have theoretically examined limited equity market participation to explain broad asset pricing features. One class of these studies exogenously specifies a group of investors excluded from the stock market.[8] For instance, Basak and Cuoco (1998) study asset prices and optimal consumption/investment policies in a restricted economy where one of two investors is the only stockholder and compare it with the unrestricted economy where both agents are stockholders. They show that limited market participation helps resolve the equity premium puzzle. Guvenen (2009) studies the implications of limited market participation for asset pricing dynamics in a setup where two investors differ in their elasticity of intertemporal substitution under a real business cycle framework. His model generates some empirically observed stylized facts of asset prices as well as wealth inequality between market participants and non-participants. This class of models, however, does not generate changes in the market composition through entry/exit which is essential for our article to explain the dynamics of asset prices.

The other class of studies endogenizes market participation decisions. Most papers in this class of studies have life-cycle features of market participation decisions and focus on explaining unconditional asset moments, participation rate, and investors' life-cycle behavior. However, they are silent on the implication for the dynamics of asset pricing.[9] Our paper builds on these papers, emphasizes the importance of the distinction of aggregate versus stockholders' consumption and delivers a novel channel for asset dynamics. A few papers endogenize market participation without the life-cycle feature.[10] For example, Allen

[8]E.g., Basak and Cuoco (1998), Polkovnichenk (2004), Guo (2004), Guvenen (2009), Chien, Cole, and Lustig (2011), Chien, Cole, and Lustig (2012), and Toda and Walsh (2019).

[9]E.g., Constantinides, Donaldson, and Mehra (2002), Gomes and Michaelides (2005), Alan (2006), Gomes and Michaelides (2008), Fagereng, Gottlieb, and Guiso (2017), and Vestman (2019) among others.

[10]E.g., Williamson (1994), Haliassos and Michaelides (2003), Cao, Wang, and Zhang (2005), Favilukis (2013) among others.

and Gale (1994) study the effect of limited market participation and liquidity preference on stock volatility. Calvet, Gonzalez-Eiras, and Sodini (2004) study the effect of financial innovation on asset prices, which leads to a change in market participation. Both papers consider the CARA preference. We consider preferences that give rise to a state-dependent portfolio choice through the wealth effect, which affects asset pricing dynamics. More recently, in a setup with idiosyncratic income shocks and homogeneous risk aversion, Bonaparte, Korniotis, and Kumar (2018) also present a model which generates investors' entry and exit. While they focus on matching the model-implied unconditional moments to the data, they do not examine either the dynamics of the equilibrium asset prices or consumption risk components as in the present paper.

3 Economy

3.1 Basic setup

Time and Uncertainty structure: We consider a continuous pure-exchange economy. The uncertainty in this economy is represented by a filtered probability space (Ω, $\mathcal{F}$, $\mathbb{P}$). Ω is the set of all possible states. $\mathcal{F} = \{\mathcal{F}_t\}_{t \in T}$ is the filtration that represents the investors' information available at time t where $T \in [0, \infty)$. The probability measure $\mathbb{P}$ is defined on $(\Omega, \mathcal{F}_\infty)$ where $\mathcal{F}_\infty = \bigcup_{t=0}^{\infty} \mathcal{F}_t$ represents the investors' common beliefs. The filtration $\mathcal{F}$ is generated by two-dimensional standard Brownian motion $W = [W_{d,t}, W_{y,t}]$. The two Brownian motion shocks are correlated (i.e., $dW_{d,t} dW_{y,t} = \rho dt$).

Investors and Preferences: We follow the specification of demographics in Blanchard (1985) and Gârleanu and Panageas (2015). The economy is populated by a continuum of investors who live until the stochastic time of death which is exponentially distributed with hazard rate, $\nu > 0$. Therefore, every period a fraction ν of the population faces death, and a new cohort of mass ν is born, resulting in the constant population size that is normalized to one.[11]

$$\int_{-\infty}^{t} \nu e^{-\nu(t-s)} ds = 1 \tag{1}$$

There are N types of investors all having the recursive utility developed in Epstein and Zin (1989) and Duffie and Epstein (1992). Investors which belong to a type i who are born at time s are maximizing

$$V_{i,s} = \mathbb{E}_s[\int_{s}^{\infty} f(C^i_{s,u}, V^i_{s,u}) du] \tag{2}$$

where $f(C, V)$ is the normalized aggregator for consumption C, which is one perishable consumption good that serves as the numéraire, and indirect utility V. For the Epstein-Zin utility, $f(C, V)$ is given by

$$f(C, V) = \frac{\tilde{\delta}}{1 - \psi^{-1}} \frac{C^{1-\psi^{-1}} - ((1-\gamma_i)V)^{\theta_i}}{((1-\gamma_i)V)^{\theta_i - 1}} \tag{3}$$

where $\tilde{\delta}$ is the effective subjective discount rate, $\tilde{\delta} = \delta + \nu$, which is the subjective discount rate δ plus the death rate ν. ψ is the Elasticity of Intertemporal Substitution (EIS) and γ_i is the coefficient of risk aversion of investors with a type i. For investors with a type $i = 1, ..., N$, risk aversion coefficients are $\gamma_1, ..., \gamma_N$, respectively, with $0 < \gamma_1 < ... < \gamma_N$.[12], $\theta_i = \frac{1-\psi^{-1}}{1-\gamma_i}$, and $\mathbb{E}_s$ denotes the expectation taken at time s.

Non-financial income: Y_t is aggregate non-financial income (labor income):

$$\frac{1}{N} \sum_{i=1}^{N} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} Y^i_{s,t} ds = Y_t \tag{4}$$

where Y_t evolves as $dY_t = \mu_y Y_t dt + \sigma_y Y_t dW_{y,t}$ where $\mu_y > 0$ is the expected labor income growth rate, $\sigma_y > 0$ is the labor income growth volatility. All investors are assumed to

[11]Our previous version was circulated without the OLG setup. The choice of the OLG setting is to guarantee a nondegenerate stationary equilibrium. However, this important stationary benefit comes with slightly complicated notations. Our main results in this article qualitatively hold without the OLG assumption for a short horizon simulation.

[12]Heterogeneous risk aversion with the same EIS is considered in the literature (e.g., Coen-Pirani, 2004, 2005; Buss, Uppal, and Vilkov, 2013; Chabakauri, 2015b). While it is plausible to assume heterogeneity in EIS (e.g., Vissing-Jørgensen, 2002a; Guvenen, 2009; Gârleanu and Panageas, 2015) and subjective time discount preferences (e.g., Cvitanić, Jouini, Malamud, and Napp, 2012; Bhamra and Uppal, 2014; Luo, 2018), our aim is to generate new implications on asset pricing with minimum assumptions. Hence, we only assume heterogeneity in risk aversion.

receive, for simplicity, the same level of stochastic exogenous non-financial income (labor income):[13] $Y^i_{s,t} = Y_t$

With this setup, as in the heterogeneous investors literature (e.g., Chabakauri, 2015a; Gârleanu and Panageas, 2015), state variables for an investor's maximization are financial wealth $X^i_{s,t}$, non-financial income Y_t, $N-1$ consumption shares $w_{j,t} = \frac{\int_{-\infty}^{t} \nu e^{-\nu(t-s)} C^j_{s,t}}{\sum_{i=1}^{N} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} C^i_{s,t} ds}$ which has the following dynamics: $dw_{j,t} = w_{j,t}[\mu_{w_{j,t}} dt + \sigma^d_{w_{j,t}} dW_{d,t} + \sigma^y_{w_{j,t}} dW_{y,t}]$[14] $\forall j = 1, ..., N-1$. Therefore, indirect utility is a function of those $N+1$ state variables: $V^i_{s,t} = V(X^i_{s,t}, Y_t, \mathbf{w_t})$ where $\mathbf{w_t} = [w_{1,t}, ..., w_{N-1,t}]$. In equilibrium, asset parameters are time-varying because of time-varying investment opportunities that arise from shocks to the economy and a change in the endogenous consumption distribution. However, a dynamic problem with full rational agents who take into account the stochastic parameters is difficult to solve and complicated due to a large number of state variables in our setting. Therefore, we employ the anticipated utility approach of Kreps (1998), following Cogley and Sargent (2008), Jagannathan and Liu (2019), and Johannes, Lochstoer, and Mou (2016). Under the anticipated utility approach, agents are assumed to treat parameters as constant at each point in time in their optimized dynamic decisions. Therefore, agents are rational in that they maximize the life-time utility given constraints and uncertainties that arise from stochastic financial wealth and non-financial income. But, they are not fully rational in the sense that they do not take into account the fact that parameters change over time. This approach allows us to focus on how changes in the composition of market participants led by fundamental shocks affect asset pricing equilibrium dynamics. We leave analytical solutions of the model with full rationality for future research.

Financial assets: An investor can allocate her wealth to two assets: a riskless asset $\frac{dB_t}{B_t} =$

[13] In the online appendix OA.1, we extend the model to a setup with an idiosyncratic labor income. Simulation of the extended model shows that our key results do not change with idiosyncratic labor income. The implication of uninsurable idiosyncratic risk in human capital for asset pricing is well studied in Constantinides and Duffie (1996) and Ai and Bhandari (2018).

[14] $\mu_{w_{j,t}}$, $\sigma^d_{w_{j,t}}$, and $\sigma^y_{w_{j,t}}$ are to be determined in equilibrium.

$r_{f,t}(D_t, Y_t, \mathbf{w_t})dt$ where the parameter $r_{f,t}$ denotes the risk-free rate and a risky asset which is a claim to an exogenous dividend D_t that follows: $dD_t = \mu_d D_t dt + \sigma_d D_t dW_{d,t}$ where $\mu_d > 0$ is the expected dividend growth rate, and $\sigma_d > 0$ is the dividend growth volatility. The equilibrium equity returns dynamics has the form:[15]

$$\frac{dS_t + D_t dt}{S_t} = \mu_t(D_t, Y_t, \mathbf{w_t})dt + \sigma_t^d(D_t, Y_t, \mathbf{w_t})dW_{d,t} + \sigma_t^y(D_t, Y_t, \mathbf{w_t})dW_{y,t} \quad (5)$$

where S_t is the stock price, μ_t is the expected stock returns, and σ_t^d and σ_t^y are the sensitivity of equity returns with respect to dividend and labor income shocks, respectively. As we will show later in Section 4, the risk-free rate, expected stock returns, and stock volatility are endogenously determined in equilibrium. They are a function of dividend D_t, non-financial income Y_t, consumption distribution $\mathbf{w_t}$: $r_{f,t}(D_t, Y_t, \mathbf{w_t})$, $\mu_t(D_t, Y_t, \mathbf{w_t})$, $\sigma_t(D_t, Y_t, \mathbf{w_t})$. To make the notation easier to follow, we omit the argument $(D_t, Y_t, \mathbf{w_t})$ for asset parameters hereafter.

3.2 Investor's optimization problem

In solving for an investor's optimization problem, we impose short-selling constraints. The importance of short-selling constraints and its association with investors' decision to participate in the stock market is well examined in Curcuru, Heaton, Lucas, and Moore (2004) and Athreya, Ionescu, and Neelakantan (2018).

In order to avoid the case where non-market participation is generated only due to short-selling constraints, we also impose a fixed market participation cost as in the literature.[16] In particular, we assume fixed cost as a fraction of non-financial income, following Gomes and Michaelides (2005, 2008). This can be interpreted as a opportunity cost of gathering

[15] This conjecture for the equilibrium stock price dynamics is confirmed in **Proposition 1**.

[16] See Allen and Gale (1994), Williamson (1994), Heaton and Lucas (1996), Vissing-Jørgensen (2002b), Haliassos and Michaelides (2003), Alan (2006), and Fagereng, Gottlieb, and Guiso (2017) for example.

information about the stock market.[17]

Therefore, a type i's investor optimization problem can be written as follows: $\{C^i_{s,t}, \pi^i_{s,t}\} = \arg\max_{(c,\pi\geq 0)} \mathbb{E}_t[\int_t^\infty f(C^i_{s,u}, V^i_{s,u})du]$. A trading strategy, with short-selling constraints, satisfies the following dynamic budget constraints:

$$dX^i_{s,t} = [\pi^i_{s,t}(\mu_t - r_{f,t}) + \nu X^i_{s,t} + r_{f,t}X^i_{s,t} + Y_t(1 - \mathbb{1}_{\{\pi^i_{s,t}>0\}}F) - C^i_{s,t}]dt \\ + \pi^i_{s,t}(\sigma^d_t dW_{d,t} + \sigma^y_t dW_{y,t}) \tag{6}$$

where $\pi^i_{s,t}$ is the dollar amount invested in the risky asset. As in Blanchard (1985) and Gârleanu and Panageas (2015), investors receive life insurance payments $\nu X^i_{s,t}$. At the time of death, insurance firms collect the financial wealth. This contract is optimal given the absence of bequest motives. $\mathbb{1}_{\{\pi^i_{s,t}>0\}}$ is the indicator that takes one for stockholders, otherwise zero.

A maximization problem with portfolio constraints can be solved via the Lagrangian method (e.g., Yiu, 2004; Chabakauri, 2013). Let $l^i_{s,t}$ denote the time t Lagrange multiplier for short-selling constraint $\pi^i_{s,t} \geq 0$. Then, the Hamilton-Jacobi-Bellman (HJB) equation with short-selling constraints for an investor is[18]

[17] In the literature, other mechanisms to generate limited market participation are considered. Life-cycle model: Constantinides, Donaldson, and Mehra (2002), Gomes and Michaelides (2005), Alan (2006), Gomes and Michaelides (2008), Fagereng, Gottlieb, and Guiso (2017); Model uncertainty: Cao, Wang, and Zhang (2005); Borrowing constraint: Allen and Gale (1994), Heaton and Lucas (1996), Constantinides, Donaldson, and Mehra (2002), Haliassos and Michaelides (2003), Alan (2006), Gomes and Michaelides (2008), Fagereng, Gottlieb, and Guiso (2017). For parsimony, and since our focus is on the change in the composition of market participants and effect on asset pricing dynamics, we limit ourselves to short-selling constraints and transaction costs.

[18] We suppress the notation for type i, and cohort s to save space.

$$
\begin{aligned}
0 = &\max_{(c,\pi)\in\mathcal{A}} f(C_t, V_t) + [\pi_t(\mu_t - r_{f,t}) + (r_{f,t} + \nu)X_t + Y_t(1 - \mathbb{1}_{\{\pi^i_{s,t}>0\}}F) - C_t]V_{x,t} + \frac{1}{2}\pi_t^2\sigma_t^2 V_{xx,t} \\
&+ \mu_y Y_t V_{y,t} + \frac{1}{2}\sigma_y^2 Y_t^2 V_{yy,t} + \rho_t\sigma_y Y_t \sigma_t \pi_t V_{xy,t} + \sum_{j=1}^{N-1}\mu_{w_{j,t}} w_{j,t} V_{w_j,t} + \frac{1}{2}\sum_{j=1}^{N-1}\sigma^2_{w_{j,t}} w^2_{j,t} V_{w_j w_j,t} \\
&+ \sum_{j\neq k}\rho_{w_j,w_k,t}\sigma_{w_{j,t}}\sigma_{w_{k,t}} w_{j,t} w_{k,t} V_{w_j w_k,t} + \sum_{j=1}^{N-1}\rho_{w_j,t}\sigma_{w_{j,t}} w_{j,t}\sigma_t\pi_t V_{w_j x,t} \\
&+ \sum_{j=1}^{N-1}\rho_{w_j,y,t}\sigma_{w_{j,t}} w_{j,t}\sigma_y Y_t V_{w_j y,t} + l_t\pi_t \qquad \forall i = 1, ..., N, \forall t \in [0,\infty)
\end{aligned} \tag{7}
$$

subject to $E_t[V_T] \to 0$, as $T \to \infty$ where ρ_t is the correlation between equity returns and labor income growth, $\sigma_{w_{j,t}}$ is the volatility of consumption share j dynamics, $\rho_{w_j,w_k,t}$ is the correlation between consumption share j and k dynamics, $\rho_{w_j,t}$ is the correlation between consumption share j dynamics and equity returns, and, $\rho_{w_j,y,t}$ is the correlation between consumption share j dynamics and labor income growth. In the online appendix OA.2, we formally derive the HJB equation with the Lagrange multiplier to confirm (7). The first-order necessary conditions for the optimization problem are given by

$$
C_t^* = (\tilde{\delta} V_{x,t}^{-1}((1-\gamma)V_t)^{-\theta+1})^{\psi} \tag{8}
$$

$$
\begin{aligned}
\pi_t^* = &- \frac{(\mu_t - r_{f,t})V_{x,t}}{\sigma_t^2 V_{xx,t}} - \frac{\rho_t\sigma_y Y_t \sigma_t V_{xy,t}}{\sigma_t^2 V_{xx,t}} - \frac{\sum_{j=1}^{N-1}\rho_{w_j,t}\sigma_{w_j,t} w_{j,t}\sigma_t V_{w_j x,t}}{\sigma_t^2 V_{xx,t}} \\
&- \frac{l_t^*}{\sigma_t^2 V_{xx,t}}
\end{aligned} \tag{9}
$$

The last term in (9) is the adjustment from the constraint and therefore we can rewrite (9) as follow.

$$
\pi_t^* = \pi_t^{w/o} - \frac{l_t^*}{\sigma_t^2 V_{xx,t}} \tag{10}
$$

where $\pi_{i,t}^{w/o}$ refers to the first three terms in (9), which is the risky asset holding without any constraints. Furthermore, the Kuhn-Tucker optimality conditions are

$$
l_t^*\pi_t^* = 0 \tag{11}
$$

$$l_t^* \geq 0, \pi_t^* \geq 0 \tag{12}$$

Equation (11) is the complementary slackness condition and is used to solve for l_t^* whenever $l_t^* \neq 0$. Therefore, from equation (9), (11), and (12),

$$l_t^* = \begin{cases} 0 & \text{if } \pi_t^{w/o} > 0 \\ -(\mu_t - r_{f,t})V_{x,t} - \rho_t \sigma_y Y_t \sigma_t V_{xy,t} - \sum_{j=1}^{N-1} \rho_{w_j,t} \sigma_{w_j,t} w_{j,t} \sigma_t V_{w_j x,t} & \text{Otherwise} \end{cases} \tag{13}$$

We plug C_t^*, π_t^*, and l_t^* back into (7) to solve the HJB equation. When the dividend growth is perfectly correlated with labor income growth, there is a closed-form solution for this maximization problem.[19] For the non-perfect correlation case, there is no closed-form solution in general for the optimal consumption and portfolio. However, following Koo (1998) and Wang, Wang, and Yang (2016) who impose $\frac{X}{Y} \to \infty$ to solve for their optimization problem, we solve equation (14) and (15) in closed-form.[20]

The following proposition shows the optimal consumption and portfolio as functions of equilibrium parameters.

Proposition 1. *An investor's optimal consumption, stock holdings, and wealth dynamics are given by* $\forall i = 1, ..., N$

$$C_{s,t}^i = \begin{cases} ((r_{f,t} + \nu + \frac{\lambda_t^2}{2\gamma_i})(1-\psi) + \tilde{\delta}\psi) \cdot (X_{s,t}^i + H_{h,t}) & \text{if } \pi_{s,t}^{i,w/o} > 0 \\ ((r_{f,t} + \nu)(1-\psi) + \tilde{\delta}\psi) \cdot (X_{s,t}^i + H_{n,t}) & \text{Otherwise} \end{cases} \tag{14}$$

$$\pi_{s,t}^i = \begin{cases} \pi_{s,t}^{i,w/o} = \frac{\lambda_t}{\gamma_i \sigma_t}(X_{s,t}^i + H_{h,t}) - \frac{\rho_t \sigma_y}{\sigma_t} H_{h,t} & \text{if } \pi_{s,t}^{i,w/o} > 0 \\ 0 & \text{Otherwise} \end{cases} \tag{15}$$

[19] We solve for the closed-form and show that our general solution reduces to this special case when imposing $\rho = 1$. See the online appendix OA.3.

[20] Wang, Wang, and Yang (2016) solve the consumption choice in their setting both analytically with the assumption of $\frac{X}{Y} \to \infty$ and numerically when this assumption is not applied and show a non-significant difference especially when $\frac{X}{Y}$ is high (See Figure 1 in their paper).

$$
dX^i_{s,t} = \begin{cases} (\pi^i_{s,t}(\mu_t - r_{f,t}) + (r_{f,t} + \nu)X^i_{s,t} + Y_t(1-F) - C^i_{s,t})dt & \\ +\pi^i_{s,t}(\sigma^d_t dW_{d,t} + \sigma^y_t dW_{y,t}) & \text{if } \pi^{i,w/o}_{s,t} > 0 \\ ((r_{f,t} + \nu)X^i_{s,t} + Y_t - C^i_{s,t})dt & \text{Otherwise} \end{cases} \quad (16)
$$

where λ_t is the Sharpe ratio, $H_{h,t} \equiv \frac{Y_t(1-F)}{r_{f,t}+\nu+\rho_t\sigma_y\lambda_t-\mu_y}$, and $H_{n,t} \equiv \frac{Y_t}{r_{f,t}+\nu-\mu_y}$, *Proof : See online appendix A.1.1*

First, regarding the optimal consumption, equation (14) shows that the marginal propensity to consume out of labor income is not unity (i.e., $\partial C^i_{s,t}(X^i_{s,t}, Y_t)/\partial Y_t \neq 1$), different from heterogeneous CARA utility case or a representative investor setup.[21] Therefore, labor income shocks affect the optimal wealth dynamics in (16) and in turn the stock price in equilibrium. Furthermore, the consumption to total wealth ratio ($\frac{C^i_{s,t}}{X^i_{s,t}+H_{h,t}}$) of stockholders is different from that of non-stockholders as non-stockholders do not face uncertainty from the risky asset holding.

Second, with regard to portfolio holdings, the unconstrained investors' optimal stock holding $\pi^{i,w/o}_{s,t}$ has an intertemporal hedging demand which stems from non-financial income risk. The intertemporal hedging demand, together with a positive correlation between stock returns and non-financial income $\rho_t > 0$ disincentivizes investors from a positive holding. This effect of non-financial income risk on portfolio holding is supported empirically by Guiso, Jappelli, and Terlizzese (1996), Angerer and Lam (2009), and Betermier, Jansson, Parlour, and Walden (2012) who show that investors' stock holdings are negatively associated with the non-financial income risk. For some investors, at each point in time, a negative intertemporal hedging demand could dominate a positive speculative demand represented by the first term in (15). Short-sale constraints bind for those investors intermittently, and they are restricted from holding the stock. Without non-financial income, there is no hedging demand and thus every investor will have a positive holding, which

[21]The key difference between the current economy and the CARA economy is that there is no asset pricing dynamics in the economy populated by heterogeneous investors with CARA preferences.

leads to a full participation economy in this setting. Given the equation (15), the condition for a positive holding $\pi_{s,t}^{i,w/o} > 0$ is

$$\frac{X_{s,t}^i}{Y_t(1-F)}\lambda_t(r_{f,t} + \nu + \rho_t\sigma_y\lambda_t - \mu_y) + \lambda_t - \gamma_i\rho_t\sigma_y > 0 \quad (17)$$

It shows that given $r_{f,t}+\nu+\rho_t\sigma_y\lambda_t-\mu_y > 0$, the higher the financial wealth to labor income $X_{s,t}^i/Y_t$, the more likely the investor has a positive holding. Therefore, the financial wealth to labor income $X_{s,t}^i/Y_t$ plays a crucial role in investors' dynamic decisions on stock market participation. This condition also shows that the higher risk aversion γ_i, the less likely the investor has a positive holding. A higher expected non-financial income growth μ_y leads to a higher value of human capital, which in turn disincentivizes investors from having a positive holding due to a greater hedging concern. In Section 6, we provide empirical evidence that market entry and exit decisions are associated with the stock market wealth to aggregate non-financial income ratio along with risk aversion, consistent with the implications of equation (17). The level of correlation between equity returns and labor income growth also plays a role in market participation decision, consistent with Curcuru, Heaton, Lucas, and Moore (2004). For more details on comparative static analysis for the effect of the correlation on market participation, please see the online appendix OA.4.

Finally, (16) shows that the unconstrained investors' wealth evolves as a function of stock holding and shocks to the stock. Therefore, in good times, the wealth of relatively risk-tolerant investors, who heavily invest in the stock, increases more than that of high risk-averse investors, thereby increasing wealth inequality. This implication is consistent with the model in Gomez (2019).

4 Equilibrium

In this section, we derive and examine the equilibrium. Subsection 4.1 describes the equilibrium. Subsection 4.2 derives the equilibrium. Subsection 4.3 presents a CCAPM allowing for market entry and exit of investors.

4.1 Description of the equilibrium

Definition 1. *An equilibrium is a set of parameters* $\{r_{f,t}(D_t, Y_t, \mathbf{w_t}), \lambda_t(D_t, Y_t, \mathbf{w_t}), \sigma_t(D_t, Y_t, \mathbf{w_t})\}$, *consumption and investment policies* $\{C^i_{s,t}, \pi^i_{s,t}\}_{i\in 1,\dots,N}$ *which maximize the sum of life time expected utility (2) subject to the dynamic budget constraint (6) for each investor and satisfy the market-clearing conditions:*

1. Stock market clears: $$\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}\pi^i_{s,t}ds = S_t \tag{18}$$

2. Bond market clears: $$\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}(X^i_{s,t}-\pi^i_{s,t})ds = 0 \tag{19}$$

3. Consumption market clears:

$$\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}C^i_{s,t}ds = D_t + Y_t(1-F\cdot P_t) + \nu\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}X^i_{s,t}ds \tag{20}$$

Proof : See appendix A.1.2. where $P_t = \frac{1}{N}\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}ds$ is the market participation rate. The stock is in unit supply and the bond is in zero supply. The stock market clearing condition (18) with the bond market clearing condition (19) implies the consumption market clearing condition (20). In our setup, aggregate consumption is not sum of dividend and aggregate non-financial income, as is the case in general. This is because a fixed transaction cost (F) paid by a fraction of the population (P_t). Moreover, every investor receives life insurance payment, which is another source of consumption.

4.2 Derivation of the equilibrium

We derive the equilibrium in the following steps. First, from equation (18), the equation for the Sharpe ratio is obtained. Second, by matching the deterministic terms of the dynamics of both the left and right-hand side of (20), the equation for the risk-free rate is obtained. Third, by matching the diffusion terms of the dynamics of (20), two equations for the stock volatility are obtained. Fourth, from equation (20) and the optimal consumption in (14) together with the fact that $S_t = \frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}X^i_{s,t}ds$, the closed-form solution for the stock price is computed. **Proposition 2** summarizes the set of equations for the equilibrium parameters and stock price.

