

Factor Investing Using Capital Market Assumptions

Redouane Elkamhi, Jacky S. H. Lee, and Marco Salerno

Redouane Elkamhi

is an associate professor of finance in the Rotman School of Management at the University of Toronto in Toronto, Ontario, Canada. redouane.elkamhi@rotman.utoronto.ca

Jacky S. H. Lee

is vice president, total portfolio, at the Healthcare of Ontario Pension Plan Trust Fund in Toronto, Ontario, Canada. jlee5@hoopp.com

Marco Salerno

is a Ph.D. candidate in finance at the Rotman School of Management at the University of Toronto in Toronto, Ontario, Canada. marco.salerno@rotman.utoronto.ca

KEY FINDINGS

- This article presents a methodology to show that CMA returns can be cross-sectionally priced by a small set of underlying macroeconomic factors, which suggests that the CMA's risk and return assumptions follow a factor structure.
- The mean-variance factor allocations generated by CMAs' implied factors are intuitive and stable through time under unconstrained mean-variance optimization.
- This article presents a new approach to building an asset portfolio that respects a desired or target factor allocation with weights that are practical.

ABSTRACT

Capital market assumptions (CMAs), which are long-term risk and return forecasts for asset classes, are important pillars of the investment industry. However, applying them reliably in portfolio construction has been (and still is) a challenge in the industry. This article demonstrates that, despite the difficulties, CMAs are useful for building an investment portfolio using a factor approach. Using a small set of macroeconomic factors, the authors detail a methodology for deriving a factor model from CMAs and then use it to show that (1) these factors price the expected returns from CMAs and (2) the mean-variance factor allocations are substantially more stable than the mean-variance asset portfolios. Furthermore, this article outlines a new approach to building an asset portfolio that respects a desired factor allocation. Overall, this article helps reduce the barrier to entry for factor-based portfolio construction by providing a recipe for building factor models and performing factor-based portfolio construction using publicly available CMAs.

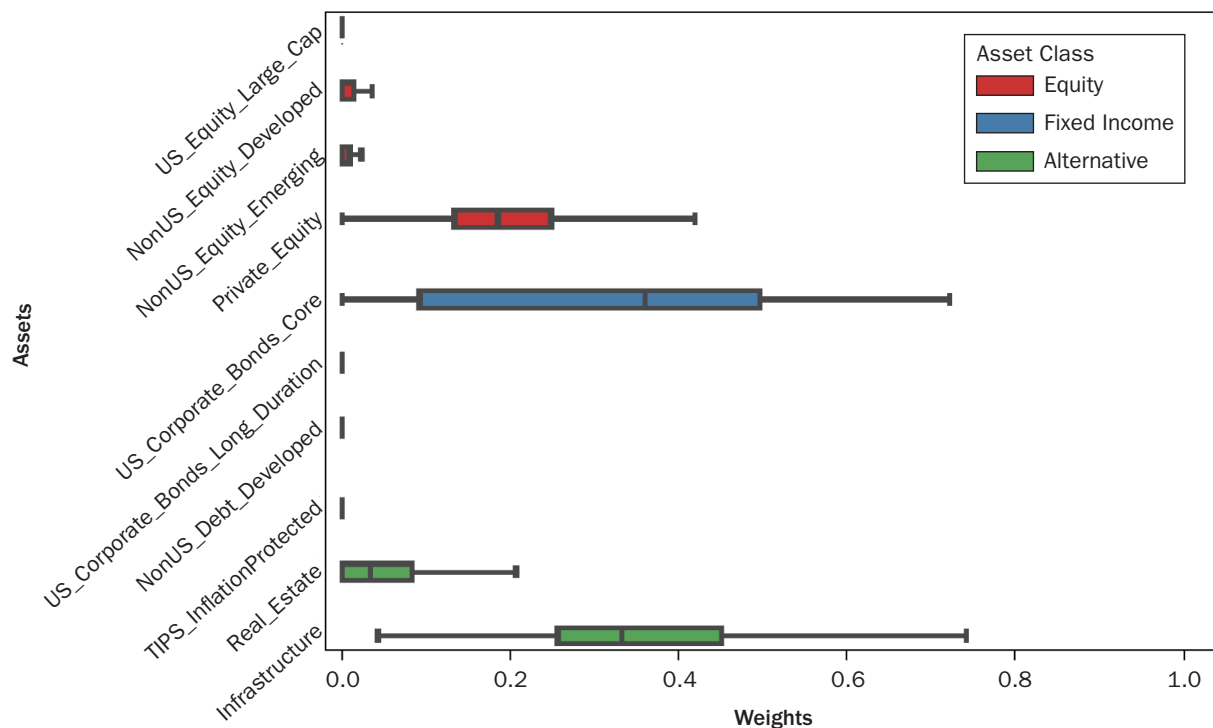
Capital market assumptions (CMAs) are a significant and integral part of the investment industry. CMAs are long-term expected return, volatility, and correlation forecasts for asset classes and serve as the primary inputs to building an investment portfolio. Traditionally, CMAs are used to determine optimal portfolio weights subject to investors' objective functions and constraints. However, using CMAs in a reliable and robust way has been (and still is) a challenge in the industry.