Proposition 2. *In the equilibrium defined by* **Definition 1**, *the set of equations for the Sharpe ratio λ_t, the risk-free rate $r_{f,t}$, the stock volatility σ_t and the stock price are:*

$$\lambda_t = \frac{\sigma_t \sum_{i=1}^{N} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} X^i_{s,t} ds + \sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} \rho_t \sigma_y H_{h,t} ds}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} (\frac{X^i_{s,t}+H_{h,t}}{\gamma_i}) ds} \tag{21}$$

$$r_{f,t} = \delta + \frac{E_t[\sum_{i=1}^{N} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} dC^i_{s,t} ds]/dt}{\sum_{i=1}^{N} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} C^i_{s,t} ds} \frac{1}{\psi} - \frac{\lambda_t^2}{2} (\frac{1+\psi}{\psi}) \sum_{i\in h_{g,t}} \frac{1}{\gamma_i} \frac{\int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C^i_{t,s} ds}{\sum_{i=1}^{N} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} C^i_{t,s} ds} \tag{22}$$

$$\sigma_t = \sqrt{(\sigma_t^d)^2 + (\sigma_t^y)^2 + 2\rho \sigma_t^d \sigma_t^y} \tag{23}$$

$$\sigma_t^d = \frac{\sigma_d D_t}{\frac{1}{N} \sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} k_{h,i,t} \pi^i_{s,t} ds - \nu S_t} \tag{24}$$

$$\sigma_t^y \tag{25}$$

$$= \frac{\sigma_y [N Y_t (1 - F \cdot P_t) - \sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)} k_{n,i,t} H_{n,t} ds - \sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} k_{h,i,t} H_{h,t} ds]}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} k_{h,i,t} \pi^i_{s,t} ds - N\nu S_t}$$

$$S_t = \frac{D_t + Y_t(1 - F\cdot P_t) - \frac{\lambda_t^2}{2\gamma_i}(1-\psi)\frac{1}{N} \sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} (X^i_{s,t} + H_{h,t}) ds}{r_{f,t}(1-\psi) + \delta\psi} \tag{26}$$

$$- \frac{r_{f,t}(1-\psi) + \delta\psi + \nu}{r_{f,t}(1-\psi) + \delta\psi} \frac{1}{N} \sum_{i=1}^{N} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} H^i_{s,t} ds$$

Proof : See appendix A.1.3. where N denotes the number of investors' types, i denotes the index for type, and s denotes the index for cohort. $h_{g,t}$ and $h_{i,t}$ denote the set of types and cohorts of investors, respectively, whose optimal portfolio is positive i.e., $\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} ds = \sum_{i=1}^{N} \int_{-\infty}^{t} \mathbb{1}_{\{\pi^i_{s,t}>0\}} ds$. $k_{h,i,t}$ and $k_{n,i,t}$ are consumption-wealth ratio for stockholders and non-stockholders, respectively.

In the online appendix OA.5, to understand the marginal effect of investors' heterogeneity and investors' entry and exit, we compare the endogenous parameters (λ_t, $r_{f,t}$, $\sigma_{s,t}$) with nested cases: (i) a representative investor economy without labor income, and (ii) a heterogeneous economy without labor income, that is a full participation economy. In doing so, we also confirm that our equilibrium parameters in closed forms reduce to the well-known expressions in nested economies studied in the literature.[22]

4.3 A Novel Conditional Consumption Capital Asset Pricing Model

Proposition 3. *In an economy where market participation is a decision/choice variable, the equilibrium equity premium is given by*

$$E_t[dR_t^e] = \underbrace{\frac{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} (\frac{C_{s,t}^i}{\gamma_i}) ds}}_{\textit{Price of risk}} \times \underbrace{Cov_t(dR_t^e, \frac{d\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds})}_{\textit{Amount of risk}} \tag{27}$$

Proof : See online appendix A.1.4

Proposition 3 shows that among all investors, it is the consumption and risk aversions of stockholders which **directly** determine the equity premium. However, it is important to note that the consumption and risk aversions of non-stockholders affect the equity premium **indirectly** through the market clearing condition. Moreover, in our economy, since investors endogenously enter and exit the market, the composition of stockholders at each time is different and hence it affects the dynamics of both components of the equity premium, which are the amount and price of risk.

A finding that stems from (27) is that we can illustrate why empiricists who use aggregate consumption to measure the amount of risk in testing representative-investor economies, find implausibly extreme and negative values of the implied price of risk (e.g., Nagel and Singleton, 2011; Roussanov, 2014). See the online appendix A.2 for a derivation of this

[22]We also solve for the equilibrium with CRRA preferences without labor income using the Martingale approach and verify that our general solution with labor income converges to this special case. See the online appendix OA.6.

result. We describe in detail this finding in Section 5.1.3.

5 Simulation

To simulate our model, the continuous model is discretized and simulated in monthly time increments for 10 years.[23] Table 1 reports the annualized parameter values used in the simulation. Panel A shows the first and second moments of the real per capita dividend growth, non-financial income growth, and their correlation. Our estimates are consistent with the literature.[24] Our choice of investors' preferences is reported in Panel B. We set the birth/mortality rate to $\nu = 2\%$ as in Gârleanu and Panageas (2015). For the subjective time preference rate δ, we choose 0.2%, as in Bansal and Yaron (2004). The EIS ψ is set to 0.5, following the general consensus of the EIS level (e.g., Vissing-Jørgensen, 2002a; Trabandt and Uhlig, 2011; Jin, 2012; Rudebusch and Swanson, 2012; Epstein, Farhi, and Strzalecki, 2014). Investors' risk aversion is uniformly distributed from 1 to 50. For comparison, in Chan and Kogan (2002), the risk aversion distribution ranges from 1 to 100. The assumed range in our model implies a stockholders' consumption weighted harmonic mean of risk aversion of 5.9, which is a generally accepted level in the literature. Panel C reports the initial value of aggregate dividend D_t as a function of normalized per capita non-financial income Y_t.

Throughout the analysis of our model, we use the stock market wealth to aggregate non-financial income ratio $\frac{\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}X_{s,t}^{i}ds}{Y_t} = \frac{S_t}{Y_t}$ as a state variable for the following reasons: First, financial wealth $X_{s,t}^{i}$ and labor income Y_t are state variables in the optimization problem for the portfolio and consumption choice as it is the case also in Koo (1998) and Wang (1996). Second, as in equation (17), it is the financial wealth to aggregate labor income ratio which affects the entry and exit decisions. Third, a high level of $\frac{S_t}{Y_t}$ coincides

[23]This time horizon is commonly used in the literature (e.g, Gârleanu and Panageas, 2015; Heyerdahl-Larsen and Illeditsch, 2019). In the online appendix OA.7, however, we also show that our key results are robust to 70 years horizon.

[24]See the footnotes in Table 1.

Table 1: Model Parameters

Table 1 presents the annualized model parameters used to simulate the model. The moments of dividend and non-financial income are chosen based on the annual U.S. real per capita data from the National Income and Product Account (NIPA). A detailed description of the data is in the online appendix A.3.

Parameter	Symbol	Value
Panel A: Dividend and Non-financial income parameters		
Dividend growth mean (%)	μ_d	3
Dividend growth volatility (%)	σ_d	9
Non-financial income growth mean (%)	μ_y	2
Non-financial income growth volatility (%)	σ_y	4
Correlation between dividend and non-financial shock (%)	ρ[1]	29
Panel B: Investor-related parameters		
Birth/Mortality rate (%)	ν[2]	2
Subjective time preference (%)	δ[3]	0.2
Elasticity of Intertemporal Substitution	ψ[4]	0.5
Lowest risk aversion coefficient	γ_1	1
Highest risk aversion coefficient	γ_N	50
Number of investors	N	30
Fixed transactions costs (%)	F	1
Panel C: Initial value		
Initial aggregate dividend stream	D_0	0.08
Initial per capita non-financial income	Y_0[5]	1

[1] Largely consistent with Dittmar, Palomino, and Yang (2016), which report 40% based on the Bureau of Economic Analysis and CRSP data.
[2] Consistent with other papers with the OLG feature (e.g., Gârleanu and Panageas, 2015; Heyerdahl-Larsen and Illeditsch, 2019).
[3] We follow Bansal and Yaron (2004). They use the value of 0.998 for the time discount factor, which translates into 0.2% for the subjective time preference rate.
[4] Consistent with Vissing-Jørgensen (2002a), Trabandt and Uhlig (2011), Jin (2012), and Rudebusch and Swanson (2012).
[5] Initial value of per capita non-financial income (Y_0) is normalized to 1.

with a high level of aggregate consumption and therefore it well captures the state of the aggregate economy.[25] Lastly, this ratio is used in Gomes and Michaelides (2008) and also closely related to the consumption to wealth ratio in Lettau and Ludvigson (2001), the stock market wealth to consumption ratio in Duffee (2005), and the labor income to consumption in Santos and Veronesi (2006). More importantly, we validate the key moments of our state variable in the model with the data counterpart: the average level and standard deviation of $\frac{S_t}{Y_t}$ in the simulation are 0.88 and 0.70 respectively, versus 0.98 and 0.58 in the

[25]The regression of log aggregate consumption on $log\frac{S_t}{Y_t}$ produces the coefficient of 0.148 with R^2 of 0.18.

data.

In what follows, we first examine the conditional equilibrium in Subsection 5.1 and the unconditional equilibrium in 5.2. In the online appendix OA.8, for the interested readers, we conduct a comparative static analysis for the equilibrium parameters with a particular focus on the role of the effect of market entry and exit.

5.1 Equilibrium dynamics

5.1.1 Conditional portfolio and market entry/exit

We first examine the entry and exit decisions of investors over the economic state. To this end, we generate in total 1,000 paths each with 10 years, resulting in 120,000 total monthly observations. The left panel of Figure 1 displays the sample path of the market participation rate $p_t \equiv \frac{1}{N}\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}} \nu e^{-\nu(t-s)}ds = \frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t} \nu e^{-\nu(t-s)}\mathbf{1}\{\pi^i_{s,t} > 0\}ds$ and the economic state $\frac{S_t}{Y_t}$. The shaded area denotes a recession defined as the lowest 10th percentile of the state. We find that the participation level is positively associated with the state of the economy. This suggests that a better state of the economy induces an entry of investors. In contrast, investors exit the market in the lower state of the economy. This

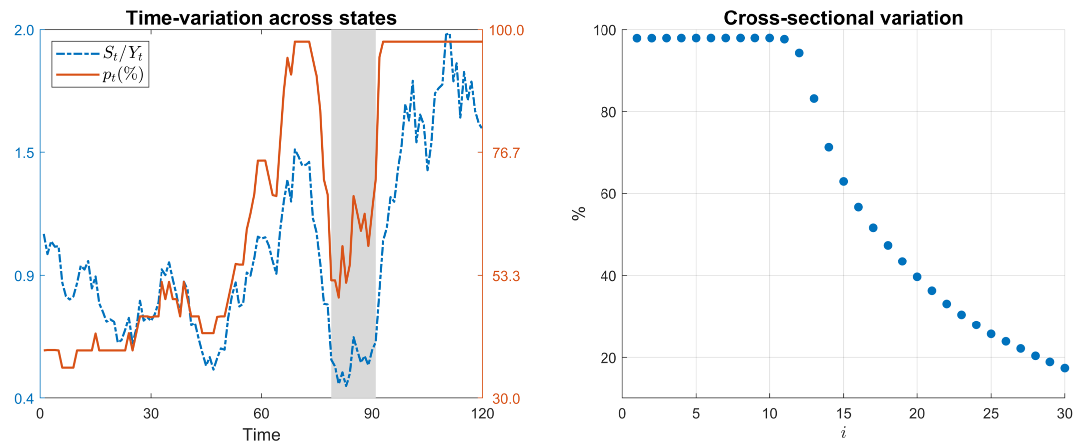

Figure 1: **Market Entry and Exit**

The left figure illustrates the relationship between the state S_t/Y_t (dash line, left y-axis) and the market participation level p_t (straight line, right y-axis) with shaded area denoting a recession defined as the lowest 10th percentile of the state. The right panel is the average participation rate of each investor type from the 120,000 simulated monthly observations.

procyclical (countercyclical) entry (exit) of investors is in line with the empirical findings we will present in Section 6. Note that jumps in entry/exit are observed due to the finite number of types of risk aversions. While our model implies the entry and exit of investors over the economic state, entry and exit behaviors are only pronounced for a certain group of investors. The right panel of Figure 1 presents the cross-sectional variation in the entry/exit decision. The result shows that investors from 1st to 11th are almost always stockholders. Investors with high risk aversions rarely enter the stock market. Therefore, entry and exit decisions are made only by the middle groups of risk aversion distribution. This is consistent with the empirical findings of Brunnermeier and Nagel (2008) and Bonaparte, Korniotis, and Kumar (2018) that only a group of investors enter or exit the market.

We emphasize that although the overall participation exhibits a mild time variation due to groups of investors who are either in or out of the market almost always, as observed in the data, a small change in market participants can have a substantial effect on the asset pricing dynamics. This is because investors who enter/exit the market have different risk aversion and optimal consumption. Therefore, the change in stockholders' composition affects investors' average risk aversion and the degree of risk-sharing which determines asset prices.

5.1.2 Amount of risk

We now study how the amount of risk varies over time. There is a consensus in the literature on the weakly countercyclical to procyclical variation in the aggregate amount of risk.[26] In good states, financial income (dividends) increases more than non-financial income, and therefore financial income accounts for a larger proportion of aggregate consumption. As a result, a change in aggregate consumption becomes more sensitive to stock returns, resulting in a high covariance between equity returns and consumption growth. This empirical finding poses a serious challenge to the ability of CCAPM in explaining asset

[26] E.g., Duffee (2005), Roussanov (2014), and Xu (2021) among others.

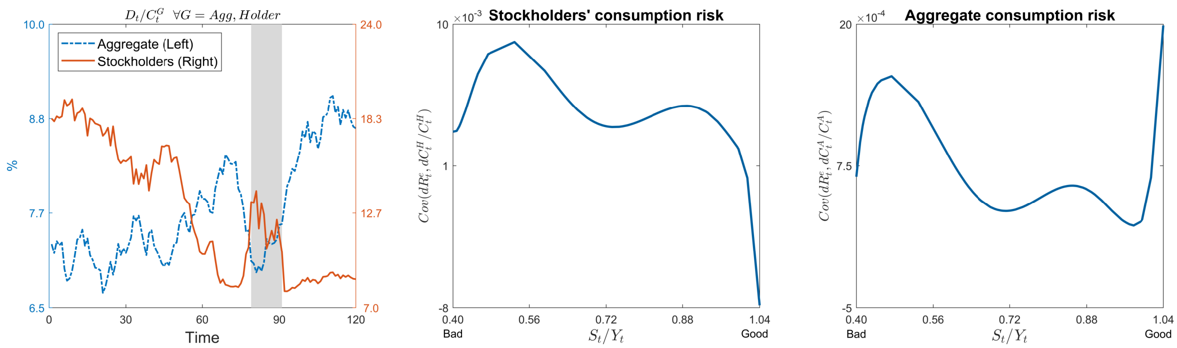

Figure 2: **Dividend Share and Amount of Risk**

This figure illustrates one sample path of dividend share in either aggregate consumption (dash-line, left y-axis) or stockholders' consumption (straight, right y-axis) with shaded area denoting a recession defined as the lowest 10th percentile of the state (left panel), the conditional covariance estimates of equity returns with stockholders' (middle panel) and aggregate (right panel) consumption growth conditioning on the stock market wealth to aggregate labor income ratio based on the simulated data. The conditional covariances are estimated by the Epanechnikov nonparametric kernel estimation.

dynamics. In order to understand how this article contributes to unlocking this challenge, we first decompose the consumption into dividends and other sources of consumption for both aggregate households and stockholders separately as follows.

$$Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e) = \frac{D_t}{C_t^G} Cov_t(\frac{dD_t}{D_t}, dR_t^e) + \frac{C_t^{G,D-}}{C_t^G} Cov_t(\frac{dC_t^{G,D-}}{C_t^{G,D-}}, dR_t^e) \;\; \forall G = A, H \tag{28}$$

where C_t^A denotes aggregate consumption i.e., $\frac{1}{N}\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds$, C_t^H denotes stockholders' consumption i.e., $\frac{1}{N}\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds$ is stock*h*olders' total consumption, and C_t^{G,D^-} $(= C_t^G - D_t)$ is the non-dividend part of consumption.

The left panel of Figure 2 illustrates one sample path of the dividend share in aggregate consumption and stockholders' consumption.[27] Consistent with previous empirical findings, we find that the share of dividends in aggregate consumption is procyclical in our economy. In bad times, the dividend stream accounts for around 7% of the total consumption stream that compares to 9% in good times.

More importantly, when it comes to stockholders' consumption, the share of dividends in stockholders' consumption exhibits a countercyclical variation with around 14% in bad

[27] Throughout this section, we keep the same path of exogenous shocks to dividend and non-financial income for comparison of figures.

times and 8.5% in good times.[28] Although dividend stream decreases due to a negative shock, stockholders' consumption also decreases due to an exit of investors, resulting in an ineffective risk-sharing. Therefore, the dividend accounts for a larger proportion of the stockholders' consumption. This result implies that it is not a fundamental shock itself, but the change in market participants due to the fundamental shock which essentially affects the degree of stockholders' risk-sharing.

The opposite dynamics of the share of dividends for aggregate consumption and stockholders' consumption lead to the strongly countercyclical amount of stockholders' consumption risk and the weakly countercyclical to procyclical amount of aggregate consumption risk as portrayed in the middle and right panel of Figure 2. These different dynamics of the amount of risk for aggregate versus stockholders generated by our model is a new finding in this article, and we provide the empirical support in the empirical section (Figure 6).

To summarize, our model shows that the distinction between stockholders' consumption and aggregate consumption reconciles consumption-based asset pricing models with the return dynamics in the data. On the one hand, the procyclical share of dividend in aggregate consumption contributes to the weakly countercyclical to procyclical variation in the aggregate amount of risk. On the other hand, the procyclical entry of investors leads to procyclical risk-sharing among stockholders and in turn countercyclical stockholders' amount of risk, which is necessary to explain the asset pricing dynamics.[29]

5.1.3 Price of risk

Habit models generate a countercyclical equity premium by relying on a countercyclical risk aversion of a representative investor. Chan and Kogan (2002) rationalize the counter-

[28]Large increases or decreases in the dividend share in stockholders' consumption in Figure 2 are driven by an entry or exit of our finite number of investors at each time.

[29]A recent study by Xu (2021) examines the aggregate amount of risk based on the return decomposition into the cash flow part and the discount rate. In the online appendix OA.9, we do the same analysis for both aggregate and stockholders' consumption.

cyclical price of risk in a heterogeneous time-invariant risk-averse investors setting. Their explanation hinges on the changes in consumption re-distribution across investors. However, it is unclear whether the price of risk still varies countercyclically if these investors are allowed to optimally enter or exit the market and hence the composition of the market participants changes. This is because if risk-averse investors leave the market, overall stockholders' risk aversion becomes lower. In our setup, the price of risk is the stockholders' consumption-weighted harmonic mean of risk aversion driven by two counterbalancing effects: (i) time-varying entry/exit and (ii) a time-varying cross-sectional consumption re-distribution effect.

$$\Gamma_t \equiv \frac{\sum_{i \in h_{g,t}} \int_{s \in h_{i,t}} \nu e^{-\nu(t-s)} C^i_{s,t} ds}{\sum_{i \in h_{g,t}} \int_{s \in h_{i,t}} \nu e^{-\nu(t-s)} (\frac{C^i_{s,t}}{\gamma_i}) ds} \tag{29}$$

Figure 3 plots the price of risk with the state and the market participation level. The two shaded areas with hatching are examples of time periods when there is no change in market entry/exit. The last shaded area denotes a recession defined as the lowest 10th percentile of the state. The left panel shows that our model-implied price of risk is procyclical. The right

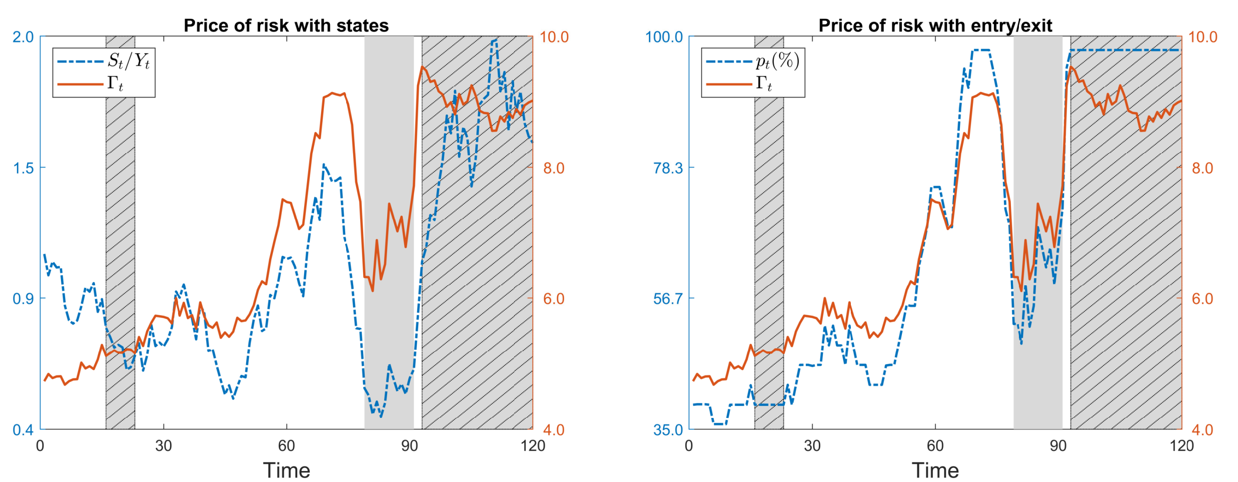

Figure 3: **Price of risk**

The left panel of figure depicts the relationship between the state S_t/Y_t (dash line, left y-axis) and the price of risk Γ_t (straight line, right y-axis). The right panel of figure is the relationship between the market participation level p_t (dash line, left y-axis) and the price of risk Γ_t (straight line, right y-axis). The two shaded areas with hatching are examples of time periods when there is no change in market entry/exit. The shaded area without hatching denotes a recession defined as the lowest 10th percentile of the state.

panel shows that it is a market entry/exit induced by a change in state that explains the procyclical variation in the price of risk. A better state induces an entry of investors who are more risk-averse than the existing stockholders, resulting in the change in the composition of stockholders and therefore increase in the average risk aversion of stockholders. Note that contrasting the left and right panel reveals that when there is no market entry/exit, as shown in the first two shaded areas with hatching, the price of risk varies countercyclially. This is due to the consumption re-distribution effect, consistent with Chan and Kogan (2002). To more formally confirm this interpretation, we run the regression of the time-series price of risk on the participation level and the stockholders' consumption share which represents the consumption share of risk-tolerant agents. Untabulated the result shows that the coefficients of the market participation and the consumption share are 29.29 and -21.77, respectively, with the R-squared of 0.982, confirming the opposite effect of consumption re-distribution and the endogenous market entry/exit on the level of price of risk.[30]

Interestingly, a procyclical variation in the price of risk does not make it difficult to produce a countercyclical equity premium. As we show in Section 5.1.5, we generate a countercyclical equity premium as observed in the data. This is because, as discussed in the previous section, the amount of stockholders' consumption risk is strongly countercyclical due to the ineffective risk-sharing during bad states.

Finally, we perform an additional simulation exercise to shed some light on the recent findings of extreme values and strong countercyclical variation in the implied price of risk in Duffee (2005), Nagel and Singleton (2011), and Roussanov (2014) among others.[31] To explain their findings through the lens of our model, we estimate the amount of risk using aggregate consumption based on the simulated data. For this exercise, we adopt

[30]The online appendix Figure OA.8 also illustrates the opposite effect of consumption re-distribution and the endogenous market entry/exit from 1,000 sample paths for the economy.

[31]e.g., The implied risk aversion ranges from -250 to 600 in Roussanov (2014) and from -88 to -4 in Duffee (2005).

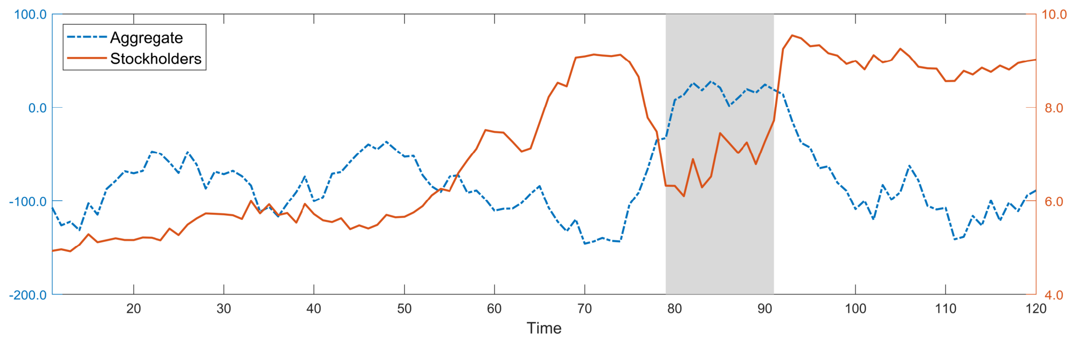

Figure 4: **Implied Price of risk from Aggregate consumption**
This figure plots one sample path of the implied price of risk using aggregate consumption (straight line, left y-axis). For comparison, we also plot the model-implied price of risk, the harmonic mean of stockholders' risk aversion (dashed line, right y-axis). The shaded area denotes a recession defined as the lowest 10th percentile of the state.

the GARCH-in-mean method, for example, as in Duffee (2005).[32] Figure 4 depicts the implied price of risk using aggregate consumption together with the model-implied price of risk, which is stockholders' harmonic mean of risk aversion for comparison. It shows that the implied price of risk using aggregate consumption has negative values over the large sample distribution and varies in a strongly countercyclical way, ranging from -151 to 28, similar to previous studies, but in contrast to the model-implied price of risk. This finding suggests that since aggregate amount of risk is not directly linked to the equilibrium equity premium, relying on the aggregate consumption would deliver a counterfactual result for the price of risk: implausible levels and a negative risk-return trade-off.

5.1.4 Stock volatility dynamics

In this section, we explore the conditional stock volatility. Both the level and dynamics of our stock volatility are largely driven by the volatility parameter associated with dividend shock σ_t^d. Given equation (24), we show that

$$\frac{\sigma_t^d}{\sigma_d} = \frac{D_t}{C_t^H} / (\frac{1}{N} \sum_{i \in h_{g,t}} \int_{s \in h_{i,t}} \nu e^{-\nu(t-s)} \frac{C_{s,t}^i}{C_t^H} \frac{\pi_{s,t}^i}{X_{s,t}^i + H_{h,t}} ds - \frac{\nu S_t}{C_t^H}) \tag{30}$$

[32]Other methodologies such as the GMM in Duffee (2005) generate virtually identical result.

where $C_t^H = \frac{1}{N}\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}C_{s,t}^i ds$. Thus, the excess volatility (i.e., $\sigma_t^d > \sigma_d$) is generated when the dividend share in stockholders' consumption (numerator) is greater than the risky asset share in total wealth with the adjustment term due to insurance payment (denominator). First, regarding the numerator, as we discussed in Section 5.1.2, the dividend share in aggregate consumption $\frac{D_t}{C_t^A}$ is procyclical. Therefore, a full participation makes it difficult to explain the countercyclical stock volatility. In contrast, the dividend share in the stockholders' consumption is countercyclical. Second, when it comes to the denominator, that is the risky asset share in total wealth with the adjustment term due to insurance payment, it is generally acyclical for the following reasons. (i) Investors optimally reduce the risky asset holding in bad times. (ii) The consumption of risk-tolerant investors drops the most, leading the average to be more tilted towards the risky asset share of risk-averse investors. (iii) The adjustment term varies countercyclically, counterbalancing the effects of the first. As illustrated in Figure 5, a countercyclical dividend share in the stockholders' consumption (numerator, left panel) drives a strongly countercyclical stock volatility as in the data (right panel). A more detailed discussion is provided in the online

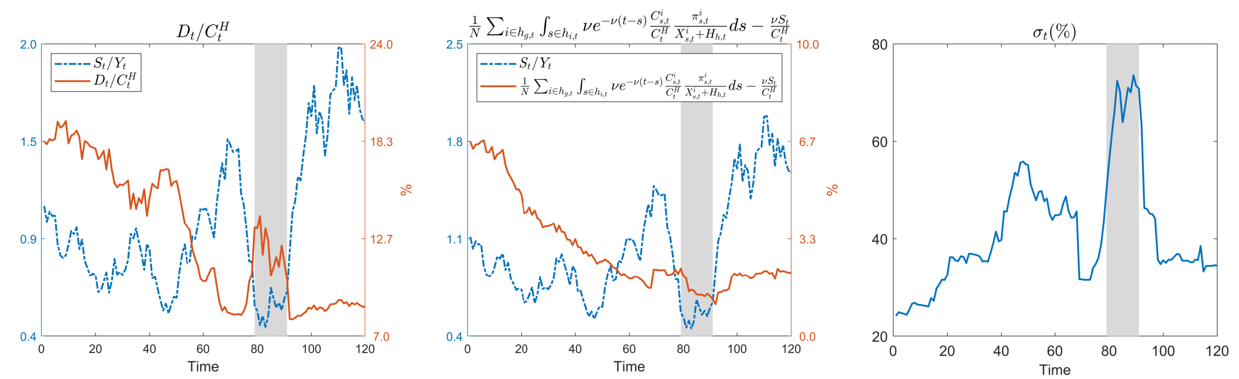

Figure 5: **Stock volatility**

The left panel shows the relationship between the state S_t/Y_t (dash line, left y-axis) and the dividend share in the stockholders' consumption D_t/C_t^H (straight line, right y-axis). The middle panel shows the relationship between the state S_t/Y_t (dash line, left y-axis) and the risky asset share in total wealth with the adjustment term due to insurance payment $\frac{1}{N}\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}\frac{C^i_{s,t}}{C^H_t}\frac{\pi^i_{s,t}}{X^i_{s,t}+H_{h,t}}ds-\frac{\nu S_t}{C^H_t}$ (straight line, right y-axis). The right figure is the corresponding conditional stock volatility in this economy. The shaded area denotes a recession defined as the lowest 10th percentile of the state.

appendix OA.10.

5.1.5 Conditional equilibrium parameters

In this section, we discuss the conditional equilibrium parameters. Table 2 summarizes the model-implied dynamic of the equity premium, price of risk, amount of risk as well as stock volatility, Sharpe ratio, stockholder's consumption volatility. In doing so, we compute the average level of equilibrium parameters across states.

Our model generates the observed dynamics of the equilibrium parameters: countercyclical equity premium of 5.53% versus 3.84% in bad and good times, respectively, stock volatility of 33.92% versus 24.39%, Sharpe ratio of 16.35% versus 15.67%, and consumption volatility of 4.74% versus 4.28%. Most importantly, we emphasize that our model generates the countercyclical equity premium in spite of the procyclical price of risk. This is because the countercyclical amount of risk is strong enough as in the data to dominate the procyclical price of risk in our model. When it comes to the average level of the price of risk, it is 5.89, which can translate into the risk aversion coefficient of a representative

Table 2: Dynamics of equilibrium parameters

Table 2 reports the dynamics of equilibrium parameters based the 120,000 simulated monthly observations. Unconditional average values of parameters as well as conditional averages in bad states and good states are reported. The bad (good) states are defined as the lowest (highest) 10% percentiles of the state variable. The state variable is the stock market wealth-aggregate labor income ratio ($\frac{S_t}{Y_t}$). Parameter values for the simulation are in Table 1. C_t^H denotes the consumption of stock*h*olders. **Notations**:"Counter": Countercyclical; "Pro": Procyclical.

	Data Dynamics	Model-implied Dynamics	Bad (%)	Good (%)	Average (%)
Equity premium $E_t(dR_t^e)$	Counter	Counter	5.53	3.84	4.64
Price of Risk Γ_t^H	Pro[1]	Pro	5.36	6.82	5.89
Amount of Risk $Cov_t(dR_t^e, \frac{dC_t^H}{C_t^H})$	Counter[2]	Counter	1.04	0.58	0.82
Equity volatility σ_t	Counter	Counter	33.92	24.39	27.95
Sharpe ratio λ_t	Counter	Counter	16.35	15.67	16.53
Consumption volatility $\sigma(\frac{dC_t^H}{C_t^H})$	Counter	Counter	4.74	4.28	4.58
Market participation rate $E(p_t)$	Pro[3]	Pro	46.75	86.11	62.78

[1] See Table 6.
[2] See Figure 6.
[3] See Section 6, Brunnermeier and Nagel (2008), and Bonaparte, Korniotis, and Kumar (2018).

investor. Therefore, our explanation for the equity premium puzzle hinges on the amount of risk rather than a high price of risk.

5.1.6 Price-dividend ratio

In this section, we assess whether the price-dividend ratio in the model is consistent with empirical observations. It is well-known that the price-dividend ratio is procyclical in the data (e.g., Fama and French, 1989). Theoretically, however, it is challenging to generate a procyclical variation in the price-dividend ratio with the EIS less than one (e.g., Ju and Miao, 2012; Chabakauri, 2015b) with few exceptions (e.g., Guvenen, 2009).[33] Our price-dividend ratio is procyclical with a correlation of 0.3951 with aggregate consumption.[34] This procyclical variation in the price-dividend ratio follows from the fact that (i) a procyclical risky asset holding, which leads to a procyclical stock price, and (ii) the excess volatility.

Campbell and Shiller (1988) document that future stock market returns are in part predicted by the price-dividend ratio. We test the predictability using 1,000 sample paths with different long-horizons in our model. Panel A of Table 3 presents the results. Our model-implied price-dividend ratio produces the correct negative sign as in the data, implying high valuations are associated with low expected returns. Moreover, both the coefficients and R^2 rise with horizons, which is a stylized-pattern in the data. R^2 values also match the data counterpart reasonably well.[35] For example, at the 5-year horizon, R^2 in the model is 37% versus 26% in the data. This high predictability comes from the fact that when the price-dividend ratio is low (high), high (low) expected returns are associated with future high (low) returns.

[33] A general consensus on the level of the elasticity of intertemporal substitution less than one (e.g., Vissing-Jørgensen, 2002a; Trabandt and Uhlig, 2011; Jin, 2012; Rudebusch and Swanson, 2012).

[34] The online appendix Figure OA.9 shows one sample path of our price-dividend ratio along with aggregate consumption and price-aggregate labor ratio, suggesting a procyclical variation in the price-dividend ratio.

[35] The bottom panel in the online appendix Figure OA.9 shows one sample path of 10-years realized returns and forecast from the price-dividend ratio.

Table 3: Stock Return Predictability and backward-looking test

Panel A reports coefficients and R^2 from the long-horizon forecasting regression: The k-year cumulative rolling ex post excess returns are regressed on the past log price-dividend ratio using the simulated data. The result for the data is from Guvenen (2009). Panel B reports R^2 from the backward looking price dividend ratio test: log price-dividend ratio is regressed on from 1 to L-year lagged consumption growth. A detailed description of the data is in the online appendix A.3.

	Year	1	2	3	5
Panel A: $r^e_{[t\to t+k]} = \alpha + \beta log(\frac{S}{D})_t + \epsilon_{t\to t+k}$					
Model	Coeff.	-0.51	-0.99	-1.36	-1.80
Data	Coeff.	-0.22	-0.39	-0.47	-0.77
Model	R^2	0.10	0.21	0.28	0.37
Data	R^2	0.09	0.14	0.15	0.26
Panel B: $log(\frac{S}{D})_t = \alpha + \sum_{j=1}^{L} \beta_j \Delta c_{t-j} + \epsilon_t$					
Model	R^2	0.018	0.023	0.024	0.014
Data	R^2	0.024	0.013	0.005	0.018

Finally, Bansal, Kiku, and Yaron (2012) point out that the price-dividend ratio in the habit model of Campbell and Cochrane (1999) is not forward-looking as the price-dividend ratio is predicted by the lagged consumption growth. We conduct the same test as in Bansal, Kiku, and Yaron (2012). Panel B of Table 3 shows that at the 1-year horizon, R^2 in our model is 0.2% and rises up to 1.4% at the 5-year horizon, which are very similar to the data counterparts. The model-implied R^2s are reasonably low, compared to around 20% to 40% for the habit model, reported in Bansal, Kiku, and Yaron (2012). This suggests that our price-dividend ratio is forward-looking. In summary, the price-dividend ratio in our model produces well-documented patterns in the data.

5.2 Unconditional moments of asset returns and consumption growth

In this section, we present the unconditional moments. Panel A of Table 4 reports the unconditional moments of consumption growth and their data counterparts. Our model-implied consumption volatility of 4.1% is closer to 2.2% in the data with labor income than 9% in an otherwise identical economy without labor income.[36] Panel B of Table 4 shows

[36]This is because without labor income, the consumption volatility equals the dividend volatility which is 9%.

Table 4: Unconditional Moments of Consumption Growth and Asset Returns

Table 4 presents the annualized consumption, stock returns moments, and market participation rate. A detailed description of the data is in the online appendix A.3.

	U.S. data	Model
Panel A: Consumption moments (%)		
Mean of aggregate consumption growth	2.0	2.1
Volatility of aggregate consumption growth	2.2	4.1
Panel B: Asset returns moments (%)		
Equity premium	5.7	4.6
Equity volatility	20.1	28.0
Sharpe ratio	28	16.5
Mean of log price-dividend ratio	2.9	2.8
Volatility of log price-dividend ratio	49	69.1
Mean of risk-free rate	0.7	3.6
Volatility of risk free rate	3.2	0.2
Panel C: Market Participation rate (%)		
Mean of market participation rate	55.7[1]	62.8

[1] This is based on both direct and indirect holding from 2019 SCF.

the unconditional moments of excess equity returns, log price-dividend ratio, and risk-free rate. Our model generates a high equity premium of 4.6%, the average log price-dividend ratio of 2.8 close to 2.9 in the data, and the standard deviation of the log price-dividend ratio is 69 versus 49 in the data. When it comes to the risk-free rate, the model generates a risk-free rate level around 3.6% versus 1% in the data. This is similar to 4.02% in Bansal and Yaron (2004) in the case where the EIS ψ equals 0.5, and the predictable component of consumption growth is shut down.[37] As for the second moment, the risk-free rate is not as volatile as the one observed in the data and almost constant as in Campbell and Cochrane (1999). This is because the EIS is the same for all investors and heterogeneous risk aversion does not generate an ample variability of the risk-free rate in the recursive utility. Finally, Panel C shows around 63% of investors invest in the stock market in our model which is close to the proportion of direct and indirect stock holdings from the SCF data 55.7%.

[37] See Panel C of Table 2 in Bansal and Yaron (2004).

6 Empirical analysis

In this section, we provide empirical support for the main findings of our paper. We first use the Survey of Income and Participation Program (SIPP) data to examine investors' entry and exit behaviors in the data. Next, we examine the amount and price of risk using the Consumer Expenditure Survey (CEX) data.

6.1 SIPP data

Following Brunnermeier and Nagel (2008) and Bonaparte, Korniotis, and Kumar (2018), we study the market entry and exit behavior of investors with a special focus on the changes in the composition of stockholders. To this end, we exploit the SIPP household data from the 1984 panel to the 2008 panel. The SIPP is a panel data where we can follow the same investors' market entry or exit at a different point in time. We run a panel regression of investors' entry or exit on the stock market wealth to aggregate non-financial income (S_t/Y_t), risk aversion, a change in financial wealth and labor income, and other household demographic characteristics. For the risk aversion measure, although it is challenging to estimate a reliable measure, we assume that risk aversion is proportional to the probability of a household reporting no tolerance for investment risk using the Survey of Consumer Finances (SCF). This is in line with the literature using the reporting no tolerance in the SCF as risk aversion measure (e.g., Haliassos and Bertaut, 1995; Bucciol and Miniaci, 2011). Under this assumption, we first estimate a Probit regression of households reporting unwillingness to take a financial risk on a set of observable characteristics in the SCF and use those estimates for the SIPP households to measure risk aversion of each SIPP household.[38]

Table 5 reports the results. Column (1) and (3) show that investors' heterogeneous risk

[38]The Probit regression is reported in Table OA.3. This methodology is similar to Malloy, Moskowitz, and Vissing-Jørgensen (2009) which use the Probit regression of stock ownership on the set of observable characteristics from the SCF and use it to the CEX households to obtain a more sophisticated definition of stockholders.

Table 5: Determinants of entries and exits from SIPP data

Table 5 reports the panel regression of either entry or exit on S_t/Y_t, risk aversion, and other characteristics. The sample includes 138,039 respondents covered by the Survey of Income and Program Participation (SIPP) for the 1984, 1985, 1986, 1987, 1990, 1991, 1992, 1993, 1996, 2001, 2004, and 2008 panels. $\text{Entry}_{i,t}$ is a dummy variable that takes the value of 1 if a respondent newly participates in the stock market either directly or indirectly through retirement investment accounts. $\text{Exit}_{i,t}$ is a dummy variable that takes the value of 1 if a respondent exits the stock market. For risk aversion measure, we assume that risk aversion is the probability that households have no tolerance for investment risk. More details on this measure are discussed in Section 6. *Wealth* is the sum of stock, mutual fund, bond, saving account, and checking account. *Number of children* is the number of children of a respondent, *Married* is a dummy variable which takes the value of 1 if a respondent is married. *High* and *College* are the dummy variables which take the value of 1 if a respondent's highest grade is high school and college, respectively. T-statistics based on standard errors clustered by year are reported in parentheses, where ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Dependent variable:	$\text{Entry}_{i,t}$		$\text{Exit}_{i,t}$	
	(1)	(2)	(3)	(4)
S_t/Y_t	0.003	-0.019	-0.007*	0.018**
	(1.21)	(-1.67)	(-2.10)	(2.38)
$S_t/Y_t \times \gamma_{i,t}$		0.039*		-0.042***
		(2.10)		(-3.13)
$\gamma_{i,t}$	-0.303***	-0.375***	0.257***	0.335***
	(-8.57)	(-5.62)	(10.97)	(7.86)
Δ *log(Wealth)*$_{i,t}$	0.004***	0.004***	-0.004***	-0.004***
	(7.45)	(7.69)	(-9.60)	(-9.88)
Δ*log(labor)*$_{i,t}$	-0.005***	-0.005***	0.004***	0.004***
	(-6.22)	(-6.08)	(10.65)	(10.32)
Number of children$_{i,t}$	-0.0002	-0.0002	-0.004***	-0.004***
	(-0.12)	(-0.12)	(-4.00)	(-3.98)
Married$_{i,t}$	0.0004	0.001	-0.001	-0.002
	(0.08)	(0.14)	(-0.25)	(-0.31)
High$_{i,t}$	-0.010**	-0.013**	0.013***	0.016***
	(-2.23)	(-2.70)	(3.30)	(4.74)
College$_{i,t}$	-0.041***	-0.046***	0.044***	0.049***
	(-4.86)	(-4.44)	(4.81)	(5.01)
Age$_{i,t}$	-0.013***	-0.009**	0.007***	0.004*
	(-5.33)	(-2.66)	(8.34)	(2.10)
Age$^2_{i,t}$	0.0001***	0.0001***	-0.0001***	-0.0001**
	(3.67)	(1.87)	(-5.12)	(-2.39)
Individual FE	Yes	Yes	Yes	Yes
Number of Obs.	319,452	319,452	319,452	319,452
Adj. R^2	0.012	0.012	0.035	0.035

aversion is statistically significantly associated with their entry and exit decisions at 1% level. This result suggests that heterogeneous risk aversion is the key to explain investors' entry and exit behaviors. Specifically, the less risk-averse, the more likely investors enter the market. Also, the more risk-averse, the more likely investors exit the market. This result holds after controlling for investors' wealth changes, idiosyncratic labor income changes and also life-cycle features. This finding lends support to our model setup where market participation decision depends on heterogeneous investors' risk aversion. In Column (2) and (4), we interact risk aversion with the stock market wealth to aggregate non-financial income (S_t/Y_t). The positive and statistically significant interaction term in Column (2) shows that when stock market valuations are high, risk-averse investors tend to enter the market. Also, the negative interaction term in Column (4) shows that when stock market valuations are low, risk-averse investors are more likely to exit the market. These empirical findings imply that the composition of stockholders varies over times through heterogeneous investors' market entry and exit, which depend on an economic state, consistent with the finding of our theory.[39]

6.2 CEX Data

While we can exploit the panel setting of the SIPP to study the investors' entry and exit behaviors, the SIPP does not provide information on consumption. Thus, to examine both the amount and price of risk dynamics, we complement our empirical analysis by relying on another micro-level household data set, the Consumer Expenditure Survey (CEX), for the period from March 1984 to December 2018.

Amount of risk: Figure 6 depicts the nonparametric estimates of the conditional covariances between stockholders or aggregate consumption growth and market returns over the stock market wealth to aggregate non-financial income ratio (S_t/Y_t) as a conditioning variable. The figure shows that while the stockholders' amount of risk notably exhibits a countercyclical variation, the aggregate amount of risk is weakly procyclical to countercyclical. We statistically evaluate whether differences in point estimates of the amount of risk evaluated at S_t/Y_t in a good time and a bad time are significant. For stockholders' amount

[39] A robustness result using the NBER recession indicator instead of our state variable (S_t/Y_t) is presented in the online appendix Table OA.4.

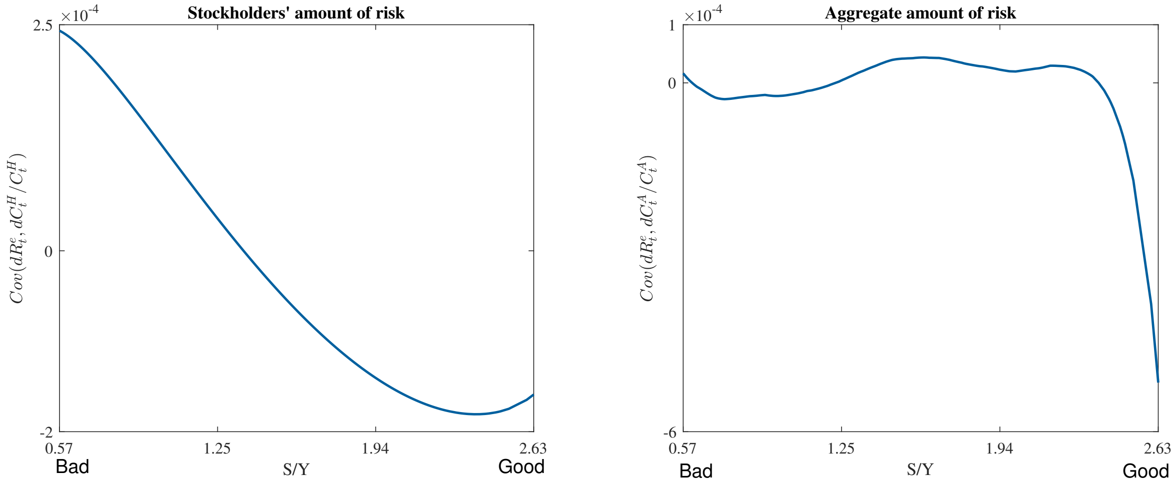

Figure 6: **Empirically estimated Amount of risk**

This figure plots the empirically estimated conditional covariance of equity returns with stockholders' (Left) and aggregate (Right) consumption growth using the stock market capitalization to aggregate non-financial income ratio (S/Y). The bold solid lines are the nonparametric estimate of conditional covariance based on the Epanechnikov kernel estimation at monthly frequency. The shaded backgrounds represent the rescaled kernel density of the conditioning variable. A detailed description of the data is in the online appendix A.3. The result using the consumption-wealth ($\widehat{cay}$) by Lettau and Ludvigson (2001) is in the online appendix figure OA.1.

of risk, untabulated one-sided tests based on stationary bootstrap with 10,000 replications produce *p*-values of 0.008, 0.0132, and 0.0083 for the difference between the maximum value of S_t/Y_t and minimum, 95th percentiles of S_t/Y_t and 5th percentiles, and 90th percentiles and 10th percentile, respectively. For aggregate amount of risk, *p*-values are 0.8697, 0.0768, and 0.0672. This result is consistent with our theory prediction.

In Panel A of Table 6, using OLS regression, we find that the dividend share in aggregate consumption is procyclical, consistent with previous empirical findings. By contrast, the dividend share in stockholders' consumption is countercyclical, consistent with our theoretical finding. Next, we regress the amount of risk on the dividend share for both aggregate and stockholders' consumption. The result shows that dividend share in consumption is positively associated with the amount of risk for both aggregate and stockholders' consumption, in line with our theory.

Price of risk: Next, we confirm the changes in market participants over the economic state in the CEX data. Panel B of Table 6 shows that coefficients for both the stockholders' con-

Table 6: Empirical test of the model

Table 6 reports the OLS regression results as a test of the theory model. $D_t/C_t^G \ \ \forall G = A, H$ is dividend share in either aggregate or stockholders consumption. S_t/Y_t is the stock market wealth to aggregate non-financial income ratio. $Cov_t(dC_t^G/C_t^G, dR_t^e) \ \forall G = A, H$ is the conditional covariances between either aggregate or stockholders consumption growth and excess market returns estimated non-parametrically based on the Epanechnikov kernel estimation. C_t^H/C_t^A is the stockholders consumption share in aggregate consumption. p_t is the market participation rate. $\sum_{i\in G} C_{i,t}/\sum_{i\in G}(C_{i,t}/\gamma_i) \ \ \forall G = A, H$ is the consumption-weighted harmonic mean of stockholders or aggregate risk aversion. For the data, the Consumer Expediture (CEX) Survey from March 1984 to December 2018 is used for stockholders' consumption and the NIPA data for aggregate consumption. Aggregate stock market is the CRSP value-weighted NYSE/AMEX/NASDAQ index. For risk aversion measure, we assume that risk aversion is the probability of reporting that households have no tolerance for investment risk. A detailed description of the data is in the online appendix A.3. T-statistics based on the Newey and West (1987) are in parentheses. ***,**,* denote the statistical significance at 1%, 5%, and 10%, respectively. The lag for the standard errors is automatically selected based on Newey and West (1994).

Dependent variable	Independent variable			Adj. R^2
	S_t/Y_t	D_t/C_t^A	D_t/C_t^H	
Panel A: Amount of risk dynamics				
D_t/C_t^A	0.026***			0.656
	(10.74)			
D_t/C_t^H	-0.466***			0.123
	(-4.41)			
$Cov_t(\Delta C_t^A/C_t^A, dR_t^e)$		9.4×10^{-4}***		0.700
		(16.95)		
$Cov_t(\Delta C_t^H/C_t^H, dR_t^e)$			7.6×10^{-5}***	0.125
			(5.63)	

Dependent variable	Independent variable			Adj. R^2
	S_t/Y_t	C_t^H/C_t^A	p_t	
Panel B: Price of risk dynamics				
C_t^H/C_t^A	0.030***			0.110
	(4.23)			
p_t	0.016***			0.073
	(3.60)			
$\sum_{i\in H} C_{i,t}/\sum_{i\in H}(C_{i,t}/\gamma_{i,t})$		-3.977***	5.098***	0.037
		(-4.33)	(3.50)	

sumption share in aggregate consumption and the market participation rate are highly significant and come in with a positive sign. This suggests that consumption of stockholders who are relatively risk-tolerant drops the most in bad times. Also, the decision to enter the stock market is procyclical, consistent with our theory and the previous empirical result based on the SIPP. With regard to the price of risk, we measure the risk aversion of each household in the same way as before for the SIPP households. We regress the price of risk on both the stockholders' consumption share and market participation. The result shows that the sign on the stockholders' consumption share is negative. It implies that within the same level of market participation, an increase in the consumption share of stockholders who are relatively risk-tolerant leads the mean of risk aversion to be more tilted towards risk-tolerant investors, resulting in a lower price of risk. In contrast, an entry of investors is associated with an increase in the mean of risk aversion, implying that those who enter the market are likely more risk-averse than existing stockholders. This finding empirically illustrates the market entry/exit effect and the consumption re-distribution effect on the price of risk in opposite direction, supporting our theory.

To summarize, we empirically find that: (1) a strong countercyclical stockholders' amount of risk versus a procyclical or weak countercyclical aggregate amount of risk, (2) procyclical (countercyclical) dividend share in aggregate (stockholders) consumption, (3) procyclical market entry, and (4) the opposites effect of changes in market participants (consumption re-distribution) on the price of risk.

7 Conclusion

In this article, we present an overlapping generations equilibrium model with heterogeneous risk-averse investors. Our model generates procyclical market entry and countercyclical exit, which lead to changes in stockholders' composition and hence ineffective (effective) risk-sharing in bad (good) states. We show that due to procyclical risk-sharing, the stockholders' amount of risk is strongly countercyclical. We also show that the aggregate amount of risk is weakly procyclical to countercyclical, in line with empirical evidence. With respect to the price of risk, we find that its time-variation is slightly procyclical in our setting because relatively more risk-tolerant stockholders are present in the market. We highlight that it is the countercyclical stockholders' amount of risk that explains the countercyclical equity premium.

The model delivers a new testable hypothesis on the novel CCAPM under investors' entry and exit. We presented some empirical findings supporting our theory. However, a natural follow-up analysis is to test this novel CCAPM equation following the mainstream conditional tests that have evaluated and rejected the representative-investor CCAPM. We leave this for future research.

Finally, we provide various extensions and clarifications in the online appendix to address potential limitations of the model. However, further features could always be considered to make the setting richer and more realistic.

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Supporting Information

The Online Appendix of this article can be found here: please click here

A. Appendix

A.1 Proof

A.1.1 Proof of Proposition 1

Substituting C_t, π_t, and l_t in (8) (9), and (13) back into the equation (7) gives

$$
\begin{aligned}
0 \;=\; & \frac{\tilde{\delta}(1-\gamma)V_t}{1-\psi^{-1}}(\tilde{\delta}^{\psi-1}((1-\gamma)V_t)^{-\theta\psi+\psi-1}V_{x,t}^{1-\psi}\psi^{-1}-1)+((r_{f,t}+\nu)X_t+Y_t(1-\mathbb{1}_{\{\pi^i_{s,t}>0\}}F))V_{x,t} \\
& -\frac{\lambda_t^2V_{x,t}^2}{2V_{xx,t}}+\mu_yY_tV_{y,t}+\frac{1}{2}\sigma_y^2Y_t^2V_{yy,t}-\frac{\lambda_tV_{x,t}\rho_t\sigma_yY_tV_{xy,t}}{V_{xx,t}}-\frac{\rho_t^2\sigma_y^2Y_t^2V_{xy,t}^2}{2V_{xx,t}}+\sum_{j=1}^{N-1}\mu_{w_{j,t}}w_{j,t}V_{w_j,t} \\
& +\frac{1}{2}\sum_{j=1}^{N-1}\sigma_{w_{j,t}}^2w_{j,t}^2V_{w_jw_j,t}-\frac{\sum_{j=1}^{N-1}\rho_{w_j,t}\sigma_{w_{j,t}}w_{j,t}\sigma_tV_{w_jx,t}(\lambda_tV_{x,t}+\rho_t\sigma_yY_tV_{xy,t})}{\sigma_tV_{xx,t}} \\
& -\frac{(\sum_{j=1}^{N-1}\rho_{w_j,t}\sigma_{w_{j,t}}w_{j,t}\sigma_tV_{w_jx,t})^2}{2\sigma_t^2V_{xx,t}}+\sum_{j=1}^{N-1}\rho_{w_j,y,t}\sigma_{w_{j,t}}w_{j,t}\sigma_yY_tV_{w_jy,t} \\
& +\sum_{j\neq k}\rho_{w_j,w_k,t}\sigma_{w_{j,t}}\sigma_{w_{k,t}}w_{j,t}w_{k,t}V_{w_jw_k,t}+\frac{{l_t}^2}{2\sigma_t^2V_{xx,t}}
\end{aligned}
\tag{A.1}
$$

Due to the nonlinearity of π_t, the first-order condition together with the HJB equation is a non-linear system. Our approach to the dynamic programming is the anticipated utility approach of Kreps (1998), following Cogley and Sargent (2008), Jagannathan and Liu (2019), and Johannes, Lochstoer, and Mou (2016). Therefore, agents solve the dynamic programming as if equilibrium parameters and thus the consumption to total wealth ratio do not change.

Unbinding constraint: At time t, if the constraint is not binding (i.e., $\pi_t^{w/o} > 0$), the Lagrange multiplier is zero (i.e., $l_t = 0$) from the complementary slackness condition. Please note that this does not mean that the constraints will never bind at time $T > t$. Constraints can bind at different time in the future depending on the states which are incorporated into the HJB equation as state variables in (7). We can solve the PDE (A.1) in a case where the constraint is not binding with $l_t = 0$. We conjecture the functional form of the value function as follows given the anticipated utility approach.

$$
V_t = \frac{(a+\sum_{j=1}^{N-1}c_{j,t}w_{j,t})(X_t+Y_t)^{1-\gamma}}{1-\gamma} \equiv \frac{p_tq_t^{1-\gamma}}{1-\gamma} \tag{A.2}
$$

where $p_t \equiv a+\sum_{j=1}^{N-1}c_{j,t}w_{j,t}$ and $q_t \equiv X_t+bY_t$. This functional form of the value function implies the following partial derivatives with respect to state variables.

$V_{x,t} = p_t q_t^{-\gamma}, V_{xx,t} = -\gamma p_t q_t^{-\gamma-1},$

$V_{y,t} = b p_t q_t^{-\gamma}, V_{yy,t} = -\gamma b^2 p_t q_t^{-\gamma-1}, V_{xy,t} = -\gamma b p_t q_t^{-\gamma-1}, V_{w_j,t} = \dfrac{c_j q_t^{1-\gamma}}{1-\gamma},$

$$V_{w_j w_j,t} = 0, V_{w_j w_k,t} = 0, V_{x w_j,t} = c_j q_t^{-\gamma}, V_{y w_j,t} = b c_j q_t^{-\gamma} \tag{A.3}$$

Substituting expressions in (A.3) into the HJB equation and rearranging terms give

$$\begin{aligned}
0 \ &= (X_t + bY_t)^2 [\frac{\tilde{\delta}}{1-\psi^{-1}}(\tilde{\delta}^{\psi-1}(a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})^{-\theta\psi}\psi^{-1} - 1) + \frac{\lambda_t^2}{2\gamma} \\
&+ \frac{(\sum_{j=1}^{N-1} \rho_{w_j,t}\sigma_{w_j,t} w_{j,t} c_j)^2}{2\gamma(a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})^2} + \frac{\sum_{j=1}^{N-1} \mu_{w_j,t} w_{j,t} c_j}{(a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})(1-\gamma)} + \frac{\sum_{j=1}^{N-1} \rho_{w_j,t}\sigma_{w_j,t} w_{j,t} c_j \lambda_t}{\gamma(a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})}] \\
&+ ((r_{f,t} + \nu) X_t + Y_t(1-F))(X_t + bY_t) - \frac{1}{2}\sigma_y^2 Y_t^2 \gamma b^2 + \frac{\rho_t^2 \sigma_y^2 Y_t^2 \gamma b^2}{2} \\
&+ (X_t + bY_t) Y_t [\mu_y b - \lambda_t \rho_t \sigma_y b - \frac{\sum_{j=1}^{N-1} \rho_{w_j,t}\sigma_{w_j,t} w_{j,t} c_j \rho_t \sigma_y \gamma b}{\gamma(a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})} + \frac{\sum_{j=1}^{N-1} \rho_{w_j,y,t}\sigma_{w_j,t} w_{j,t} \sigma_y b c_j}{a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t}}]
\end{aligned} \tag{A.4}$$

When the correlation between dividend growth and labor income growth ρ is equal to 1 (implying also that the correlation between equity returns and labor income growth $\rho_t = 1$ is equal to 1), we can solve the above PDE in a closed form solution.[40] For a non-perfect correlation between dividend and labor income growth $\rho \neq 1$, there is no closed form solution. However, as discussed in the body section, we follow the assumption that X_t/Y_t goes to infinity as used in Koo (1998) and Wang, Wang, and Yang (2016) and solve for this expression in closed form. Each term in (A.4) can be factorized as follows.

$$\begin{aligned}
0 \ &= X_t^2 (d_t + r_{f,t} + \nu) \\
&+ X_t Y_t [2bd_t + (r_{f,t} + \nu) b + 1 - F + \mu_y b - \lambda_t \rho_t \sigma_y b \\
&- \frac{\sum_{j=1}^{N-1} \rho_{w_j,t}\sigma_{w_j,t} w_{j,t} c_j \rho_t \sigma_y \gamma b}{\gamma(a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})} + \frac{\sum_{j=1}^{N-1} \rho_{w_j,y,t}\sigma_{w_j,t} w_{j,t} \sigma_y b c_j}{a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t}}] + Y_t o(z)
\end{aligned} \tag{A.5}$$

where $d_t \equiv \frac{\tilde{\delta}}{1-\psi^{-1}}(\tilde{\delta}^{\psi-1}(a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})^{-\theta\psi}\psi^{-1} - 1) + \frac{\lambda_t^2}{2\gamma} + \frac{(\sum_{j=1}^{N-1} \rho_{w_j,t}\sigma_{w_j,t} w_{j,t} c_j)^2}{2\gamma(a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})^2}$ $+ \frac{\sum_{j=1}^{N-1} \mu_{w_j,t} w_{j,t} c_j}{(a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})(1-\gamma)} + \frac{\sum_{j=1}^{N-1} \rho_{w_j,s,t}\sigma_{w_j,t} w_{j,t} c_j \lambda_t}{\gamma(a_i + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})}$, $z \equiv \frac{X_t}{Y_t}$, and $o(z)$ is a function such that $\lim_{z\to\infty} \frac{o(z)}{z} = 0$. After dividing all terms by $X_{i,t}$, in (A.5), as z goes to infinity, the above PDE can be solved

[40] See the online appendix OA.3.

by

$$d_t = -(r_{f,t} + \nu), \quad b = \frac{1 - F}{r_{f,t} + \nu + \lambda_t \rho_t \sigma_y - \mu_y}, \quad c_1^* = \cdots = c_{N-1}^* = 0 \tag{A.6}$$

$d_t = -(r_{f,t} + \nu)$ is equivalent to

$$a = (\tilde{\delta}^{1-\psi} \psi((-r_{f,t} - \nu - \frac{\lambda_t^2}{2\gamma}) \frac{1 - \psi^{-1}}{\delta} + 1))^{-\frac{1}{\theta\psi}} \tag{A.7}$$

Then, the value function is

$$V(X_t, Y_t) = \frac{(\tilde{\delta}^{1-\psi} \psi((-r_{f,t} - \nu - \frac{\lambda_t^2}{2\gamma}) \frac{1-\psi^{-1}}{\tilde{\delta}} + 1))^{-\frac{1}{\theta\psi}}}{1 - \gamma} (X_t + \frac{Y_t(1-F)}{r_{f,t} + \nu + \rho_t \sigma_y \lambda_t - \mu_y})^{1-\gamma} \tag{A.8}$$

The optimal policies are given by

$$C_t^* = (\tilde{\delta}^{\psi} a^{-\theta\psi}) p_t = ((r_{f,t} + \nu + \frac{\lambda_t^2}{2\gamma})(1 - \psi) + \tilde{\delta}\psi)(X_t + \frac{Y_t(1-F)}{r_{f,t} + \nu + \rho_t \sigma_y \lambda_t - \mu_y}) \tag{A.9}$$

$$\pi_t^* = \frac{\lambda_t}{\gamma \sigma_t}(X_t + \frac{Y_t(1-F)}{r_{f,t} + \nu + \rho_t \sigma_y \lambda_t - \mu_y}) - \frac{\rho_t \sigma_y}{\sigma_t} \frac{Y_t(1-F)}{r_{f,t} + \nu + \rho_t \sigma_y \lambda_t - \mu_y} \tag{A.10}$$

$$l_t^* = 0 \tag{A.11}$$

Note that the consumption to wealth ratio is constant (i.e., $\tilde{\delta}^{\psi} a^{-\theta\psi}$) although they are time-varying in equilibrium. This is due to the anticipated utility approach adopted in our paper. **Binding constraint:** At time t, if the constraint is binding (i.e., $\pi_t^{w/o} \leq 0$), the Lagrange multiplier is nonzero and from the equation (13), its value is $l_t^* = -(\mu_{s,r} - r_{f,t})V_{x,t} - \rho_t \sigma_y Y_t \sigma_t V_{xy,t} - \sum_{j=1}^{N-1} \rho_{w_j,s,t} \sigma_{w_{j,t}} w_{j,t} \sigma_{s,t} V_{w_j x,t}$. Substituting l_t^* into the equation (A.1) and rearranging terms gives

$$\begin{aligned} 0 \;=\; & \frac{\tilde{\delta}(1-\gamma)V_t}{1 - \psi^{-1}} (\tilde{\delta}^{\psi-1}((1-\gamma)V_t)^{-\theta_i\psi + \psi - 1} V_{x,t}^{1-\psi} \psi^{-1} - 1) + ((r_{f,t} + \nu)X_t + Y_t)V_{x,t} \\ & + \mu_y Y_t V_{y,t} + \frac{1}{2}\sigma_y^2 Y_t^2 V_{yy,t} + \sum_{j=1}^{N-1} \mu_{w_{j,t}} w_{j,t} V_{w_j,t} + \frac{1}{2} \sum_{j=1}^{N-1} \sigma_{w_{j,t}}^2 w_{j,t} V_{w_j w_j,t} \\ & + \sum_{j=1}^{N-1} \rho_{w_j,y,t} \sigma_{w_{j,t}} w_{j,t} \sigma_y Y_t V_{w_j y,t} + \sum_{j \neq k} \rho_{w_j,w_k,t} \sigma_{w_{j,t}} \sigma_{w_{k,t}} w_{j,t} w_{k,t} V_{w_j w_k,t} \end{aligned} \tag{A.12}$$

In the same way of unbinding constraint case, we conjecture the functional form and solve the HJB equation with $\frac{X_t}{Y_t} \to \infty$ The value function is then given by

$$V(X_t, Y_t) = \frac{(\tilde{\delta}^{1-\psi} \psi(-(r_{f,t} + \nu) \frac{1-\psi^{-1}}{\tilde{\delta}} + 1))^{-\frac{1}{\theta\psi}}}{1 - \gamma} (X_t + \frac{Y_t}{r_{f,t} + \nu - \mu_y})^{1-\gamma} \tag{A.13}$$

Based on the above value function, the optimal consumption and stock-holding are

$$C_t^* = ((r_{f,t} + \nu)(1 - \psi) + \tilde{\delta}\psi)(X_t + \frac{Y_t}{r_{f,t} + \nu - \mu_y}) \tag{A.14}$$

$$\pi_t^* = 0 \tag{A.15}$$

$$l_t = \frac{(\tilde{\delta}^{1-\psi}\psi((-r_{f,t} - \nu - \frac{\lambda_t^2}{2\gamma})\frac{1-\psi^{-1}}{\tilde{\delta}} + 1))^{-\frac{1}{\theta\psi}}}{(X_t + H_{h,t})^\gamma}(-(\mu - r_{f,t}) + \frac{\rho_t\sigma_y\sigma_t\gamma H_{h,t}}{X_t + H_{h,t}}) \tag{A.16}$$

where $H_{h,t} = \frac{Y_t}{r_{f,t}+\nu+\rho_t\sigma_y\lambda_t-\mu_y}$, $H_{n,t} = \frac{Y_t}{r_{f,t}+\nu-\mu_y}$ ■

A.1.2 Proof of Market clearing condition

Stock market clearing condition is

$$\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}\pi_{s,t}^i ds = S_t \tag{A.17}$$

Bond market clearing condition is

$$\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}(X_{s,t}^i - \pi_{s,t}^i)ds = 0 \tag{A.18}$$

Equation (A.17) and (A.18) imply that

$$\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}X_{s,t}^i ds = S_t \tag{A.19}$$

Dynamics of the LHS and RHS are

$$\frac{1}{N}\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}[\pi_{s,t}^i(\mu_t - r_{f,t}) + \nu X_{s,t}^i + r_{f,t}X_{s,t}^i + Y_t(1-F) - C_{s,t}^i]]ds$$
$$+\frac{1}{N}\sum_{i\notin h_{g,t}}\int_{s\notin h_{i,t}}\nu e^{-\nu(t-s)}[\nu X_{s,t}^i + r_{f,t}X_{s,t}^i + Y_t - C_{s,t}^i]]ds = \mu_t S_t - D_t \tag{A.20}$$

From (A.20), solving for the aggregate consumption gives

$$\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}C_{s,t}^i ds \tag{A.21}$$
$$= D_t + Y_t - F\frac{1}{N}\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}Y_t ds + \nu\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}X_{s,t}^i ds$$
$$= D_t + Y_t(1 - F\cdot P_t) + \nu\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}X_{s,t}^i ds$$

where $P_t = \frac{1}{N}\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}ds$

A.1.3 Proof of Proposition 2

Sharpe ratio

Using (A.10), the optimal stock holding can be written as:

$$\pi^i_{s,t} = \frac{\lambda_t}{\gamma_i \sigma_t}(X^i_{s,t} + \frac{Y_t(1-F)}{r_{f,t} + \nu + \rho_t\sigma_y\lambda_t - \mu_y}) - \frac{1}{\sigma_t}\frac{\rho_t\sigma_y Y_t(1-F)}{r_{f,t} + \nu + \rho_t\sigma_y\lambda_t - \mu_y}$$

$$\forall X_{i,t} > 0, Y_t > 0, \tag{A.22}$$

The stock market clearing condition is equivalent to the following equation.

$$\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}\pi^i_{s,t}ds = \frac{1}{N}\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}\pi^i_{s,t}ds = \frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}X^i_{s,t}ds \tag{A.23}$$

where $h_{g,t}$ denotes the set of types which has at least one stockholder cohort and $h_{i,t}$ denotes the set of cohorts which are stockholders among type i investors at time t. The above equation implies

$$\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}(\frac{\lambda_t}{\gamma_i\sigma_t}(X^i_{s,t} + \frac{Y_t(1-F)}{r_{f,t} + \nu + \rho_t\sigma_y\lambda_t - \mu_y}) - \frac{1}{\sigma_t}\frac{\rho_t\sigma_y Y_t(1-F)}{r_{f,t} + \nu + \rho_t\sigma_y\lambda_t - \mu_y})ds$$

$$= \sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}X^i_{s,t}ds \tag{A.24}$$

This can be re-written as

$$\frac{\lambda_t}{\sigma_t}\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}(\frac{X^i_{s,t} + H_{h,t}}{\gamma_i})ds - \sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}(\frac{\rho_t\sigma_y H_{h,t}}{\sigma_t})ds$$

$$= \sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}X^i_{s,t}ds \quad \text{where} \quad H_{h,t} = \frac{Y_t(1-F)}{r_{f,t} + \nu + \rho_t\sigma_y\lambda_t - \mu_y} \tag{A.25}$$

Solving for the Sharpe ratio λ_t gives

$$\lambda_t = \frac{\sigma_t\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}X^i_{s,t}ds + \sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}\rho_t\sigma_y H_{h,t}ds}{\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}(\frac{X^i_{s,t}+H_{h,t}}{\gamma_i})ds} \tag{A.26}$$

If we consider no labor income $Y_t = 0$ and without OLG setting, (A.26) becomes

$$\lambda_t = (\sum_{i=1}^{N}\frac{C^*_{i,t}}{\sum_{i=1}^{N}C^*_{i,t}}\frac{1}{\gamma_i})^{-1}\sigma_d \tag{A.27}$$

This is the same as the one in Cvitanić, Jouini, Malamud, and Napp (2012) without heterogeneity in terms of belief and time discount rate and also in Chabakauri (2013) without constraint.

Risk-free rate

From (A.9), the optimal consumption is

$$C^i_{s,t} = (\tilde{\delta}^{\psi}a^{-\theta_i\psi})p_t = (\delta^{\psi}a^{-\theta_i\psi})(X^i_{s,t} + bY_t) \quad \forall i = 1,..,N \tag{A.28}$$

Then, the dynamics of (20) is given by

$$\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}(\tilde{\delta}^{\psi}a^{-\theta_i\psi})(dX_{s,t}^i+bdY_t)ds = dD_t + dY_t(1-F\cdot P_t)+\nu dS_t \tag{A.29}$$

This can be re-written as

$$\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}(\tilde{\delta}^{\psi}a^{-\theta_i\psi})(dX_{s,t}^i+bdY_t)ds = \frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}dC_{s,t}^i ds \tag{A.30}$$

We can obtain the optimal dynamics of the financial wealth by plugging the optimal consumption and portfolio into (6).

$$dX_{s,t}^i = \begin{cases}(\pi_{s,t}^i(\mu_t - r_{f,t}) + (r_{f,t}+\nu)X_{s,t}^i + Y_t(1-F) - C_{s,t}^i)dt & \\ +\pi_{s,t}^i(\sigma_t^d dW_{d,t} + \sigma_t^y dW_{y,t}) & \text{if } \pi_{s,t}^{w/o} > 0 \\ ((r_{f,t}+\nu)X_{s,t}^i + Y_t - C_{s,t}^i)dt & \text{Otherwise}\end{cases} \tag{A.31}$$

Collecting the deterministic terms of (A.30) yields

$$\begin{aligned}&\frac{1}{N}\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}(\tilde{\delta}^{\psi}a^{-\theta_i\psi})[\frac{\lambda_t^2(X_{s,t}^i+H_{h,t})}{\gamma_i} - \lambda_t\rho_t\sigma_y H_{h,t} + (r_{f,t}+\nu)X_{s,t}^i + Y_t(1-F)\\ &-((r_{f,t}+\nu+\frac{\lambda_t^2}{2\gamma_i})(1-\psi)+\tilde{\delta}\psi)(X_{s,t}^i+H_{h,t}) + \mu_y H_{h,t}]ds\\ &+\frac{1}{N}\sum_{i\notin h_{g,t}}\int_{s\notin h_{i,t}}\nu e^{-\nu(t-s)}(\tilde{\delta}^{\psi}a^{-\theta_i\psi})[(r_{f,t}+\nu)X_{s,t}^i + Y_t\\ &-((r_{f,t}+\nu)(1-\psi)+\tilde{\delta}\psi)(X_{s,t}^i+H_{n,t}) + \mu_y H_{n,t}]ds = \frac{1}{N}E_t[\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}dC_{s,t}^i ds]/dt\end{aligned} \tag{A.32}$$

Rearranging the terms gives the following equation.

$$\begin{aligned}&\frac{1}{N}\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}(\tilde{\delta}^{\psi}a^{-\theta_i\psi})(X_{s,t}^i+H_{h,t})(\frac{\lambda_t^2}{\gamma_i}(1-\frac{1}{2}(1-\psi)) + (r_{f,t}+\nu)\psi - \tilde{\delta}\psi)ds\\ &+\frac{1}{N}\sum_{i\notin h_{g,t}}\int_{s\notin h_{i,t}}\nu e^{-\nu(t-s)}(\tilde{\delta}^{\psi}a^{-\theta_i\psi})(X_{s,t}^i+H_{n,t})((r_{f,t}+\nu)\psi - \tilde{\delta}\psi)ds\\ &=\frac{1}{N}E_t[\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}dC_{s,t}^i ds]/dt\end{aligned} \tag{A.33}$$

Solving the above equation for $r_{f,t}$ yields the closed form solution for the risk-free rate.

$$r_{f,t} = \delta + \frac{E_t[\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}dC_{s,t}^i ds]/dt}{\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}C_{s,t}^i ds}\frac{1}{\psi} - \frac{\lambda_t^2}{2}(\frac{1+\psi}{\psi})\sum_{i\in h_{g,t}}\frac{1}{\gamma_i}\frac{\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}C_{t,s}^i ds}{\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}C_{t,s}^i ds} \tag{A.34}$$

If we consider no labor income $Y_t = 0$ without OLG setting, (A.34) becomes

$$r_{f,t} = \delta + \mu_d \frac{1}{\psi} - \frac{1+\psi}{2\psi} (\sum_{i=1}^{N} \frac{C^*_{i,t}}{\sum_{i=1}^{N} C^*_{i,t}} \frac{1}{\gamma_i})^{-1} \sigma_d^2 \tag{A.35}$$

When the preferences are the CRRA, then $\psi = 1/\gamma_i$, the risk-free is

$$r_{f,t} = \delta + \mu_d D_t (\sum_{i=1}^{N} \frac{C^*_{i,t}}{\gamma_i})^{-1} - (\sum_{i=1}^{N} \frac{C^*_{i,t}}{\gamma_i})^{-3} \sum_{i=1}^{N} \frac{1}{2} \frac{C^*_{i,t}}{\gamma_i} (\frac{1}{\gamma_i} + 1)(\sigma_d D_t)^2 \tag{A.36}$$

This is the same as the one in Cvitanić, Jouini, Malamud, and Napp (2012) without heterogeneity in terms of belief and time discount rate and also in Chabakauri (2013) without constraint.

Stock volatility

Collecting the diffusion terms of (A.30) yields

$$\frac{1}{N} \sum_{i \in h_{g,t}} \int_{s \in h_{i,t}} \nu e^{-\nu(t-s)} (\tilde{\delta}^{\psi} a^{-\theta_i \psi}) [\pi^i_{s,t} (\sigma^d_t dW_{d,t} + \sigma^y_t dW_{y,t}) + \sigma_y H_{h,t} dW_{y,t}] \tag{A.37}$$

$$+ \frac{1}{N} \sum_{i \notin h_{g,t}} \int_{s \notin h_{i,t}} \nu e^{-\nu(t-s)} (\tilde{\delta}^{\psi} a^{-\theta_i \psi}) \sigma_y H_{n,t} dW_{y,t}$$

$$= \sigma_d D_t dW_{d,t} + \sigma_y Y_t (1 - F \cdot p_t) dW_{y,t} + \nu S_t \sigma^d_t dW_{d,t} + \nu S_t \sigma^y_t dW_{y,t}$$

This gives the following two equations for the stock volatility.

$$\frac{1}{N} \sum_{i \in h_{g,t}} \int_{s \in h_{i,t}} \nu e^{-\nu(t-s)} (\tilde{\delta}^{\psi} a^{-\theta_i \psi}) \pi^i_{s,t} \sigma^d_t ds = \sigma_d D_t + \nu S_t \sigma^d_t \tag{A.38}$$

$$\frac{1}{N} \sum_{i \in h_{g,t}} \int_{s \in h_{i,t}} \nu e^{-\nu(t-s)} (\tilde{\delta}^{\psi} a_i^{-\theta_i \psi}) (\pi^i_{s,t} \sigma^y_t + \sigma_y H_{h,t}) ds \tag{A.39}$$

$$+ \frac{1}{N} \sum_{i \notin h_{g,t}} \int_{s \notin h_{i,t}} \nu e^{-\nu(t-s)} (\tilde{\delta}^{\psi} a_i^{-\theta_i \psi}) \sigma_y H_{n,t} ds = \sigma_y Y_t (1 - F \cdot P_t) + \nu S_t \sigma^y_t$$

Then,

$$\sigma^d_t = \frac{\sigma_d D_t}{\frac{1}{N} \sum_{i \in h_{g,t}} \int_{s \in h_{i,t}} \nu e^{-\nu(t-s)} (\tilde{\delta}^{\psi} a^{-\theta_i \psi}) \pi^i_{s,t} ds - \nu S_t} \tag{A.40}$$

$$\sigma^y_t \tag{A.41}$$

$$= \frac{\sigma_y [N Y_t (1 - F \cdot P_t) - \sum_{i \notin h_{g,t}} \int_{s \notin h_{i,t}} \nu e^{-\nu(t-s)} (\tilde{\delta}^{\psi} a_i^{-\theta_i \psi}) H_{n,t} ds - \sum_{i \in h_{g,t}} \int_{s \in h_{i,t}} \nu e^{-\nu(t-s)} (\tilde{\delta}^{\psi} a_i^{-\theta_i \psi}) H_{h,t} ds]}{\sum_{i \in h_{g,t}} \int_{s \in h_{i,t}} \nu e^{-\nu(t-s)} (\tilde{\delta}^{\psi} a_i^{-\theta_i \psi}) \pi^i_{s,t} ds - N \nu S_t}$$

Finally, the equilibrium stock volatility is

$$\sigma_{s,t} = \sqrt{(\sigma^d_t)^2 + (\sigma^y_t)^2 + 2\rho \sigma^d_t \sigma^y_t} \tag{A.42}$$

If we consider no labor income $Y_t = 0$ without OLG setting,

$$\sigma_{s,t} = \sigma_d(\sum_{i=1}^{N} \frac{C^*_{i,t}}{\sum_{i=1}^{N} C^*_{i,t}} \frac{1}{\gamma_i})^{-1} \sum_{i=1}^{N} \frac{X_{i,t}}{\sum_{i=1}^{N} X_{i,t}} \frac{1}{\gamma_i} \tag{A.43}$$

Stock price

Consumption clearing condition is

$$\frac{1}{N} \sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)}((r_{f,t} + \nu + \frac{\lambda_t^2}{2\gamma_i})(1-\psi) + \tilde{\delta}\psi)(X^i_{s,t} + H_{h,t})ds \tag{A.44}$$

$$+ \frac{1}{N} \sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)}((r_{f,t} + \nu)(1-\psi) + \tilde{\delta}\psi)(X^i_{s,t} + H_{n,t})ds = D_t + Y_t(1 - F \cdot P_t) + \nu S_t$$

Given that $\frac{1}{N}\sum_{i=1}^{N} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} X^i_{s,t} ds = S_t$, we can obtain the following equation

$$(r_{f,t}(1-\psi) + \delta\psi + \nu)S_t = D_t + Y_t(1 - F \cdot P_t) + \nu S_t \tag{A.45}$$

$$- \frac{\lambda_t^2}{2\gamma_i}(1-\psi)\frac{1}{N} \sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)}(X^i_{s,t} + H_{h,t})ds$$

$$- (r_{f,t}(1-\psi) + \delta\psi + \nu)\frac{1}{N} \sum_{i=1}^{N} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} H^i_{s,t} ds \tag{A.46}$$

By solving for S_t and rearranging term, S_t can be expressed as

$$S_t = \frac{D_t + Y_t(1 - F\cdot P_t) - \frac{\lambda_t^2}{2\gamma_i}(1-\psi)\frac{1}{N}\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)}(X^i_{s,t} + H_{h,t})ds}{r_{f,t}(1-\psi) + \delta\psi} \tag{A.47}$$

$$- \frac{r_{f,t}(1-\psi) + \delta\psi + \nu}{r_{f,t}(1-\psi) + \delta\psi} \frac{1}{N} \sum_{i=1}^{N} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} H^i_{s,t} ds$$

A.1.4 Proof of Proposition 3

Consider the conditional covariance between stock returns and stockholders' consumption growth.

$$Cov_t(dR^e_t, \frac{d\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C^i_{s,t} ds}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C^i_{s,t} ds}) \tag{A.48}$$

For covariance, we only need to consider the following diffusion terms.

$$dR^e_t - E_t[dR^e_t] = \sigma^d_t dW_{d,t} + \sigma^y_t dW_{y,t} \tag{A.49}$$

$$d\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}C^i_{s,t}ds - E_t[d\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}C^i_{s,t}ds]$$

$$= \sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}(\delta^\psi a^{-\theta_i\psi})[\pi^i_{s,t}(\sigma^d_t dW_{d,t} + \sigma^y_t dW_{y,t}) + \sigma_y H_{h,t}dW_{y,t}]ds \tag{A.50}$$

Plugging (A.49) and (A.50) into (A.48) yields

$$Cov_t(dR^e_t, \frac{d\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}C^i_{s,t}ds}{\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}C^i_{s,t}ds}) = \frac{1}{\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}C^i_{s,t}ds}\times$$

$$\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}[(\delta^\psi a^{-\theta_i\psi})(\pi^i_{s,t}\sigma^d_t(\sigma^d_t + \rho\sigma^y_t) + \pi^i_{s,t}\sigma^y_t(\rho\sigma^d_t + \sigma^y_t)$$

$$+ \sigma_y H_{h,t}(\rho\sigma^d_t + \sigma^y_t))dt]ds \tag{A.51}$$

Substituting $\pi^i_{s,t}$ into the above equation gives

$$= \frac{\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}(\delta^\psi a^{-\theta_i\psi})(\frac{\sigma_t\lambda_t}{\gamma_i}(X^i_{s,t} + H_{h,t}) - \sigma_t\rho_t\sigma_y H_{h,t} + \sigma_y H_{h,t}(\rho\sigma^d_t + \sigma^y_t))dtds}{\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}C^i_{s,t}ds} \tag{A.52}$$

After rearranging and canceling out some terms, the equation becomes

$$= \frac{\lambda_t\sigma_t dt\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}(\frac{C^*_{i,t}}{\gamma_i})ds}{\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}C^i_{s,t}ds} \tag{A.53}$$

Solving for $\lambda_t\sigma_{s,t}$ in the (A.53) gives

$$\lambda_t\sigma_{s,t}dt = E_t[dR^e_t] = \frac{\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}C^i_{s,t}ds}{\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}(\frac{C^*_{i,t}}{\gamma_i})ds}\times$$

$$Cov_t(dR^e_t, \frac{d\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}C^i_{s,t}ds}{\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}C^i_{s,t}ds}) \quad \blacksquare \tag{A.54}$$

A.2 Implications for the implied price of risk from aggregate consumption

A.2.1 Lemma 1

Lemma 1. *In an economy where market participation is time-varying, the association between the equilibrium equity premium and the conditional covariance of aggregate consump-*

tion growth with stock returns is given by

$$E_t[dR_t^e] = \frac{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} (\frac{C_{i,t}^*}{\gamma_i}) ds} \times Cov_t(dR_t^e, \frac{d\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds}) - \frac{\sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)} k_{n,i,t}\sigma_y H_{n,t}\rho_t\sigma_t ds dt}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} (\frac{C_{i,t}^*}{\gamma_i}) ds} \tag{A.55}$$

Proof : See Appendix A.2.2

Note that the above equation has a second term, different from (27) because the consumption of non-stockholders does not affect the equity premium directly. Previous empirical studies which test the conditional consumption-based asset pricing have modeled the equity premium as follows.

$$E_t[dR_t^e] = \alpha + \Gamma_t Cov_t(dR_t^e, \frac{d\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds}) \tag{A.56}$$

By equating (A.55) with (A.56), we can recover what the estimated price of risk Γ_t in (A.56) from the lenses of our theoretical model:

$$\hat{\Gamma}_t \equiv \frac{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} (\frac{C_{i,t}^*}{\gamma_i}) ds} - a_t = \frac{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{i,t}^* ds} \Gamma_t^H - a_t \tag{A.57}$$

where $a_t \equiv \frac{\sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)} k_{n,i,t}\sigma_y H_{n,t}\rho_t\sigma_t ds dt + \alpha}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} (\frac{C_{i,t}^*}{\gamma_i}) ds Cov_t(dR_t^e, \frac{d\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds})}$ and

$\Gamma_t^H \equiv \frac{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} (\frac{C_{i,t}^*}{\gamma_i}) ds}$

Although $\hat{\Gamma}_t$ is not interpretable in a formal way as opposed to Γ_t^H (stockholders' average risk aversion), it has the following implications. First, a procyclical market participation leads $\frac{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{i,t}^* ds}$ to vary in a countercyclical way, which in turn leads $\hat{\Gamma}_t$ to vary in a more countercyclical way or at least less procyclical than Γ_t^H. This provides an explanation for the large countercyclical implied price of risk in the empirical literature using aggregate consumption. Second, if the second term a_t is large enough, it will generate a negative implied price of risk as documented in the empirical literature. Section 5.1.3 shows that our model reproduces a large countercyclical and negative price of risk as in previous studies (e.g., Duffee, 2005; Nagel and Singleton, 2011; Roussanov, 2014).

A.2.2 Proof of Lemma 1

Let us consider the conditional covariance between stock returns and aggregate consumption growth. The aggregate consumption can be decomposed into the consumption of stockholders and that of non-stockholder.

$$Cov_t(dR_t^e, \frac{d\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds})$$
$$= \frac{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds} Cov_t(dR_t^e, \frac{d\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds})$$
$$+ \frac{\sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds} Cov_t(dR_t^e, \frac{d\sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds}) \tag{A.58}$$

In the same way as before, we only need to consider the diffusion terms from the dynamics of the non-stockholders' consumption.

$$d\sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds - E_t[d\sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds]$$
$$= \sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)} (\delta^\psi a^{-\theta_i \psi}) \sigma_y H_{n,t} dW_{y,t} ds \tag{A.59}$$

Substituting (A.49), (A.53), and (A.59) into (A.58) yields

$$Cov_t(dR_t^e, \frac{d\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds})$$
$$= \frac{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds} \frac{\lambda_t \sigma_t \sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} (\frac{C_{i,t}^*}{\gamma_i}) ds dt}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds}$$
$$+ \frac{\sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds} \frac{\sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)} (\delta^\psi a^{-\theta_i \psi}) (\sigma_y H_{n,t} (\rho \sigma_t^d + \sigma_t^y)) ds dt}{\sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)} C_{s,t}^i ds} \tag{A.60}$$

After rearranging terms, the equation becomes

$$Cov_t(dR_t^e, \frac{d\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds})$$
$$= \frac{\lambda_t \sigma_t \sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} (\frac{C_{i,t}^*}{\gamma_i}) ds dt}{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds} + \frac{\sum_{i\notin h_{g,t}} \int_{s\notin h_{i,t}} \nu e^{-\nu(t-s)} (\delta^\psi a^{-\theta_i \psi}) \sigma_y H_{n,t} \rho_t \sigma_t ds dt}{\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds} \tag{A.61}$$

Solving (A.61) for $\lambda_t \sigma_d dt$ yields

$$\lambda_t \sigma_{s,t} dt = E_t[dR_t^e] =$$

$$\frac{\sum_{i=1}^{N} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i \in h_{g,t}} \int_{s \in h_{i,t}} \nu e^{-\nu(t-s)} (\frac{C_{i,t}^*}{\gamma_i}) ds} Cov_t(dR_t^e, \frac{d \sum_{i=1}^{N} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\sum_{i=1}^{N} \int_{-\infty}^{t} \nu e^{-\nu(t-s)} C_{s,t}^i ds})$$

$$- \frac{\sum_{i \notin h_{g,t}} \int_{s \notin h_{i,t}} \nu e^{-\nu(t-s)} k_{n,i,t} \sigma_y H_{n,t} \rho_t \sigma_t ds dt}{\sum_{i \in h_{g,t}} \int_{s \in h_{i,t}} \nu e^{-\nu(t-s)} (\frac{C_{i,t}^*}{\gamma_i}) ds} \quad \blacksquare \tag{A.62}$$

A.3 Data

In this article, the U.S. dividend, non-financial income, financial market, and consumption data are used for the simulation in Section 5 and empirical analysis in Section 6. In this section, we describe the data we use.

A.3.1 Dividend and non-financial income data

Dividend and non-financial income data for the longest period from 1930 to 2016, similar to Mehra and Prescott (1985), Kandel and Stambaugh (1991), Abel (1999), Bansal and Yaron (2004), and Beeler and Campbell (2012) are used for the choice of parameter values in Table 1. Both data are collected from the National Income and Product Account (NIPA) of the U.S. by the Bureau of Economic Analysis (BEA). Nominal values are deflated using the personal consumption expenditures deflator. U.S. population data are also used to obtain per capita value.

A.3.2 Excess equity returns and risk-free rate

Equity returns and risk-free rate from 1930 to 2016 are used in Table 4. We construct stock returns by log growth of real value of all NYSE/Amex/Nasdaq stocks from the CRSP. To construct ex-ante real risk free, we follow the methodology in Beeler and Campbell (2012). We create a proxy for the ex-ante risk-free rate by forecasting the ex-post quarterly real return on three-month Treasury bills with past one-year inflation and the most recent available three-month nominal bill yield. The detail on the methodology is described in the online appendix in Beeler and Campbell (2012).

A.3.3 NIPA data

Consumption data: Aggregate consumption data from the NIPA by the BEA for the period from 1930 to 2016 are used in Table 4. Consumption is defined as the sum of nondurable and services as durable is not closely linked to consumers' intertemporal choice of consumption and portfolio. Nominal consumption values are deflated using the personal consumption expenditures deflator. We construct the log per capita consumption growth based on the population data.

Dividend and labor income data: Dividend and labor income (disposable personal income) are from the NIPA of the U.S. Nominal values are deflated using the personal consumption expenditures deflator. U.S. population data are also used to obtain the per capita value of both dividend and labor income.

A.3.4 Households Survey data

CEX data: In this article, We highlight the importance of distinction between aggregate consumption and stockholders' consumption. In Section 6, we use stockholders' consumption from the Consumer Expenditure (CEX) by the Bureau of Labor Statistics (BLS) from March 1984 to December 2018 to test the key implications of our theoretical model. The way the interview is conducted is the BLS interviews a selected family every 3 months over four times. After the last interview (fourth), the sample family is dropped from the survey and a new sample family is introduced. Therefore, the composition of interviewed households in a month is different from the next month, and thus, we can calculate the quarterly consumption growth at a monthly frequency. Finance asset holding information is collected in the last interview.[41] As a definition of consumption, we use items in CEX which match the definition of nondurables and services in the NIPA. We exclude housing expenses (but not costs of household operations), medical care costs, and education costs due to its substantial durable components. For the sample choice. We apply the same rules as in Malloy et al. (2009). We drop household-quarters in which a household reports negative consumption. Extreme outliers having consumption growth ($C_{i,t+1}/C_{i,t}$) more than 5.0 and less than 0.2 are drop. Moreover, nonurban households and households residing in student housing are dropped.

To identify the stockholders, we refer to the question of "As of today, what is the total value of all directly-held stocks, bonds, and mutual funds?". Our definition of stockholders is the intersection of the positive holdings of "stocks, bonds, mutual funds and other such securities" and a predicted probability of owning stocks at least 0.5 as in the sophisticated definition of stockholders as in Malloy et al. (2009). In order to compute the probability of owning stocks for CEX households, we use the Survey of Consumer Finances (SCF) as described below where one can accurately observe holdings of stocks and mutual funds, following Malloy et al. (2009). By running a probit regression of whether a household holds stocks or mutual funds on a set of characteristics using the SCF, we obtain coefficients of characteristics and apply them to the CEX households.

SCF data: The SCF is a cross-sectional survey of U.S. families conducted by the Federal

[41]For a more detailed information, see https://www.bls.gov/opub/hom/cex/data.htm

Reserve Board every three years. The survey data cover a wide variety of information on families' balance sheets, pensions, income, and demographic characteristics. Unlike CEX data, the SCF directly asks households whether respondents have any stock (Variable name:hstocks) or mutual funds excluding MMMFs (hnmmf). However, since the survey is conducted on a triennial basis, it is difficult to use the data for the conditional asset pricing test. Using the SCF data from 1989, 1992, 1995, 1998, 2001, 2004, 2007, 2010, 2013, and 2016, We run a probit regression on a set of observable characteristics that are also available in the CEX: age, age squared, number of kids, an indicator for high school and more than college education for household, an indicator for race not being white, the log of income before taxes (set to zero if income = 0), an indicator for income =0, the log of checking and savings accounts (set to zero if checking and savings = 0), an indicator for checking and savings account = 0, an indicator for positive dividend income, year dummies, and a constant. The regression is a cross-sectional regression as a household appears in SCF only once. We also use the SCF data to estimate risk aversion of each household. From the SCF data, we run a probit regression of a dummy variable which takes one if a household reporting no tolerance for financial risk on the same set of independent variables to compute the probability of owning stocks in addition to the log of one plus financial asset holdings. The estimates of the coefficients from the Probit model in the SCF data are applied to the CE data to obtain the probability of reporting no tolerance for financial risk, which is assumed to be risk aversion of each household. All Probit regression results are reported in Table OA.3.

Internet Appendix

Contents

OA.1 Idiosyncratic non-financial income

In this section, we extend the baseline model by introducing idiosyncratic non-financial income. To ease notation, we consider a setup without the OLG feature. This simplifies analysis without changing the key economic mechanism of our model.

OA.1.1 Basic setup

Each investors' non-financial income evolves as

$$\frac{dY_{i,t}}{Y_{i,t}} = \mu_y dt + \sigma_y dW_{y_{i,t}} \quad \forall i = 1, ..., N \tag{OA.63}$$

where $dW_{y_{i,t}}$ is idiosyncratic non-financial income shock for each investor and its correlation structure is modeled flexibly as follows $dW_{y_{i,t}} = \rho_d dW_d + \rho_y dW_y + \sqrt{1-\rho_d^2-\rho_y^2} dW_{i,t}$ where dW_d, dW_y, and $dW_{i,t}$ are independent Brownian motions. ρ_d governs the correlation between dividend and labor income and ρ_y governs the correlation among non-financial income shocks.[42] Then, the correct conjecture for the equilibrium equity returns dynamics is:

$$\frac{dS_t + D_t dt}{S_t} = \mu_{s,t} dt + \sigma_{s,t}^d dW_{d,t} + \sum_{i=1}^{N} \sigma_{s,t}^{y_i} dW_{y_{i,t}} \tag{OA.64}$$

Note that both stockholders and non-stockholders labor income shocks are priced in equilibrium as before due to the all markets clearing condition. Then, the correlation between stock returns and an investor i's labor income growth is $\rho_{s_{i,t}} \equiv Corr_t(\sigma_{s,t}^d dW_{d,t} + \sum_{i=1}^N \sigma_{s,t}^{y_i} dW_{y_{i,t}}, \sigma_y dW_{y_{i,t}}) = \frac{\sigma_{s,t}^d \rho_d + \sum_{j\neq i} \sigma_{s,t}^{y_j}(\rho_d^2+\rho_y^2)+\sigma_{s,t}^{y_i}}{\sigma_{s,t}}$ and the stock volatility is $\sigma_{s,t} = \sqrt{{\sigma_{s,t}^d}^2 + \sum_{i=1}^N {\sigma_{s,t}^{y_i}}^2 + 2\rho_d \sigma_{s,t}^d \sum_{i=1}^N \sigma_{s,t}^{y_i} + (\rho_d^2+\rho_y^2)\sum_{i\neq j} \sigma_{s,t}^{y_i}\sigma_{s,t}^{y_j}}$

OA.1.2 Optimal policies

After solving the HJB equation as before in this setup, the optimal polices are

$$C_{i,t}^* = \begin{cases} ((r_{f,t} + \frac{\lambda_t^2}{2\gamma_i})(1-\psi) + \delta\psi) \cdot (X_{i,t} + H_{h_{i,t}}) & \text{if } \pi_{i,t}^{w/o} > 0 \\ (r_{f,t}(1-\psi) + \delta\psi)(X_{i,t} + H_{n_{i,t}}) & \text{Otherwise} \end{cases} \tag{OA.65}$$

$$\pi_{i,t}^* = \begin{cases} \pi_{i,t}^{w/o} = \frac{\lambda_t}{\gamma_i \sigma_{s,t}}(X_{i,t} + H_{h_{i,t}}) - \frac{\rho_{s_{i,t}}\sigma_y}{\sigma_{s,t}} H_{h_{i,t}} & \text{if } \pi_{i,t}^{w/o} > 0 \\ 0 & \text{Otherwise} \end{cases} \tag{OA.66}$$

[42]The correlation between dividend shock and non-financial income shock i is $dW_d dW_{y_{i,t}} = \rho_d dt$ and the correlation between non-financial income shock i and j is $dW_{y_{i,t}} dW_{y_{j,t}} = (\rho_d^2 + \rho_y^2)dt$. Depending on the value of ρ_y, $\rho_d^2 + \rho_y^2$ can be greater than ρ_d.

$$dX^*_{i,t} = \begin{cases} (\pi^*_{i,t}(\mu_{s,t} - r_{f,t}) + r_{f,t}X_{i,t} + Y_{i,t} - C^*_{i,t})dt \\ +\pi^*_{i,t}(\sigma^d_{s,t}dW_{d,t} + \sum_{i=1}^N \sigma^{y_i}_{s,t}dW_{y_{i,t}}) & \text{if } \pi^{w/o}_{i,t} > 0 \\ (r_{f,t}X_{i,t} + Y_{i,t} - C^*_{i,t})dt & \text{Otherwise} \end{cases} \tag{OA.67}$$

where $H_{h_{i,t}} \equiv \frac{Y_{i,t}}{r_{f,t}+\rho_{s_{i,t}}\sigma_y\lambda_t-\mu_y}$, and $H_{n_{i,t}} \equiv \frac{Y_{i,t}}{r_{f,t}-\mu_y}$

OA.1.3 Equilibrium

After solving the equilibrium as in the main section, the set of equations for the Sharpe ratio λ_t, the risk-free rate $r_{f,t}$, the stock volatility $\sigma_{s,t}$ and the stock price are given by:

$$\lambda_t = \frac{\sigma_{s,t}\sum_{i=1}^N X_{i,t} + \sigma_y \sum_{i\in h^*_t}\rho_{s_{i,t}}H_{h_{i,t}}}{\sum_{i\in h^*_t}\frac{X_{i,t}+H_{h_{i,t}}}{\gamma_i}} \tag{OA.68}$$

$$r_{f,t} = \delta + \frac{\mu_d D_t + \mu_y \sum_{i=1}^N Y_{i,t}}{D_t + \sum_{i=1}^N Y_{i,t}}\frac{1}{\psi} - \sum_{i\in h^*_t}\frac{C_{i,t}}{D_t + \sum_{i=1}^N Y_{i,t}}(\frac{\lambda^2_t}{\gamma_i}\frac{1+\psi}{2\psi}) \tag{OA.69}$$

$$\sigma_{s,t} = \sqrt{{\sigma^d_{s,t}}^2 + \sum_{i=1}^N {\sigma^{y_i}_{s,t}}^2 + 2\rho_d\sigma^d_{s,t}\sum_{i=1}^N\sigma^{y_i}_{s,t} + (\rho^2_d + \rho^2_y)\sum_{i\neq j}\sigma^{y_i}_{s,t}\sigma^{y_j}_{s,t}} \tag{OA.70}$$

$$\sigma^d_{s,t} = \frac{\sigma_d D_t}{\sum_{i\in h^*_t}k_{h,i,t}\pi^*_{i,t}} \tag{OA.71}$$

$$\sigma^{y_i}_{s,t} = \begin{cases} \sigma_y Y_{i,t}[1 - k_{h,i,t}/(r_{f,t} + \rho_{s_{i,t}}\sigma_y\lambda_t - \mu_y)]/\sum_{i\in h^*_t}k_{h,i,t}\pi^*_{i,t} & \text{if } i \in h^*_t \\ \sigma_y Y_{i,t}[1 - k_{n,t}/(r_{f,t} - \mu_y)]/\sum_{i\in h^*_t}k_{h,i,t}\pi^*_{i,t} & \text{Otherwise} \end{cases} \tag{OA.72}$$

$$S_t = \frac{D_t + \sum_{i=1}^N Y_{i,t} - \sum_{i\in h^*_t}\frac{\lambda^2_t}{2\gamma_i}(1-\psi)(X_{i,t} + H_{h_{i,t}})}{r_{f,t} - (\frac{\mu_d D_t + \mu_y\sum_{i=1}^N Y_{i,t}}{D_t + \sum_{i=1}^N Y_{i,t}} - \sum_{i\in h^*_t}\frac{C_{i,t}}{D_t+\sum_{i=1}^N Y_{i,t}}\frac{\lambda^2_t}{\gamma_i}\frac{1+\psi}{2})} - (\sum_{i\in h^*_t}H_{h_{i,t}} + \sum_{i\notin h^*_t}H_{n_{i,t}}) \tag{OA.73}$$

We simulate this setup using the same parameter values reported in Table 1 with $\rho_d = 0.43$, $\rho_y = 0.7$. Figure OA.2 illustrates one sample path of changes in market participation, the stockholders' amount of risk, and the price of risk for idiosyncratic labor income setup. Overall, the results here are similar to those in the body: changes in market participation is procyclical. Stockholders' amount of risk is countercyclical. The price of risk is procyclical. However, idiosyncratic labor income leads to a less variation of changes in market participation. This is because idiosyncratic labor income shocks make market participation decision less systematic. For example, an investor, who would leave the market during the bad states in the previous setup, can continue to hold a stock in this case depending on her own labor income shock. This result is clearly depicted in the second recession of the top figure of Panel B. A less volatile changes in participation in turn leads to a less procyclical and less volatile variation in the price of consumption risk as shown in the bottom figure of Panel B, most notably in the second recession of the one simulated sample path.

OA.2 Proof of the HJB equation with Lagrange multiplier

In this section, we formally derive the Hamilton-Jacobi-Bellman equation with the Lagrange mutliplier for the dynamic programming under constraints.

OA.2.1 Structure of stochastic control problem

The uncertainty and information are represented by a filtered probability space (Ω,$\mathcal{F} = \{\mathcal{F}_t\}_{t\in\tau}$,$\mathbb{P}$). $\forall \tau \in [0,\infty)$. State variables $X = (X_t)$, a subset of $\mathbb{R}^m$, are $\mathcal{F}$-adapted stochastic process representing the evolution of the variables describing the system. In our paper, state variables are financial wealth, labor income, and consumption shares of $N-1$ investors. A $\mathcal{F}$-adapted process $\alpha = \alpha_t$, a subset of $\mathbb{R}^n$, is a control law whose value is chosen at time t as a function of the state variables X_t. In a portfolio-consumption choice problem, $\alpha_t = (c_{i,t}, \pi_{i,t})$. The control law α_t satisfies the integrability conditions. There can be a constraint for the control law: $g(\alpha) \geq m$ where $g(\cdot)$ is a function from $\mathbb{R}^n$ into $\mathbb{R}$ and $m \in \mathbb{R}$. In our paper, we restrict the set of admissible controls to be non-negative i.e., $\alpha \in \mathcal{A} = \{(c,\pi) \mid c \geq 0 \;\; \& \;\; \pi \geq 0\}$. Consider a Brownian motion W and functions $\mu : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^m$ and $\sigma : \mathbb{R}^m \times \mathbb{R}^n \to \mathbb{R}^{+m}$. The dynamics of the state variables in $\mathbb{R}^m$ are given by

$$dX_t = \mu(X_t, \alpha_t)dt + \sigma(X_t, \alpha_t)dW_t \tag{OA.74}$$

Given a function f from $\mathbb{R}^m \times \mathbb{R}^n$ into $\mathbb{R}$, we define the objective function:

$$J(t,x,\alpha) = \mathbb{E}[\int_t^\infty f(X_s,\alpha_s) + \lambda_s(g(\alpha_s) - m)ds], \qquad \forall (t,x) \in [0,\infty) \times \mathbb{R}^m, \quad \alpha \in \mathcal{A} \tag{OA.75}$$

where $\lambda_s \geq 0$ is the Lagrange multiplier and $\lambda_s(g(\alpha_s) - m)$ penalizes the objective function when the constraint is violated. We re-define the objective function $y(X_s, \alpha_s) \equiv f(X_s, \alpha_s) + \lambda_s(g(\alpha_s) - m)$ and the control law $\beta \equiv (\alpha, \lambda) \in \mathbb{R}^{n+1}$. Then, the value function is defined as follows.

$$\hat{J}(t,x) = \sup_{\beta \in \mathcal{A}} J(t,x,\beta) = J(t,x,\hat{\beta}) \tag{OA.76}$$

OA.2.2 Dynamic programming principle and the HJB

The dynamic programming principle implies that for every stopping time $\theta \in \tau_{(t,\infty)}$, it holds that

$$\hat{J}(t,x) = \sup_{\beta \in \mathcal{A}} E[\int_t^\theta y(s, X_s^\beta, \beta_s)ds + \hat{J}(\theta, X_\theta^\beta)] \tag{OA.77}$$

For $\beta \in \mathcal{A}$ and a controlled state variables X_t^β, apply Itô lemma to $\hat{J}(s, X_s^\beta)$ between $s = t$ and $s = t + h$.

$$\hat{J}(t+h, X_{t+h}^\beta) = \hat{J}(t, X_t^\beta) + \int_t^{t+h} \hat{J}_t(s, X_s^\beta) + \mathcal{L}^\beta \hat{J}(s, X_s^\beta) ds + \int_t^{t+h} \hat{J}_x(s, X_s^\beta)\sigma_s^\beta dW_s \tag{OA.78}$$

where $\mathcal{L}^\beta$ is the differential operator associated to the diffusion X with control law β

$$\mathcal{L}^\beta \hat{J} = \mu(x, \beta) D_x \hat{J} + \frac{1}{2} tr(\sigma(x, \beta)\sigma'(x, \beta)) D_{xx} \hat{J} \tag{OA.79}$$

By the martingale property of the stochastic integral, taking the expectation of (OA.78) gives

$$E(\hat{J}(t+h, X_{t+h}^\beta)) = \hat{J}(t, X_t^\beta) + E(\int_t^{t+h} \hat{J}_t(s, X_s^\beta) + \mathcal{L}^\beta \hat{J}(s, X_s^\beta) ds) \tag{OA.80}$$

Plugging this into the Dynamic Programming Principle (OA.77) gives

$$\sup_{\beta \in \mathcal{A}} E[\int_t^{t+h} y(s, X_s^\beta, \beta_s) + \hat{J}_t(s, X_s^\beta) + \mathcal{L}^\beta \hat{J}(s, X_s^\beta) ds] = 0 \tag{OA.81}$$

By dividing by h and $h \to 0$ and we obtain that

$$\hat{J}_t(t, X_t^\beta) + \sup_{\beta \in \mathcal{A}} y(t, X_t^\beta, \beta_t) + \mathcal{L}^\beta \hat{J}(t, X_t^\beta) = 0 \tag{OA.82}$$

This can be re-written as

$$\hat{J}_t(t, X_t^\alpha) + \sup_{\alpha \in \mathcal{A}} f(X_t, \alpha_t) + \lambda_t(g(\alpha_t) - m) + \mathcal{L}^\alpha \hat{J}(t, X_t^\alpha) = 0 \tag{OA.83}$$

The HJB equation in our paper (7) is an application of the above HJB equation.

OA.3 Optimization problem when $\rho = 1$ to confirm our closed form

In this section, we solve the individual optimization problem, which is formulated as the HJB equation in (7) for the special case where the dividend growth is perfectly correlated with labor income growth i.e., $\rho = 1$. Since $\rho = 1$, the correlation between equity returns and labor income growth is also perfect $\rho_t = Corr_t(\sigma_t^d dW_{d,t} + \sigma_t^y dW_{y,t}, \sigma_y dW_{y,t}) = \frac{\sigma_t^d \rho + \sigma_t^y}{\sigma_t} = \frac{\sigma_t^d + \sigma_t^y}{\sqrt{\sigma_t^{d2} + \sigma_t^{y2} + 2\sigma_t^d \sigma_t^y}} = 1$. Then, the HJB equation for unconstrained investors in (A.4) is

$$
\begin{aligned}
0 \ &= (X_t + bY_t)^2 [\frac{\tilde{\delta}}{1-\psi^{-1}} (\tilde{\delta}^{\psi-1} (a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})^{-\theta\psi} \psi^{-1} - 1) + \frac{\lambda_t^2}{2\gamma} \\
&+ \frac{(\sum_{j=1}^{N-1} \rho_{w_j,t} \sigma_{w_j,t} w_{j,t} c_j)^2}{2\gamma (a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})^2} + \frac{\sum_{j=1}^{N-1} \mu_{w_j,t} w_{j,t} c_j}{(a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})(1-\gamma)} + \frac{\sum_{j=1}^{N-1} \rho_{w_j,t} \sigma_{w_j,t} w_{j,t} c_j \lambda_t}{\gamma (a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})}] \\
&+ ((r_{f,t} + \nu) X_t + Y_t)(X_t + bY_t) - \frac{1}{2} \sigma_y^2 Y_t^2 \gamma b^2 + \frac{\sigma_y^2 Y_t^2 \gamma b^2}{2} \\
&+ (X_t + bY_t) Y_t [\mu_y b - \lambda_t \sigma_y b - \frac{\sum_{j=1}^{N-1} \rho_{w_j,t} \sigma_{w_j,t} w_{j,t} c_j \sigma_y \gamma b}{\gamma (a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t})} + \frac{\sum_{j=1}^{N-1} \rho_{w_j,y,t} \sigma_{w_j,t} w_{j,t} \sigma_y b c_j}{a + \sum_{j=1}^{N-1} c_{j,t} w_{j,t}}]
\end{aligned} \tag{OA.84}
$$

After the term $\frac{\sigma_y^2 Y_t^2 \gamma b^2}{2}$ cancels out, the above PDE can be solved by

$$
\begin{aligned}
a^* &= (\tilde{\delta}^{1-\psi} \psi ((-r_{f,t} - \nu - \frac{\lambda_t^2}{2\gamma_i}) \frac{1-\psi^{-1}}{\delta} + 1))^{-\frac{1}{\theta_i \psi}} \\
b^* &= \frac{1}{r_{f,t} + \nu + \lambda_t \sigma_y - \mu_y} \\
c_1^* &= \cdots = c_{N-1}^* = 0
\end{aligned} \tag{OA.85}
$$

Then, the value function is

$$
V(X_t, Y_t) = \frac{(\tilde{\delta}^{1-\psi} \psi ((-r_{f,t} - \nu - \frac{\lambda_t^2}{2\gamma}) \frac{1-\psi^{-1}}{\tilde{\delta}} + 1))^{-\frac{1}{\theta\psi}}}{1-\gamma} (X_t + \frac{Y_t}{r_{f,t} + \nu + \sigma_y \lambda_t - \mu_y})^{1-\gamma} \tag{OA.86}
$$

This solution is the same as the value function (A.8) in closed form with putting $\rho_t = 1$

OA.4 Market participation rate and ρ

In this section, we examine the relation between market participation level and the correlation between dividend growth and non-financial income growth ρ. ρ essentially determines the correlation between equity returns and non-financial income growth and in turn the optimal stock holding in (15). Therefore, ρ is one of the important determinants of the market participation. In our body section, we report the correlation between dividend and non-financial income growth for the period of 1930 to 2016 of 43% in Table 1 which leads to 30% of participation rate at time $t = 0$. We vary the correlation level from 20% to 60% and examine the equilibrium effect on the market participation rate. Figure OA.3 plots the result. When $\rho = 0.2$, every investor is a stockholder because financial income is less correlated with non-financial income. As ρ increases, market participation level declines. As a result, when $\rho = 0.6$ only 20% of total investors hold the stock. This finding provides an empirically testable hypothesis for future research.

OA.5 Description of the equilibrium parameters

In this section, we describe the endogenous parameters (λ_t, $r_{f,t}$, $\sigma_{s,t}$) and stock price in our economy. In doing so, we study them with our nested cases without the OLG feature in order to be comparable to prior studies without OLG feature: (i) a representative investor economy without labor income, (ii) a heterogeneous economy without labor income which in turn characterizes a full participation economy. We confirm that our equilibrium parameters in closed forms reduce to the well-known expressions in nested economies studied in the literature.

OA.5.1 Sharpe ratio

From (21), if we shut down both heterogeneity and labor income, the Sharpe ratio reduces to $\lambda_t = \gamma\sigma_d$. For a heterogeneous economy without labor income income, $\lambda_t = \frac{\sum_{i=1}^{N} C_{i,t}^*}{\sum_{i=1}^{N} \frac{C_{i,t}^*}{\gamma_i}}\sigma_d$, that is the consumption-weighted harmonic mean of stockholders' risk aversion multiplied by the dividend growth volatility, which coincides with the expression in Cvitanić et al. (2012). The time-variation of the Sharpe ratio in this case only comes from the cross-sectional consumption re-distribution which generates countercyclical variation as Chan and Kogan (2002) point out. This is because in bad states, the consumption share of risk-tolerant investors who heavily invest in the risky asset drops the most, leading the average risk aversion to be tilted towards risk-averse investors. However, in our economy, there is another source of time-variation in the Sharpe ratio which is time-varying market participation. Time-varying market participation drives $\frac{\sum_{i=1}^{h_t^*} C_{i,t}^*}{\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\gamma_i}}$ in a procyclical way. This is because in bad economic times, only risk-tolerant investors optimally stay in the market, which decreases the average risk aversion of stockholders. By contrast, in good times even risk-averse investors are willing to enter the market, increasing the average risk aversion of stockholders. We elaborate on this finding in detail in Secition 5.1.3.

OA.5.2 Risk-free rate

From (22), the risk-free rate reduces to the known expression in the simplest representative economy $r_{f,t} = \delta + \frac{\mu_d}{\psi} - \frac{1+\psi}{\psi}\frac{\gamma\sigma_d^2}{2}$. For a heterogeneous economy without labor income, $r_{f,t} = \delta + \mu_d\frac{1}{\psi} - \frac{1+\psi}{2\psi}(\sum_{i=1}^{N} \frac{C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*}\frac{1}{\gamma_i})^{-1}\sigma_d^2$. Putting $\psi = 1/\gamma_i$, this expression is the same as in Cvitanić et al. (2012) which consider CRRA preferences. In our model, as (22) shows, the consumption smoothing demand $\frac{\mu_d D_t + \mu_y N \cdot Y_t}{D_t + N \cdot Y_t}\frac{1}{\psi}$ is time-varying due to the time-varying dividend share in total consumption.

OA.5.3 Stock volatility

From (23) and (24), the stock volatility in the representative economy reduces to the dividend volatility. $\sigma_{s,t} = \sigma_d$. The stock volatility in a heterogeneous economy without labor income reduces to $\sigma_{s,t} = \sigma_d \sum_{i=1}^{N} \frac{X_{i,t}}{\sum_{i=1}^{N} X_{i,t}} \frac{1}{\gamma_i} / (\sum_{i=1}^{N} \frac{C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*} \frac{1}{\gamma_i})$. This equation shows that the stock volatility is determined by the ratio of the wealth-weighted average risk tolerance to the consumption-weighted average risk tolerance. Thus, in a full participation economy without labor income, a countercyclical stock volatility can be generated only if the wealth distribution is more unequal than the consumption distribution in bad time than in good time.

In our economy, we have two parameters $\sigma_{s,t}^d$ and $\sigma_{s,t}^y$ associated with the stock volatility, but as we shall show in the simulation, the second parameter $\sigma_{s,t}^y$ contributes to the stock volatility only marginally compared to the first parameter $\sigma_{s,t}^d$. First, from equation (24), the following holds $\frac{\sigma_{s,t}^d}{\sigma_d} = \frac{D_t}{\sum_{i=1}^{h_t^*} C_{i,t}^*} / (\sum_{i=1}^{h_t^*} \frac{C_{i,t}^*}{\sum_{i=1}^{h_t^*} C_{i,t}^*} \frac{\pi_{i,t}}{X_{i,t}+H_{h,t}})$. Therefore, the excess volatility from the first parameter is generated (i.e., $\sigma_{s,t}^d > \sigma_d$) when the dividend share in stockholders' consumption is greater than the risky asset share in total wealth. The intuition is as follows. When the dividend accounts for a large proportion of the stockholders' consumption, a change in the stockholders' consumption is highly sensitive to dividend shocks. However, since the risky asset accounts for only a small proportion of total wealth, a high sensitive change in the stockholders' consumption with respect to dividend shocks translates into the high volatility associated with the dividend shocks $\sigma_{s,t}^d$. We discuss this point in detail in Section 5.1.4. Regarding the second parameter $\sigma_{s,t}^y$ in (25), this can be re-written as $\sigma_{s,t}^y = \frac{\sigma_y Y_t N}{\sum_{i=1}^{h_t^*} k_{h,i,t} \pi_{i,t}^*} (1 - \frac{1}{N} \sum_{i=1}^{N} \partial C_{i,t}^*(X_{i,t}, Y_t)/\partial Y_t)$. Note that if the average marginal propensity to consume out of labor income across all investors is less than unity (i.e., $\frac{1}{N} \sum_{i=1}^{N} \partial C_{i,t}^*(X_{i,t}, Y_t)/\partial Y_t < 1$), then $\sigma_{s,t}^y > 0$. In this case, investors invest some fraction of their labor income in the risky asset and therefore the sensitivity of the stock returns with respect to labor income shocks $\sigma_{s,t}^y$ is positive. In particular, for a CARA investor or a representative investor, $\partial C_{i,t}^*(X_{i,t}, Y_t)/\partial Y_t$ is always unity and hence $\sigma_{s,t}^y$ is always zero.

OA.5.4 Stock price

The first term in (26) is aggregate wealth in this economy including both financial wealth and human capital. Therefore, the equilibrium stock price is expressed by subtracting total human capital from aggregate wealth. This equilibrium stock price equation shows how non-stockholders and labor income affect the stock price. In a heterogeneous economy without labor income, the stock price reduces to $\frac{D_t - (\sum_{i=1}^{N} \frac{C_{i,t}}{\sum_{i=1}^{N} C_{i,t}} \frac{1}{\gamma_i})^{-2} \sigma_d^2 \frac{1-\psi}{2} \sum_{i=1}^{N} \frac{X_{i,t}}{\gamma_i}}{r_{f,t} - (\mu_d - \frac{1+\psi}{2} (\sum_{i=1}^{N} \frac{C_{i,t}^*}{\sum_{i=1}^{N} C_{i,t}^*} \frac{1}{\gamma_i})^{-1} \sigma_d^2)}$. The price-dividend ratio in this case is counterfactually more volatile than the data because the dividend shock is the only fundamental shock. On the contrary, in our economy labor income shock as well as dividend shock affects the price-dividend ratio and given the fact that labor income shock is less volatile than the dividend shock, the volatility of price-dividend ratio matches the data reasonably well, as we will show in Section 5.2. If we further simplify the economy by considering a representative economy, the equilibrium stock price is $S_t = \frac{D_t}{r_f(1-\psi)+\delta\psi+\gamma\sigma_d^2\frac{1-\psi}{2}} = \frac{D_t}{\delta+\mu_d\frac{1-\psi}{\psi}-\gamma\frac{\sigma_d^2}{2}(\frac{1-\psi}{\psi})}$, the same as in the existing studies (e.g., Yan, 2008; Cvitanić et al. 2012). If there is no uncertainty on dividend stream ($\sigma_d = 0$), the equilibrium stock price is the same as in the Gordon's dividend model ($S_t = \frac{D_t}{r_f-\mu_d} = \frac{D_0 exp(\mu_d t)}{r_f-\mu_d}$).

OA.6 Martingale approach with CRRA

In this section, we solve the equilibrium for the case where investors are not endowed with stochastic labor income and their preferences are CRRA without the OLG feature. The purpose of this section is to show that solutions from this approach verify solutions from the HJB approach. In this case, the investor is facing a dynamically complete market and therefore the optimality of $c_{i,t}$ is equivalent to the marginal utility process $e^{-\rho t}u_i^{'}(c_{i,t})$ being proportional to the equilibrium state price density as in Basak and Cuoco (1998), that is,

$$e^{-\rho t}u_i^{'}(c_{i,t}) = \psi_i \xi_t \tag{OA.87}$$

for some $\psi_i > 0$ and where ξ_t is the state price density and its dynamic process is $d\xi_t/\xi_t = -r_{f,t}dt - \lambda_t dW_{d,t}$ Since we consider the power utility function, the above equation can be rearranged as $c_{i,t}^* = (e^{\rho t}\psi_i\xi_t)^{-\frac{1}{\gamma_i}}$. And, the differential of the optimal consumption is $dc_{i,t}^* = -\frac{1}{\gamma_i}(e^{\rho t}\psi_i\xi_t)^{-\frac{1}{\gamma_i}-1}(\rho e^{\rho t}\psi_i\xi_t dt + e^{\rho t}\psi_i d\xi_t) + \frac{1}{2}\frac{1}{\gamma_i}(\frac{1}{\gamma_i}+1)(e^{\rho t}\psi_i\xi_t)^{-\frac{1}{\gamma_i}-2}e^{2\rho t}\psi_i^2 d\xi_t d\xi_t$. This can be re-written as

$$dc_{i,t}^* = -\frac{c_{i,t}^*}{\gamma_i}(\rho dt + \frac{d\xi_t}{\xi_t}) + \frac{1}{2}\frac{c_{i,t}^*}{\gamma_i}(\frac{1}{\gamma_i}+1)\frac{d\xi_t d\xi_t}{d\xi_t^2} \tag{OA.88}$$

Aggregating the above differentials across investors yields: $\sum_{i=1}^N dc_{i,t}^* = -\sum_{i=1}^N \frac{c_{i,t}^*}{\gamma_i}(\rho dt + \frac{d\xi_t}{\xi_t}) + \sum_{i=1}^N \frac{1}{2}\frac{c_{i,t}^*}{\gamma_i}(\frac{1}{\gamma_i}+1)\frac{d\xi_t d\xi_t}{d\xi_t^2}$. The consumption market clearing condition implies that

$$\begin{aligned}\sum_{i=1}^N dc_{i,t}^* = -\sum_{i=1}^N \frac{c_{i,t}^*}{\gamma_i}(\rho dt + \frac{d\xi_t}{\xi_t}) + \sum_{i=1}^N \frac{1}{2}\frac{c_{i,t}^*}{\gamma_i}(\frac{1}{\gamma_i}+1)\frac{d\xi_t d\xi_t}{d\xi_t^2} \\ = \mu_d D_t dt + \sigma_d D_t dW_{d,t}\end{aligned} \tag{OA.89}$$

By matching the diffusion terms of (OA.89) in each side, the market price of risk is

$$\lambda_t = (\sum_{i=1}^N \frac{c_{i,t}^*}{\gamma_i})^{-1}\sigma_d D_t \tag{OA.90}$$

Also, by matching the deterministic terms of (OA.89), the risk-free rate is

$$r_{f,t} = \rho + \mu_d D_t (\sum_{i=1}^{N} \frac{c^*_{i,t}}{\gamma_i})^{-1} - (\sum_{i=1}^{N} \frac{c^*_{i,t}}{\gamma_i})^{-3} \sum_{i=1}^{N} \frac{1}{2} \frac{c^*_{i,t}}{\gamma_i} (\frac{1}{\gamma_i} + 1)(\sigma_d D_t)^2 \quad \text{(OA.91)}$$

We verify that these solutions are the same as in other papers (e.g., Cvitanić et al., 2012) which study this economy and our endogenous equilibrium parameters in **Proposition 2** when $Y_t = 0$ and $\psi_i = 1/\gamma_i$.

OA.7 Robustness to the number of investors and horizon

In this section, we examine whether our key results are robust to the choice of the number of investors and the simulation horizon. Figure OA.4 presents one sample path of changes in market participation along with the stock market wealth-aggregate labor ratio as a state variable, the stockholders' amount of risk, and the price of risk for the case with $N = 40$, keeping all other parameter values changed. Figure OA.5 presents one sample path of the same key variables. However, in this case, we change the simulation horizon from 40 years to 70 years, keeping all other parameter values unchanged including the number of investor $N = 30$. Both figures show that our main results in the baseline model are robust to the horizon and the number of investors: market participation is procyclical. The amount of risk is countercyclical. The price of risk is procyclical.

OA.8 Comparative Statics of equilibrium moments

For the comparative statics exercise, We **exogenously** change the group of stockholders and investigate how r_f, EP (Equity Premium), λ, σ_s, Cov (amount of risk), and Γ (Price of risk) change accordingly. We start this exercise by imposing the least risk-averse investor as a cut-off stockholder $h = 1$, then we repeat this exercise by moving the cut-off stockholder one by one up until the point where every investor is a stockholder ($h = N$). For ease of exposition, we suppress time index throughout this exercise.

Figure OA.6 plots $r_f(h)$, $EP(h)$, $\lambda(h)$, $\sigma_s(h)$, $Cov(h)$, $\Gamma(h)$ in a given level of state $\frac{S_t}{\sum Y_i}$. In Panel A, the risk-free rate is increasing at the low market participation level. This is because there is a greater selling demand on the bond, as risk-tolerant investors, who are willing to borrow money to invest in the risky asset, are included in the market. As more risk-averse investors are included in the market, the risk-free rate is decreasing. The reason is as follows. First, more risk-averse investors are more willing to invest in the bond. Second, as discussed in Section OA.5.2, the decreasing risk-free rate is also attributed to the increasing precautionary saving demand, as more investors become stockholders and hence more exposure to a future uncertainty.

Panel B shows that as we impose more investors to stay in the market, the equity premium is decreasing and turning to increasing. To pin down the source of the variation in the equity premium, we decompose the equity premium with respect to the market risk and consumption risk in Panel C and D, respectively. In Panel C, note that when the least risk-averse investor is the only stockholder, the market price of risk λ is the highest possible level. This is because there should be a substantial compensation in order to induce this investor to bear the market risk alone. As more investors are assumed to be in the market, λ is decreasing with more buying demand. From a certain point, λ is turning to increasing as the investors who want to optimally short-sell the stock are assumed to be in the market. An increasing selling demand requires the market to compensate more to induce investors to hold the market. As for the amount of market risk - stock volatility σ_s, it has the exact same shape as the Sharpe ratio. We delve into and discuss this finding in Figure OA.7.

Panel D decomposes the equity premium into the amount $Cov(dR_t^e, \frac{d\sum_{i=1}^h C_{i,t}^*}{\sum_{i=1}^h C_{i,t}^*})$ and price of consumption risk $\Gamma^H \equiv \frac{\sum_{i=1}^h C_{i,t}^*}{\sum_{i=1}^h \frac{C_{i,t}^*}{\gamma_i}}$ as in **Proposition 3**. When it comes to the price of risk, Γ^H is increasing with market participation. This is because the more risk-averse investors we include in the market, the higher the stockholders' harmonic mean of risk aversion, and the higher the required compensation. By contrast, the amount of risk is decreasing as more investors are in the market. The intuition behind this finding is as more investors bear the market risk together, the risk is effectively shared-out (improving risk-sharing) among stockholders and the amount of risk decreases. Please note that the risk-sharing is improving at a decreasing rate because new investors included in the market are more risk-averse than the existing ones and they are not willing to take the risk as much as risk-tolerant investors. Therefore, their contribution of sharing the risk is only marginal. For more details on the consumption risk-sharing, please see Appendix OA.12.

While $EP(h)$ in this comparative statics increases for $h > h_B^*$, this result does not translate into the relationship between the equity premium and market participation across different equilibria. For example, in our base state (B) the equity premium is 4% ($EP(h_B^*) = 4\%$ and $h_B^* = 9$). In a better state (G), the endogenous market participation is $h_G^* = 11$ ($> h_B^* = 9$). The equilibrium equity premium is 3%, lower than 4% in our base state $EP(h_G^*) = 3\% < EP(h_B^*) = 4\%$ even with the inclusion of more investors in the market.

To further understand the shape of $\sigma_s(h)$ with market participation in Panel C of Figure OA.6, we explore the two parameters that govern the stock volatility as a function of the market participation (i.e., $\sigma_s^d(h)$ and $\sigma_s^y(h)$). Panel A of Figure OA.7 illustrates that it is the parameter associated with the dividend shocks $\sigma_s^d(h)$ which drives the shape of $\sigma_s(h)$, whereas $\sigma_s^y(h)$ works in the opposite way. As discussed in Section OA.5.3, σ_s^d/σ_d can be expressed as the dividend share in the stockholder's consumption divided by the stockholders' consumption-weighted mean of risky asset share in total wealth $\frac{D}{\sum_{i=1}^h C_i} / \sum_{i=1}^h \frac{C_i}{\sum_{i=1}^h C_i} \frac{\pi_i}{X_i + H_h}$. Each term is illustrated in Panel B of Figure OA.7.

First, $\frac{D}{\sum_{i=1}^h C_i}$ is decreasing as more investors are assumed to be in the market. This is because the same amount of dividend D_t is shared out by more investors. Also, as the amount of risk is decreasing at a decreasing rate, so does $\frac{D}{\sum_{i=1}^{h} C_i}$. The reason is that a newly included investor is more risk-averse than the existing stockholders. Due to a high precautionary saving motive, the new investor's consumption level is low (see Figure OA.10.) and thus her contribution of sharing dividend is only marginal. Second, $(\sum_{i=1}^{h} \frac{C_i}{\sum_{i=1}^{h} C_i} \frac{\pi_i}{X_i+H_h})^{-1}$ is positively linked to the price of consumption risk in Panel D of Figure OA.6. As more risk-averse investors are assumed to be in the market, $\sum_{i=1}^{h} \frac{C_i}{\sum_{i=1}^{h} C_i} \frac{\pi_i}{X_i+H_h}$ is decreasing because the inclusion of more risk-averse investor whose optimal portfolio is relatively low drives down the overall average. Thus, $\sum_{i=1}^{h} \frac{C_i}{\sum_{i=1}^{h} C_i} \frac{\pi_i}{X_i+H_h}$ inversely capture the consumption-weighted mean of stockholders' risk aversion. As the increasing price of consumption risk dominates the decreasing amount of consumption risk from h_B^* in Panel D of Figure OA.6, the increasing $(\sum_{i=1}^{h} \frac{C_i}{\sum_{i=1}^{h} C_i} \frac{\pi_i}{X_i+H_h})^{-1}$ dominates the decreasing $\frac{D}{\sum_{i=1}^{h} C_i}$ from h_B^* in Panel B. This leads to non-monotonic relationship for $\sigma_s^d(h)$ and in turn $\sigma_s(h)$.

Lastly, since $\sigma_s^y(h)$ is mainly driven by the average marginal propensity to consume out of labor income across all investors $\frac{1}{N}(\sum_{i=h+1}^{N} \partial C_i^*(X_i, Y)/\partial Y + \sum_{i=1}^{h} \partial C_i^*(X_i, Y)/\partial Y)$ in (25), we explore the marginal consumption with respect to labor income for comparative statics. Panel D of Figure OA.7 shows the decomposition of this term. On the one hand, the first component, non-stockholders' marginal consumption $\partial C_i^*(X_i, Y)/\partial Y = \frac{r_f(1-\psi)+\delta\psi}{r_f-\mu_y}$ (dotted line) is mainly due to the risk free rate in Panel A of Figure OA.6. If the risk-free rate goes down, non-stockholders value their future income highly and therefore the marginal consumption with respect to labor goes up. On the other hand, the second component, stockholders' marginal consumption with respect to labor income $\partial C_i^*(X_i, Y)/\partial Y = \frac{(r_f+\frac{\lambda^2}{2\gamma_i})(1-\psi)+\delta\psi}{r_f+\rho_s\sigma_y\lambda-\mu_y}$ depends on the Sharpe ratio λ due to the trade-off between investment and consumption. As such, the shape of the Sharpe ratio λ in Panel C of Figure OA.6 mimics that of the stockholders' marginal consumption with respect to labor income. Taken together, the two components shape Panel C of Figure OA.7, which in turn explains the effect of market participation on the level of $\sigma_s^y(h)$.

Note that as in the case of the equity premium, this comparative statics of increasing $\sigma_s^d(h)$ with market participation does not translate into the equilibrium result. In equilibrium, if the state changes to a better state (G), new market participation level $h_G^* = 11$ leads to $\sigma_s^d(h_G^*) = 28\%$, lower than $\sigma_s^d(h_B^*) = 33\%$ even with more investors in the market.

OA.9 Returns decomposition for the amount of risk

Returns decomposition: Another way of examining consumption risk dynamics is to decompose equity returns into the cash flow part and the discount rate as in Xu (2018). She shows that it is the cash flow part of returns which contributes to the procyclical variation in aggregate consumption risk while the non-cash flow part returns varies with aggregate consumption countercyclically. In order to illustrate the importance of separating stockholder consumption risk from aggregate consumption risk, we do this decomposition for both stockholders and aggregate household separately as follows and report the dynamic of the amount of risk as well as its components.

$$Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e) =$$

$$Cov_t(\frac{dC_t^G}{C_t^G}, \frac{dD_t}{D_t}) + Cov_t(\frac{dC_t^G}{C_t^G}, dR_t^e - \frac{dD_t}{D_t}) \qquad \forall G = A, H \tag{OA.92}$$

Table OA.1 reports the results. Panel A shows that our model generates a procyclical variation in the conditional covariance between aggregate consumption growth and dividend growth $Cov_t(\frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A}, \frac{dD_t}{D_t})$ as in the data. In good times, due to an entry of investors into the market, stockholders consumption constitutes a larger proportion of aggregate consumption. Given the fact that stockholders' consumption is highly correlated with dividend, the covariance between aggregate consumption growth and dividend becomes higher than in bad times. As Xu (2018) shows, major asset pricing models calibrated to aggregate consumption cannot generate these dynamics. When it comes to the non-dividend part of returns, our model generates the same dynamics for $Cov_t(\frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A}, dR_t^e - \frac{dD_t}{D_t})$ as observed in the data. In our calibration, the dynamics of $Cov_t(\frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A}, dR_t^e - \frac{dD_t}{D_t})$ dominates $Cov_t(\frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A}, \frac{dD_t}{D_t})$ and thus $Cov_t(\frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A}, dR_t^e)$ is weakly countercyclical.

More importantly, since the equity premium is directly driven by stockholders consumption, we also examine each component of the equation (OA.92) for stockholders. Our simulation shows that both cash flow and discount rate parts of returns contribute to the countercyclical covariance between equity returns and stockholders' consumption growth. While the representative-investor model of Xu (2018) explains each consumption risk component for aggregate consumption, her model requires a dramatically countercyclical price of consumption risk even stronger than the habit model of Campbell and Cochrane (1999). This is because the amount of aggregate consumption risk is procyclical in her model. We argue that the distinction between stockholders and aggregate households is necessary for explaining empirical moments and covariances in the data.

OA.10 More details on the stock volatility

In this section, we discuss our model-implied stock volatility in greater details. As discussed in Section 5.1.4, the stock volatility parameter associated with dividend shock σ_t^d is the ratio of the dividend share in stockholders' consumption to stockholders' consumption-weighted harmonic mean of risky asset share in total wealth. That is,

$$\frac{\sigma_t^d}{\sigma_d} = \frac{D_t}{C_t^H} / (\frac{1}{N} \sum_{i \in h_{g,t}} \int_{s \in h_{i,t}} \nu e^{-\nu(t-s)} \frac{C_{s,t}^i}{C_t^H} \frac{\pi_{s,t}^i}{X_{s,t}^i + H_{h,t}} ds) \tag{OA.93}$$

As for σ_t^y, understanding its dynamics boils down to the average marginal propensity to consume out of labor income across all investors.

$$\sigma_{s,t}^y = \frac{\sigma_y Y_t N}{\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} k_{h,i,t}\pi_{s,t}^i ds}(1 - \frac{1}{N}\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} \frac{\partial C_{s,t}^i(X_{s,t}^i, Y_t)}{\partial Y_t}) \quad \text{(OA.94)}$$

In Table OA.2, we compute unconditional average values of parameters as well as conditional averages in bad states and good states for all components related to the first and second volatility parameters as well as resulting stock volatility: σ_t, σ_t^d, σ_t^y as well as σ_t/σ_d, σ_t^d/σ_d, $\frac{D_t}{C_t^H}$, $\frac{1}{N}\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} \frac{C_{s,t}^i}{C_t^H} \frac{\pi_{s,t}^i}{X_{s,t}^i + H_{h,t}} ds$, and $\frac{1}{N}\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} \frac{\partial C_{s,t}^i(X_{s,t}^i, Y_t)}{\partial Y_t}$. First, we find that both σ_t^d and σ_t^y are countercyclical, but most of the variation of σ_t stems from σ_t^d. Second, we also find that the average of σ_t^d/σ_d and σ_t/σ_d is 2.40 and 2.29 respectively. The latter level compares to around 2 in the data. As shown in the discussion of the amount of consumption risk, the dividend share in the stockholders' consumption $\frac{D_t}{C_t^H}$ is countercyclical. By contrast, the stockholders' consumption weighted mean of risky asset share in total wealth $\frac{1}{N}\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} \frac{C_{s,t}^i}{C_t^H} \frac{\pi_{s,t}^i}{X_{s,t}^i + H_{h,t}} ds$ is mildly procyclical. This is because (i) investors optimally reduce the risky asset holding in bad times, and (ii) consumption of risk-tolerant investors drops the most, leading the average to be more tilted towards the risky asset share of risk-averse investors. Since σ_t^d/σ_d is the ratio of these two terms, the countercyclical $\frac{D_t}{C_t^H}$ together with the procyclical $\frac{1}{N}\sum_{i\in h_{g,t}} \int_{s\in h_{i,t}} \nu e^{-\nu(t-s)} \frac{C_{s,t}^i}{C_t^H} \frac{\pi_{s,t}^i}{X_{s,t}^i + H_{h,t}} ds$ leads the excess volatility to vary in a highly countercyclical way.

Finally, with respect to σ_t^y, we find that it is negative. This is due to the average marginal propensity to consume out of labor income is above one.[43]

OA.11 More details on the Price-dividend ratio

The top and middle panel of Figure OA.9 shows one sample path of the aggregate consumption and the price-aggregate labor ratio, respectively, along with the price-dividend ratio. The figure shows that the price-dividend ratio moves closely with aggregate consumption and strongly with the price-labor ratio.

The bottom panel of Figure OA.9 shows one sample path of simulated 10-year rolling cumulative excess returns and return forecast by the long-horizon regression using the log price-dividend ratio. The forecast by our log price-dividend ratio notably fits the 10-year future returns reasonably well with R^2 of 0.64 in this particular sample.

[43]Please note that "$\frac{1}{N}\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} \frac{\partial C_{s,t}^i(X_{s,t}^i, Y_t)}{\partial Y_t} > 1$" does not mean that the marginal propensity to consume out of labor income is also above one at aggregate level because $\frac{\partial \frac{1}{N}\sum_{i=1}^N \int_{-\infty}^t \nu e^{-\nu(t-s)} C_{s,t}^i ds}{\partial Y_t}$ is always one by the consumption market clearing condition.

OA.12 Further analysis on consumption risk sharing

In our model, the risk-sharing among the stockholders is limited due to limited market participation. In this section, we analyze the risk-sharing mechanism in a detailed manner. In Panel A of Figure OA.11, we plot the amount of risk (Panel A1) with the **exogenous** inclusion of more investors as in Section OA.8, the stockholders' consumption volatility (Panel A2) and the correlation between stockholders' consumption growth and stock returns (Panel A3). The result shows that as more investors are assumed to be in the market, stockholders' consumption is less volatile and less correlated with stock returns, indicating the stockholders' decreasing exposure to the consumption risk. If only one investor is the stockholder, the investor's marginal utility is highly sensitive to the shocks to the stock price, which is represented by the high amount of consumption risk, consumption volatility, and correlation with stock returns. However, as more investors are imposed to stay in the market, the risk is effectively shared out, decreasing the amount of risk. If every investor is assumed to be a stockholder, then the lowest possible amount of consumption risk is attained.

The amount of consumption risk plotted in Panel A is based on the ascending order of inclusion (h=1,2,...,30) with the risk aversion boundary from 1 to 50. To understand the risk-sharing further, we consider the following variants of the baseline case. In Case 2 (Panel B), the inclusion of investors is first the most risk-tolerant investor ($i = 1$) followed by the most risk-averse investor ($i = 30$) and the second most risk-averse investor ($i = 29$) and so on. In Case 3 (Panel C), the lowest risk aversion is 1.1 ($\gamma_1 = 1.1$). Finally, in Case 4 (Panel D), the highest risk aversion is 10 ($\gamma_N = 10$). Panel B, C, and D of Figure OA.11 show the result. First, Panel B shows that the order of the inclusion does not change the amount of consumption risk. This implies that once the most risk-tolerant investors are in the market, the degree of risk-sharing does not depend on risk aversion of investors who follow the most risk-tolerant investor. Panel C shows that even though the lowest risk aversion marginally changes from 1 to 1.1 ($\gamma_1 = 1.1$), the risk-sharing is ineffective than the baseline case. This is because risk-averse investors are not willing to take the risk and thus their contribution of risk-sharing is lower than risk-tolerant investors. However, the dramatic difference of risk-sharing between the baseline case and the Case 3 is quickly decreasing with the inclusion of more investors. Therefore, the lower bound of risk aversion is important for risk-sharing especially when the market participation rate is low. Finally, Panel D shows that if the highest risk aversion changes from 50 to 10 ($\gamma_N = 10$), the risk-sharing is slightly more effective than the baseline case at each point of the inclusion. This is because investors in Case 4 are more risk-tolerant than investors in the baseline case. Thus, investors in Case 4 are more willing to take the risk and hence their contribution of risk-sharing is high. However, in terms of the magnitude, the amount of consumption risk is virtually identical to the baseline case. This implies that a change in the upper boundary of risk aversion from 50 to 10 does not significantly change the degree of risk-sharing.

To summarize, the improving risk-sharing with the inclusion of investors are represented by decreasing covariance or correlation between stockholders consumption growth and stock return and decreasing stockholders' consumption volatility. Also, the risk aversion of the most tolerant investor (lower boundary of risk aversion) is the most important for the degree of risk-sharing because she is willing to take the risk the most among all investors and this makes it possible to share out the risk effectively.

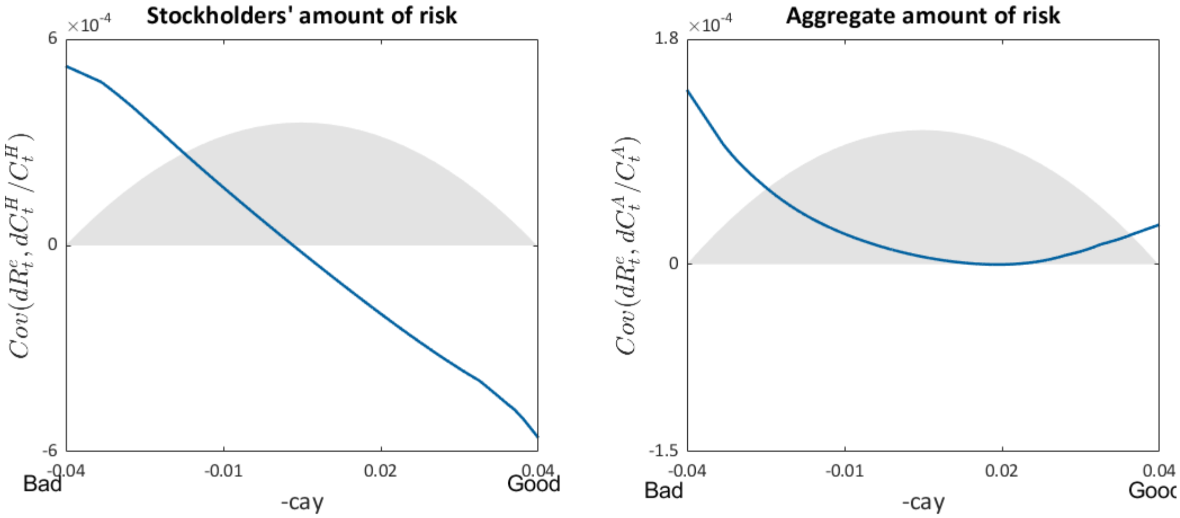

Figure OA.1: **Conditional amount of consumption risk**

This figure plots the empirically estimated conditional covariance of equity returns with stock*h*olders consumption growth $Cov_t(dR_t^e, \frac{d\mathbf{C}_t^H}{\mathbf{C}_t^H})$ (Left) and *a*ggregate consumption growth $Cov_t(dR_t^e, \frac{d\mathbf{C}_t^A}{\mathbf{C}_t^A})$ (Right) using the consumption-wealth ($\widehat{cay}$) by Lettau and Ludvigson (2001). The bold solid lines are the nonparametric estimate of conditional covariance based on the Epanechnikov kernel estimation at monthly frequency. The shaded backgrounds represent the rescaled kernel density of the conditioning variable. The source of aggregate consumption data is the national income and product accounts (NIPA) by the Bureau of economic analysis and that of stockholders' consumption is the consumer expenditure (CEX) by the Bureau of labor statistics from March 1984 to December 2018.

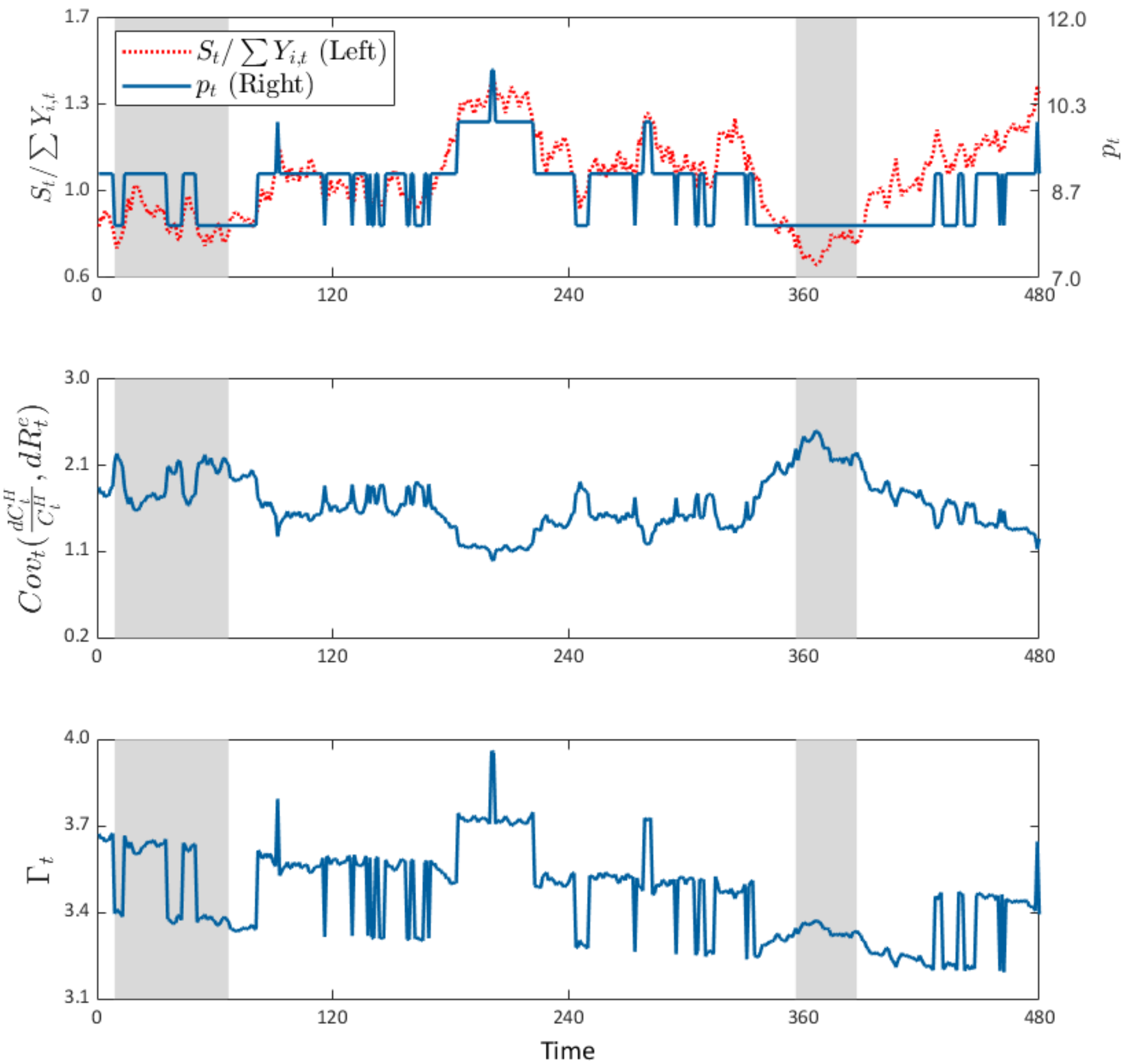

Figure OA.2: **Idiosyncratic labor income setup**

This figure illustrates one sample path of changes in market participation (right y-axis) and the state variable: the stock market wealth-aggregate labor ratio ($\frac{S_t}{\sum Y_{i,t}}$) (left y-axis) in the top figure, the covariance of stock returns with stockholders' consumption growth $Cov_t(dC_t^H/C_t^H, dR_t^e)$ in the middle figure, and the price of risk (stockholders' average risk aversion) in the bottom figure. Those results are based on the idiosyncratic labor income case. The shaded area denotes a recession defined as the lowest 10th percentile of the state variable $\frac{S_t}{\sum Y_{i,t}}$ based on simulated data. This figure is based on the parameters in Table 1 with $\rho_d = 0.43$, $\rho_y = 0.7$.

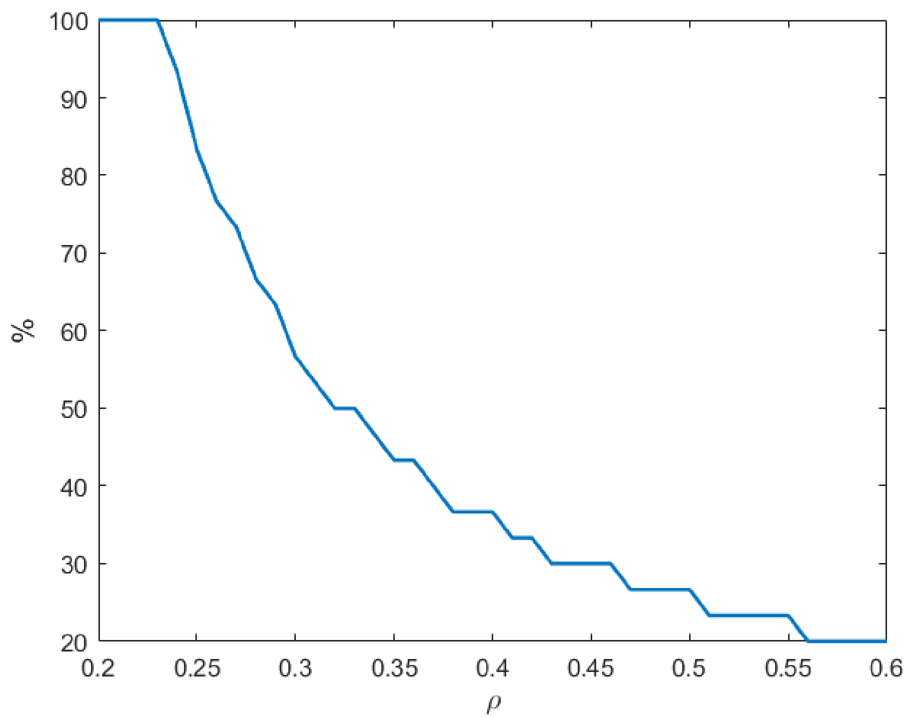

Figure OA.3: **Market participation rate with** ρ

This figure plots market participation rate with different values of correlation between dividend growth and non-financial income growth at $t = 0$. Other parameter values are reported in Table 1.

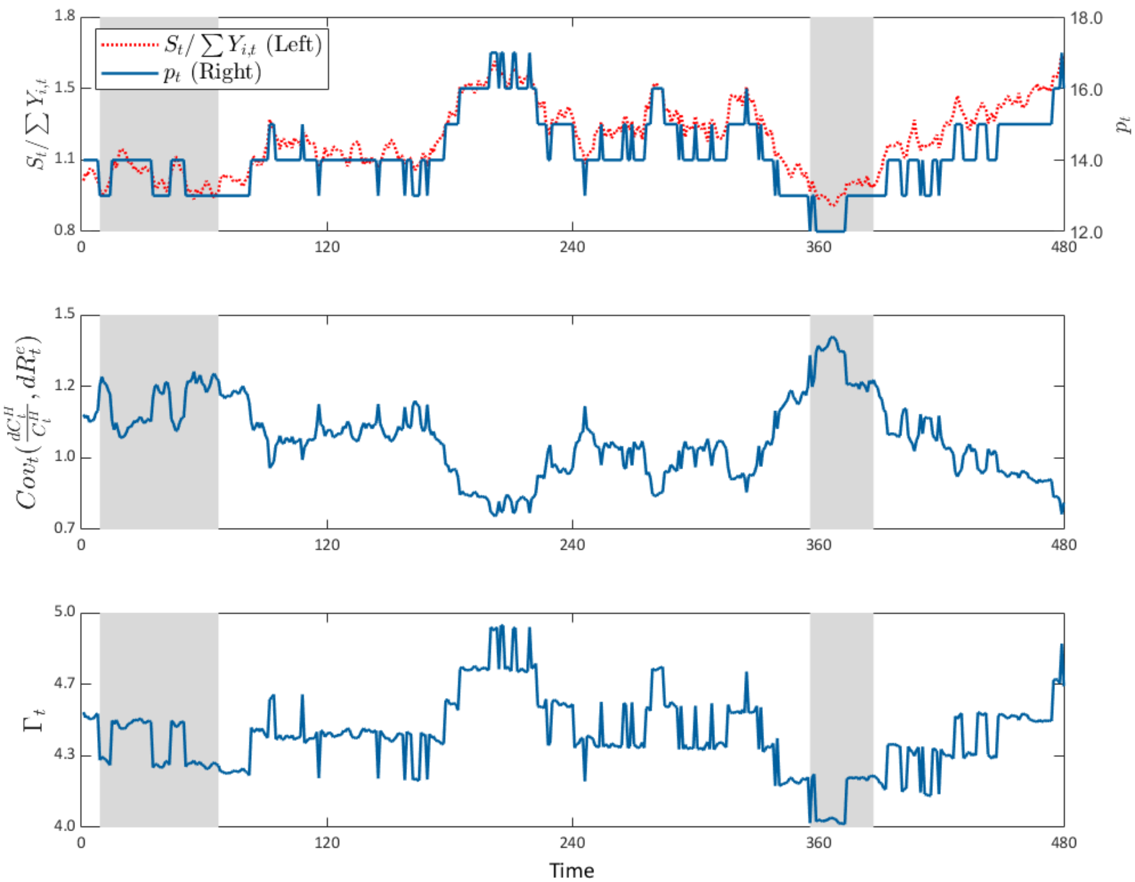

Figure OA.4: **The number of investors equal 40**

This figure illustrates one sample path of changes in market participation (right y-axis) and the state variable: the stock market wealth-aggregate labor ratio ($\frac{S_t}{\sum Y_{i,t}}$) (left y-axis) in the top figure, the covariance of stock returns with stockholders' consumption growth $Cov_t(dC_t^H/C_t^H, dR_t^e)$ in the middle figure, and the price of risk (stockholders' average risk aversion) in the bottom figure. Those results are based on the case for $N = 40$. The shaded area denotes a recession defined as the lowest 10th percentile of the state variable $\frac{S_t}{\sum Y_{i,t}}$ based on simulated data. This figure is based on the parameters in Table 1.

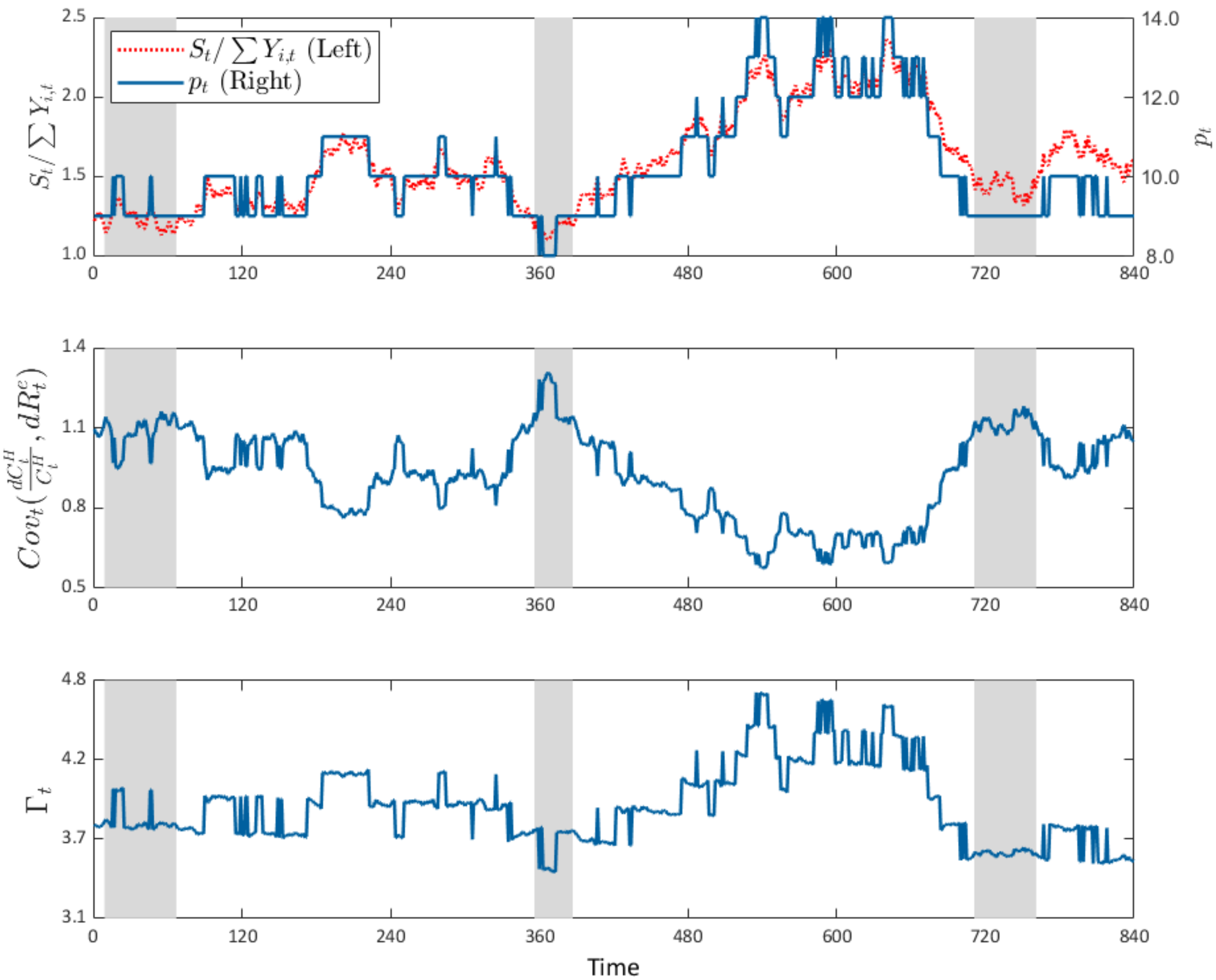

Figure OA.5: **Simulation of 70 years**

This figure illustrates one sample path of changes in market participation (right y-axis) and the state variable: the stock market wealth-aggregate labor ratio ($\frac{S_t}{\sum Y_{i,t}}$) (left y-axis) in the top figure, the covariance of stock returns with stockholders' consumption growth $Cov_t(dC_t^H/C_t^H, dR_t^e)$ in the middle figure, and the price of risk (stockholders' average risk aversion) in the bottom figure. Those results are based on the total simulation horizon of 70 years. The shaded area denotes a recession defined as the lowest 10th percentile of the state variable $\frac{S_t}{\sum Y_{i,t}}$ based on simulated data. This figure is based on the parameters in Table 1.

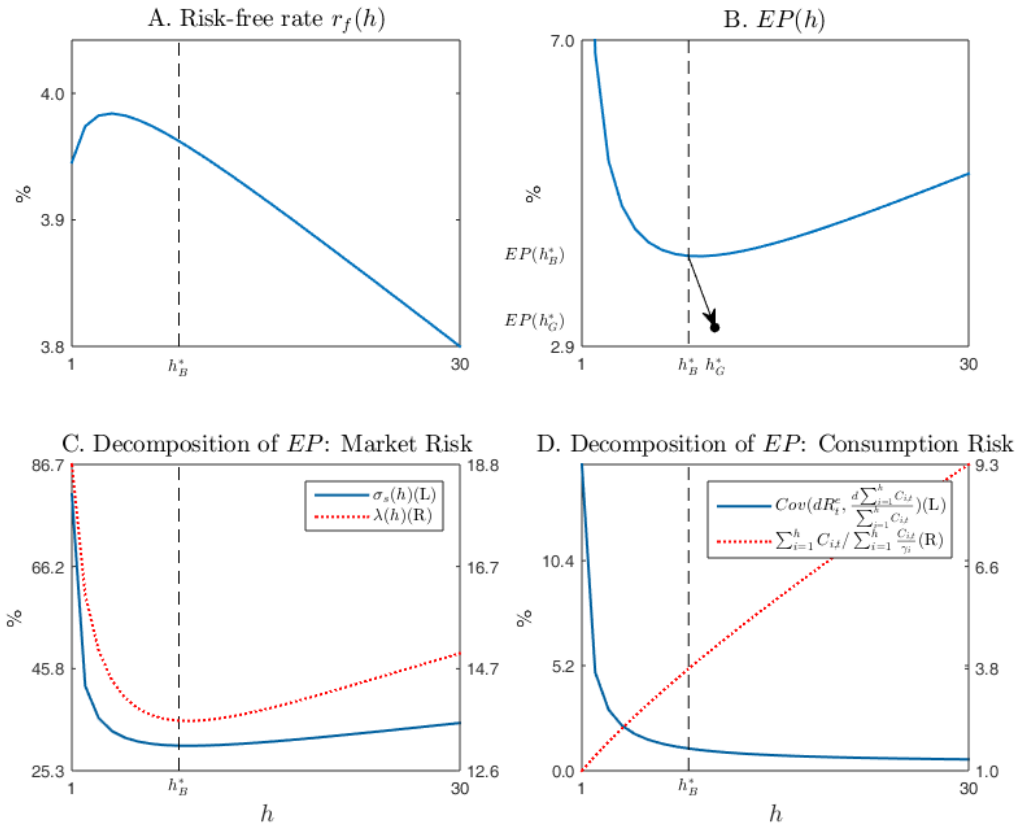

Figure OA.6: **Comparative Statics:** $r_f(h)$, $EP(h)$, $\lambda(h)$, $\sigma_s(h)$, $Cov(h)$, $\Gamma(h)$

This figure plots $r_f, EP, \lambda, \sigma_s$, $Cov(dR_t^e, d\sum_{i=1}^h C_i/\sum_{i=1}^h C_i)$, and $\Gamma \equiv \sum_{i=1}^h C_i/\sum_{i=1}^h \frac{C_i}{\gamma_i}$ as a function of the cut-off stockholder h at the base state. We exogenously include investors to the stock market in a monotonic way from the least risk-averse investor to the most risk-averse investor. That is, the set of stockholders increases as follows: $\{1\}$, $\{1, 2\}$, ..., $\{1, 2, ..., N\}$, as $h = 1, 2, ..., N$. In Panel C, the equity premium is decomposed into the amount of market risk $\sigma_s(h)$ (solid line, left y-axis) and the price of market risk $\lambda(h)$ (dotted line, right y-axis). In Panel D, the equity premium is decomposed into the $Cov(dR_t^e, d\sum_{i=1}^h C_i/\sum_{i=1}^h C_i)$ (solid line, left y-axis), and $\Gamma(h)$ (dotted line, right y-axis). The endogenous cut-off stockholder at the base state h_B^* is 9th stockholder (dashed vertical line). In Panel B, h_G^* denotes the cut-off stockholder at a good state. Parameter values for the simulation are in Table 1.

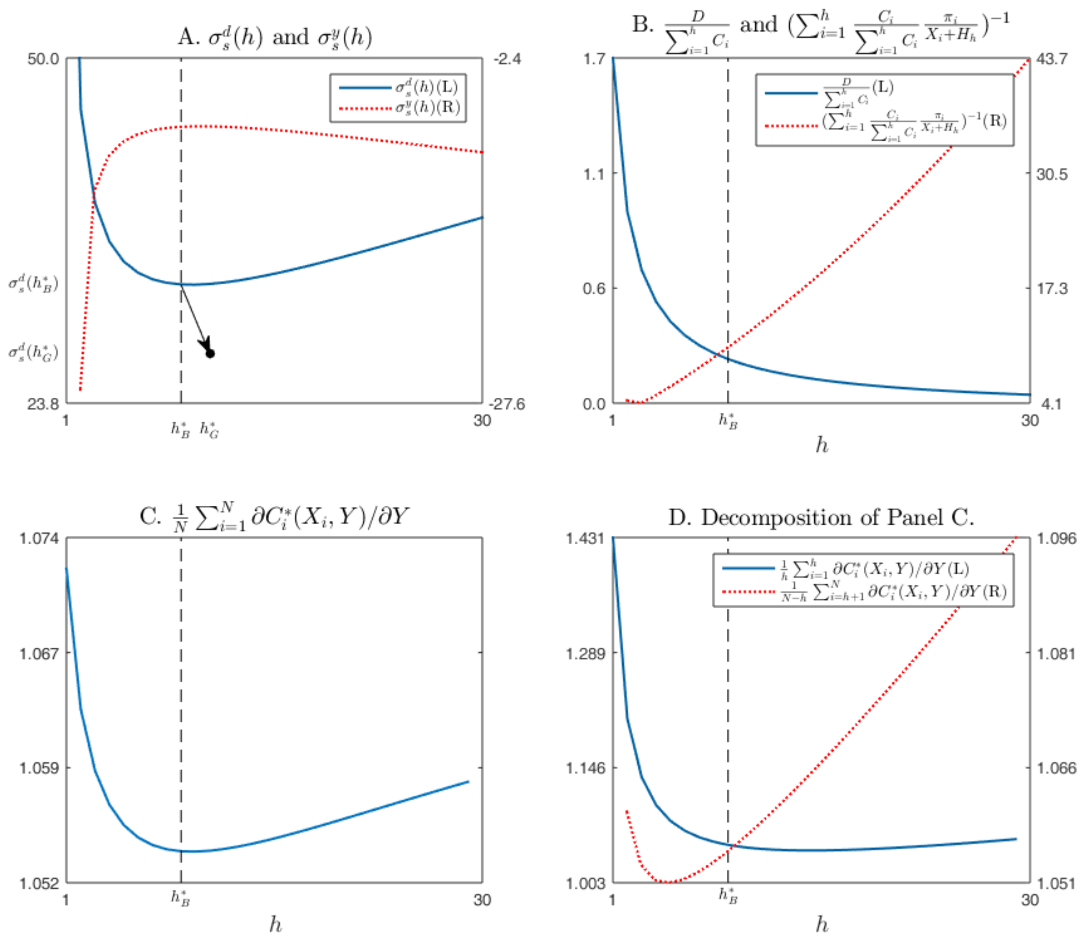

Figure OA.7: **Comparative Statics: Analysis on the stock volatility**

Panel A is σ_s^d (solid line, left y-axis) and σ_s^y (dotted line, right y-axis). Panel B is $\frac{D}{\sum_{i=1}^h C_i}$ (solid line, left y-axis), and $(\sum_{i=1}^h \frac{C_i}{\sum_{i=1}^h C_i} \frac{\pi_i}{X_i+H_h})^{-1}$ (dotted line, right y-axis). Panel C is the average of marginal consumption with respect to labor $\frac{1}{N}\sum_{i=1}^N \partial C_i^*(X_i, Y)/\partial Y$. Panel D is the stockholders' (solid line, left y-axis) and non-stockholders' (dotted line, right y-axis) average of marginal consumption with respect to labor income, respectively, as a function of the cut-off stockholder h at base state. We exogenously include investors to the stock market in a monotonic way from the least risk-averse investor to the most risk-averse investor. That is, the set of stockholders increases as follows: $\{1\}$, $\{1, 2\}$, ..., $\{1, 2, ..., N\}$, as $h = 1, 2, ..., N$. The endogenous cut-off stockholder at the base state h_B^* is 9th stockholder (dashed vertical line). In Panel B, h_G^* denotes the cut-off stockholder at a good state. Parameter values for the simulation are in Table 1.

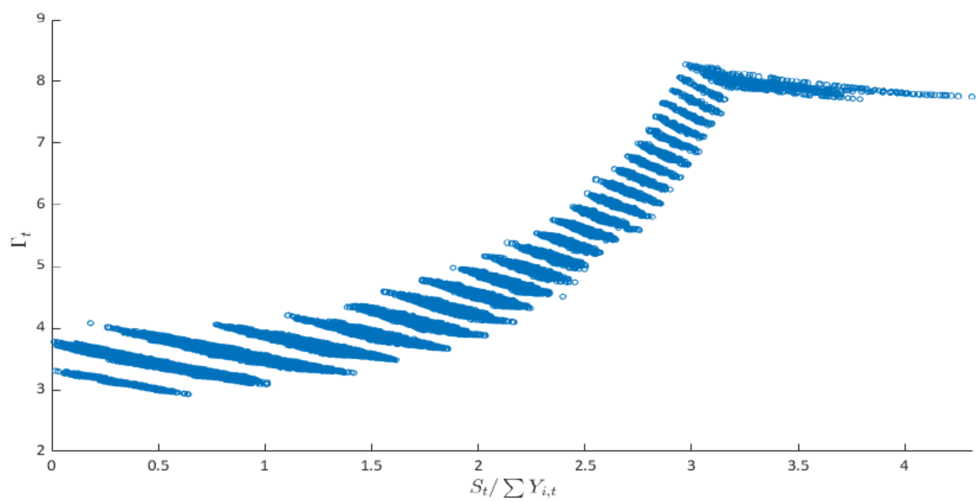

Figure OA.8: **Price of risk**

This figure plots the relationship between the price of risk and the stock market wealth-to-aggregate labor income $S_t / \sum Y_{i,t}$. To generate this, 1,000 sample paths of economy are simulated. Each path consists of 480 monthly observations (40 years), in total 480,000 months.

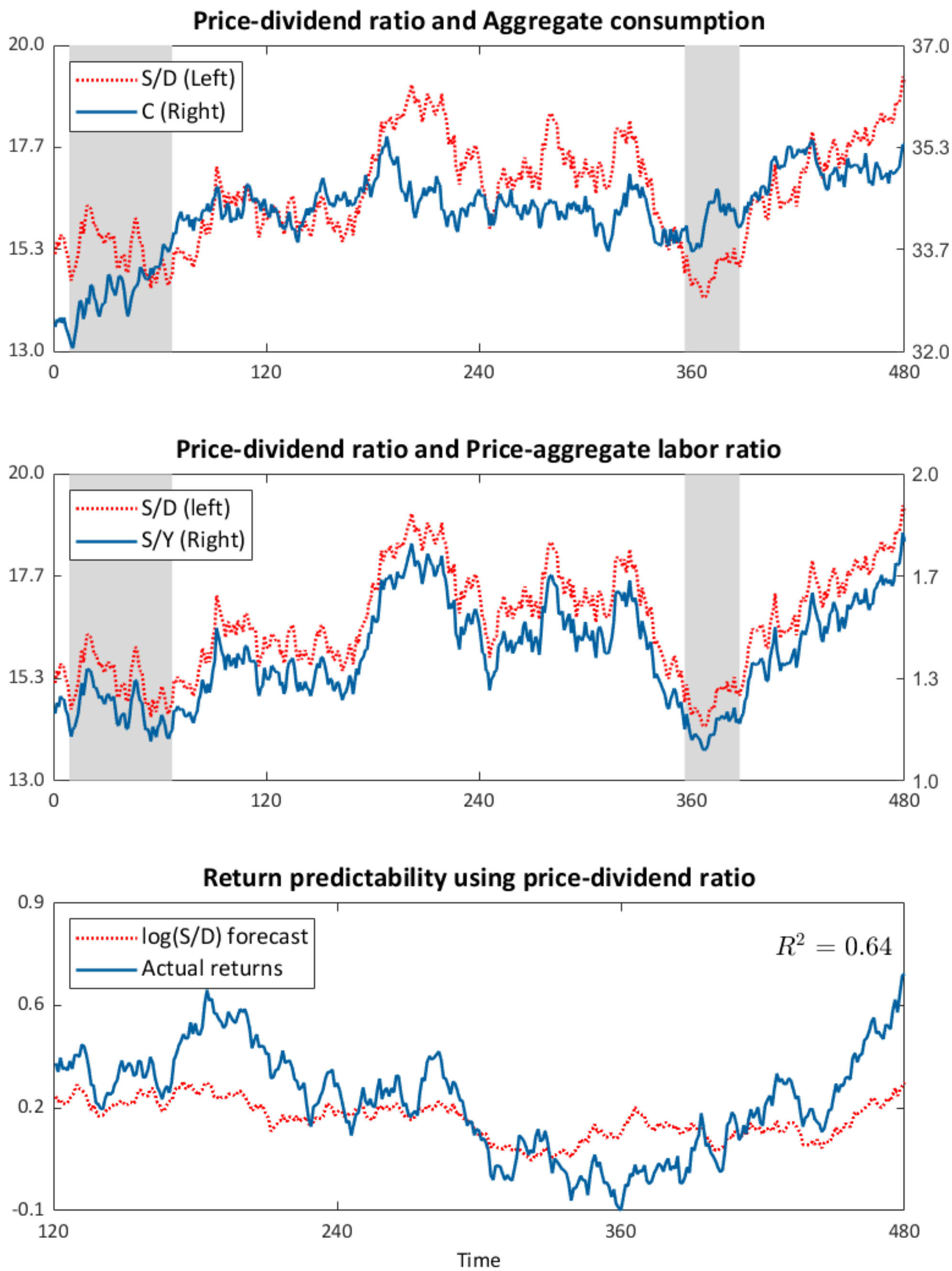

Figure OA.9: **Price-Dividend ratio**

The top (middle) figure is one sample path of the price-dividend ratio and the aggregate consumption (price-aggregate labor ratio). The bottom figure plots one sample path of 10-year cumulative realized excess returns and log price-dividend ratio forecast from the simulated data. Log price-dividend ratio forecast is based on estimates from the forecasting regression: $r^e_{[t\to t+k]} = \alpha + \beta log(\frac{S}{D})_t + \epsilon_{t\to t+k}, \forall k = 10$ years. This regression uses simulated 1,000 sample paths of economy. Parameter values for the simulation are in Table 1.

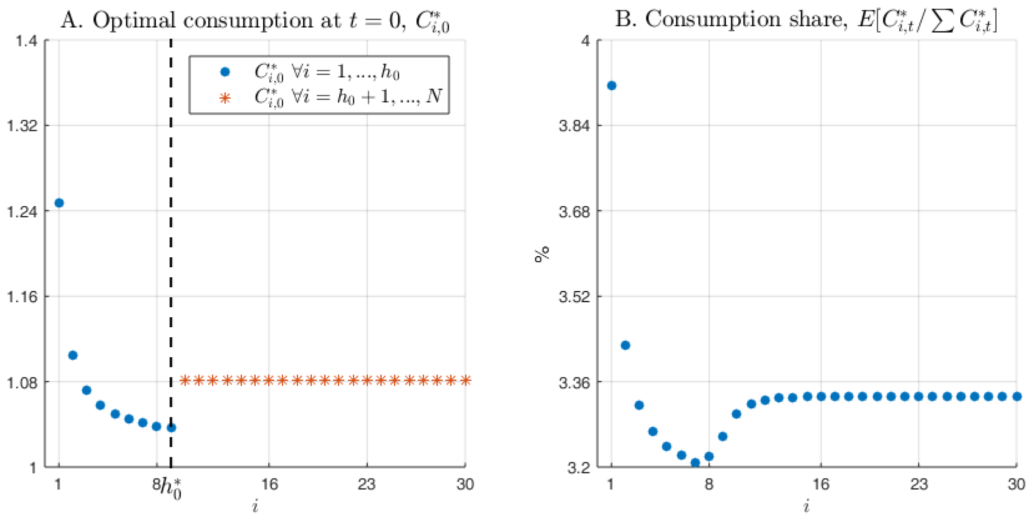

Figure OA.10: **Optimal consumption and consumption share across investors**
Panel A plots the optimal consumption for each investor at time 0 ($t = 0$) in equilibrium. The cut-off stockholder h^*_0 is 9th stockholder (dashed vertical line). Therefore, the stockholders range from the first investor to 9th investor and non-stockholders range from 10th to the last (30th). Panel B plots the unconditional consumption share of each investor. To generate this, 1,000 sample paths of economy are simulated. Each path consists of 480 monthly observations (40 years), in total 480,000 months. For both Panel A and B, parameter values for the simulation are in Table 1. Per capital labor income level is normalized to unity.

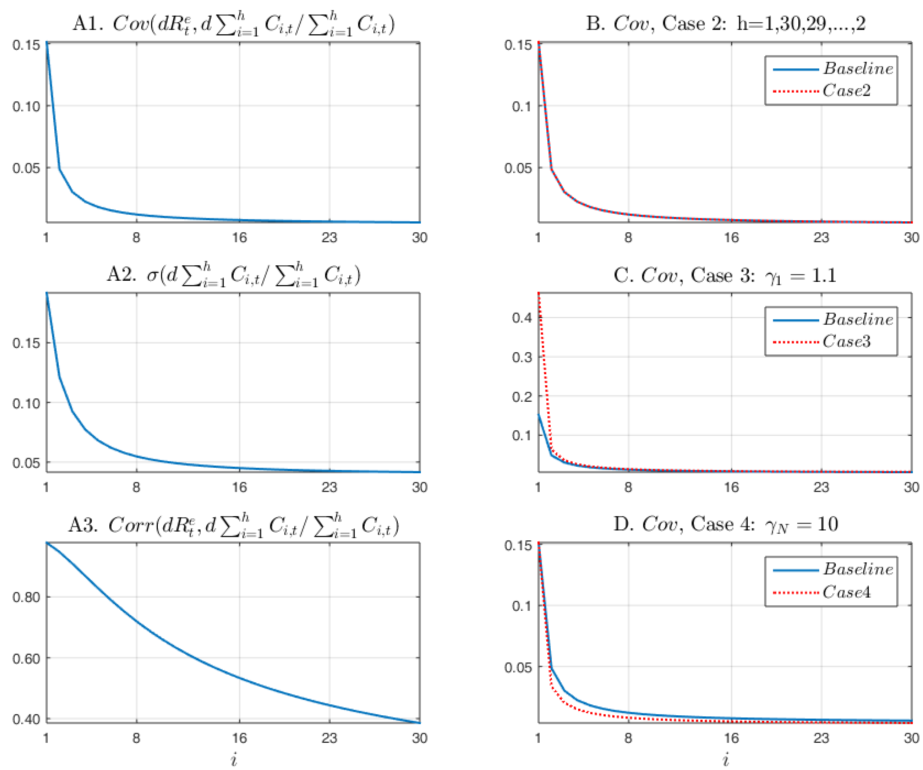

Figure OA.11: **Comparative Statics: Risk-Sharing**

Panel A plots the amount of risk (A1), the stockholders' consumption volatility (A2), and the correlation of stockholders' consumption growth with stock returns (A3) as a function of $h = i$. In this baseline case $\gamma_1 = 1$, $\gamma_N = 50$, and we exogenously include investors to the stock market in a monotonic way from the least risk-averse investor to the most risk-averse investor. That is, the set of stockholders increases as follows: $\{1\}$, $\{1, 2\}$, ..., $\{1, 2, ..., N\}$, as $h = 1, 2, ..., N$. For the Case 2, $\gamma_1 = 1$, $\gamma_N = 50$, the order of inclusion is $h = 1, 30, 29, ..., 2$ (Panel B), For the Case 3, $\gamma_1 = 1.1$, $\gamma_N = 50$, with the ascending order of inclusion (Panel C). For the Case 4, $\gamma_1 = 1$, $\gamma_N = 10$, with the ascending order of inclusion (Panel D). This figure is based on the parameters in Table 1.

Table OA.1: Consumption risk with Return decomposition

Equity returns are decomposed into the dividend growth part dD_t/D_t and non-dividend part of returns $dR_t^e - dD_t/D_t$ for the covariance between equity returns and consumption growth. Panel A reports the result for aggregate consumption and Panel B for stockholders consumption. We report the average level of each component across states and its model-implied dynamics. In doing so, we simulate 1,000 sample paths of the economy. Each path consists of 120 monthly observations (10 years), in total 120,000 months. The state variable is the stock market wealth-aggregate labor income ratio ($\frac{S_t}{Y_t}$). Average values in bad states and good states are reported in brackets [bad good]. The bad (good) states are defined as the lowest (highest) 10% percentiles of the state variable. Parameter values for the simulation are in Table 1. C_t^A denotes the aggregate consumption including both stockholders and non-stockholders' consumption. C_t^H denotes the consumption of aggregate stock*h*olders. **Notations** "Counter": Counter-cyclical; "Pro": Pro-cyclical.

	Model-implied Dynamics	Bad (%)	Good (%)	Average (%)
Panel A: Aggregate consumption				
$Cov_t(\frac{dC_t^A}{C_t^A}, \frac{dD_t}{D_t})$	Pro	0.29	0.31	0.30
$Cov_t(\frac{dC_t^A}{C_t^A}, dR_t^e - \frac{dD_t}{D_t})$	Counter	0.46	0.14	0.26
Panel B: Stockholders' consumption				
$Cov_t(\frac{dC_t^H}{C_t^H}, \frac{dD_t}{D_t})$	Counter	0.62	0.60	0.61
$Cov_t(\frac{dC_t^H}{C_t^H}, dR_t^e - \frac{dD_t}{D_t})$	Counter	1.12	0.40	0.66

Table OA.2: Conditional behavior of the Stock Volatility

Table OA.2 reports the conditional behavior of the parameters associated with the stock volatility. We simulate 1,000 sample paths of the economy. Each path consists of 120 monthly observations (10 years), in total 120,000 months. Parameter values for the simulation are in Table 1. The bad (good) states are defined as the lowest (highest) 10% percentiles of the state variable.

	Model-implied Dynamics	Bad	Good	Average
σ_t	Counter	36.11	22.50	27.44
σ_t/σ_d	Counter	3.01	1.87	2.29
σ_t^d	Counter	37.36	23.95	28.81
σ_t^d/σ_d	Counter	3.11	2.00	2.40
$\frac{D_t}{C_t^H}$	Counter	28.83	27.85	28.22
$\frac{1}{N}\sum_{i\in h_{g,t}}\int_{s\in h_{i,t}}\nu e^{-\nu(t-s)}\frac{C_{s,t}^i}{C_t^H}\frac{\pi_{s,t}^i}{X_{s,t}^i+H_{h,t}}ds$	Pro	10.02	14.53	12.67
σ_t^y	Counter	-3.26	-4.23	-3.82
$\frac{1}{N}\sum_{i=1}^{N}\int_{-\infty}^{t}\nu e^{-\nu(t-s)}\frac{\partial C_{s,t}^i(X_{s,t}^i,Y_t)}{\partial Y_t}$	Counter	1.05	1.04	1.04

Table OA.3: Probit regression of stock ownership and risk Risk appetite

Table OA.3 reports the Probit regression of households stock ownership or households' unwillingness to take financial risks on the observable characteristics. The SCF data from 1989, 1992, 1995, 1998, 2001, 2004, 2007, 2010, and 2013. The first dependent variable takes one if a household has positive holding either in stock (hstocks=1) or mutual funds excluding MMMFs (hnmmf=1) otherwise zero. The second dependent variable takes one if a household reports that they have no tolerance for investment risk otherwise zero. The regressors are age of household (age), age squared (age^2), an indicator for race not being white/Caucasian (race=1), the number of kids (kids), an *highschool* indicator for at least 12 but less than 16 years of education for head of household (educ>11 and educ<16), an *college* indicator for 16 or more years of education (educ >16), the log of real total household income before taxes (income), the log of real dollar amount in checking and savings account (log(checking+saving)) (set to zero if checking and savings = 0), and indicator for checking and savings account = 0, an indicator for dividend income (X5709=1), and year dummies. For the second dependent variable, the log of one plus stock and mutual funds holding amount is also included. Robust standard errors are used for Z-statistic and statistical significance at the 10%, 5%, and 1% levels are denoted by *, **, ***, respectively.

Independent Variable	Dependent Variable: Stock ownership	Dependent Variable: Unwillingness to take risk
age	0.029***	-0.017***
age^2	-1.3×10^{-4}***	3.3×10^{-4}***
$kids$	-0.018***	0.028***
$1_{i\in highschool}$	0.397***	-0.268***
$1_{i\in college}$	1.012***	-0.662***
$1_{i\in nonwhite}$	-0.507***	0.240***
$log(1+chk+saving)$	0.074***	-0.053***
$log(1+income)$	0.262***	-0.097***
$log(1+holding)$	-	-0.074***
1_{1992}	0.044***	0.006
1_{1995}	0.116***	-0.136***
1_{1998}	0.183***	-0.242***
1_{2001}	0.221***	-0.194***
1_{2004}	0.119***	-0.136***
1_{2007}	0.113***	-0.234***
1_{2010}	-0.130***	-0.064***
1_{2013}	-0.204***	-0.133***
1_{2016}	-0.170***	-0.268***
$Cons$	-5.719***	1.795***
Number of Obs.	238,880	238,880
Pseudo R^2	0.299	0.232

Table OA.4: Determinants of entries and exits from SIPP data

Table OA.4 reports the panel regression of either entry or exit on recession, risk aversion, and other characteristics. The sample includes 138,039 respondents covered by the Survey of Income and Program Participation (SIPP) for the 1984, 1985, 1986, 1987, 1990, 1991, 1992, 1993, 1996, 2001, 2004, and 2008 panels. $\text{Entry}_{i,t}$ is a dummy variable that takes the value of 1 if a respondent newly participates in the stock market either directly or indirectly through retirement investment accounts. $\text{Exit}_{i,t}$ is a dummy variable that takes the value of 1 if a respondent exits the stock market. Recession_t is the NBER recession dummy variable. *Wealth* is the sum of stock, mutual fund, bond, saving account, and checking account. *Number of children* is the number of children of a respondent, *Married* is a dummy variable which takes the value of 1 if a respondent is married. *High* and *College* are the dummy variables which take the value of 1 if a respondent's highest grade is high school and college, respectively. T-statistics based on standard errors clustered by year are reported in parentheses, where ***, **, and * indicate significance at the 1%, 5%, and 10% levels, respectively.

Dependent variable:	$\text{Entry}_{i,t}$		$\text{Exit}_{i,t}$	
	(1)	(2)	(3)	(4)
Recession_t	-0.058***	0.298***	0.107***	0.088***
	(-75.63)	(25.09)	(82.61)	(13.92)
$\text{Recession}_t \times \gamma_{i,t}$		-0.568***		0.029**
		(-29.97)		(2.88)
$\gamma_{i,t}$	-0.303***	-0.303***	0.257***	0.257***
	(-8.53)	(-8.53)	(10.95)	(10.95)
$\Delta\text{log(Wealth)}_{i,t}$	0.004***	0.004***	-0.004***	-0.004***
	(7.45)	(7.45)	(-9.60)	(-9.60)
$\Delta\text{log(labor)}_{i,t}$	-0.005***	-0.005***	0.004***	0.004***
	(-6.20)	(-6.20)	(10.62)	(10.62)
Number of $\text{children}_{i,t}$	-0.0002	-0.0002	-0.004***	-0.004***
	(-0.12)	(-0.13)	(-3.97)	(-3.97)
$\text{Married}_{i,t}$	0.0004	0.0004	-0.001	-0.001
	(0.08)	(0.08)	(-0.24)	(-0.24)
$\text{High}_{i,t}$	-0.010**	-0.011**	0.014***	0.014***
	(-2.25)	(-2.26)	(3.37)	(3.37)
$\text{College}_{i,t}$	-0.041***	-0.042***	0.044***	0.044***
	(-4.87)	(-4.87)	(4.83)	(4.83)
$\text{Age}_{i,t}$	-0.012***	-0.012***	0.006***	0.006***
	(-5.00)	(-4.99)	(4.98)	(4.98)
$\text{Age}^2_{i,t}$	0.0001***	0.0001***	-0.0001***	-0.0001***
	(3.64)	(3.63)	(-4.79)	(-4.79)
Individual FE	Yes	Yes	Yes	Yes
Number of Obs.	319,452	319,452	319,452	319,452
Adj. R^2	0.012	0.012	0.035	0.035