Mean-variance optimization, a celebrated tool from the modern portfolio theory developed by Markowitz (1952), has been difficult to apply on CMAs. Exhibit 1 shows that variations across different CMA forecasts can lead to large variations in mean-variance weights.¹ Investment managers recognize this portfolio optimization

¹Portfolio weights generated from mean-variance optimization are sensitive to small changes in expected return assumptions. This has been studied at length in the literature (e.g., Merton 1980; Michaud 1989; Jagannathan and Ma 2003; DeMiguel, Garlappi, and Uppal 2009). Appendix A provides a description of how we generated Exhibit 1.

EXHIBIT 1

Distribution of Weights Using CMAs



NOTES: This exhibit shows the distribution of mean-variance weights using the 2020 CMA survey produced by Horizon Actuarial Services (HAS) for 1,000 simulations. The HAS report provides the interquartile range of expected returns in the CMA survey samples (see Exhibit 10). We examine how the range of return assumptions affects portfolio optimization using a simulation procedure described in Appendix A. For each asset class, the median weight is shown by a vertical line segment; the colored box shows the interquartile range (25%–75% percentiles). The black lines that extend from the colored box cover 99% of the distribution. We do not allow for short sales for any asset class.

issue (e.g., Aliaga-Diaz et al. 2019; Jacobsen et al. 2019) and attempt to address it by imposing constraints on portfolio weights, enhancing optimization methodologies to account for estimation errors, using ad hoc portfolio construction rules, and even applying a different approach, such as factor- or risk-based portfolio construction techniques, all together.

Factor investing has gained tremendous traction in recent years in the arena of asset allocation. Both the financial literature and the industry have explored the benefits of using factors in portfolio allocation (e.g., Ang 2014; Blyth, Szigety, and Xia 2016; Bass, Gladstone, and Ang 2017; Lorenzen and Jarner 2017; Lawler et al. 2020). These research efforts have motivated the implementation of factor investing by industry giants such as BlackRock and several sophisticated pension funds in Europe and North America. Factors are the quintessential, persistent drivers of asset return. The challenge in applying a factor framework to portfolio construction lies in understanding how factors behave and drive assets' risk and return. This involves the difficult task of building a factor model for each relevant public and private asset class. To date, institutional investors need to build advanced and proprietary models to implement a factor framework. As a result, this limits the benefits of factor investing to resourceful professionals.

This backdrop stimulates the research question of whether we could use publicly available CMAs for factor investing such that it can be used by more investors. Interestingly, a vast majority of CMA reports discuss the importance of macroeconomic factors (e.g., economic growth, real interest rate, and inflation) as the main drivers of assets' return. This provides a hint that hidden factor structures might be used for pricing these assets and therefore in determining these capital market assumptions, whether intentional or not. Therefore, deriving such factor models out of CMAs can be very valuable in meeting the goal of using CMAs for factor-based portfolio allocation.

We begin our investigation by deriving a methodology to assess whether macroeconomic factors—which we define according to economic intuition—can sufficiently price the cross section of CMAs' asset returns. Furthermore, using our methodology, we derive the implied factor loadings for assets, which can be used as a multi-asset factor model. If our factors can price the cross section of CMA asset returns to a large degree, then the performance of the mean–variance factor and asset portfolios would be similar. It follows that those factors would be appropriate for factor portfolio construction.²

Using our methodology, we perform an empirical study by defining three macroeconomic factors—representing economic growth, real rate, and inflation—and a private-specific factor aimed at capturing the private assets' illiquidity premium orthogonal to the market risk premiums. Our macroeconomic factor selection is guided by a combination of industry choices (Bass, Gladstone, and Ang 2017; Lorenzen and Jarner 2017; Jacobsen et al. 2019), empirical evidence (Fama 1981; Lee 1992; Bansal and Shaliastovich 2013; Elkamhi, Lee, and Salerno 2020), and discussions in CMAs reports. The choice to include a private-specific factor is motivated by the literature (Moskowitz and Vissing-Jørgensen 2002; Amihud et al. 2015). Our factors are defined as factor-mimicking portfolios.³ The use of factor-mimicking portfolios is common in practice (Greenberg, Babu, and Ang 2016; Bender, Le Sun, and Thomas 2018) and has the advantage of being well defined, transparent, and tradable.

Using our factor-mimicking portfolios, we compute the assets' factor loadings using the publicly available covariances contained in the CMA reports.⁴ We show that the factor loadings are consistent with common intuition and previous studies (e.g., Cornell 2010). For example, equities present high positive loadings on the economic growth factor, slightly negative loadings on the inflation factor, and no exposure to the real rate factor.

We examine the goodness-of-fit of our factors by observing that they price the cross section of asset returns with generally small errors. This result is encouraging because we define our factor-mimicking portfolios to follow common industry assumptions and existing studies (e.g., Greenberg, Babu, and Ang 2016), rather than by minimizing asset pricing errors.⁵ In a sense, our factor definitions are truly out of sample. This finding suggests that, although the CMA reports provide return forecasts for individual asset classes, such returns are largely internally consistent with a latent priced factor structure.

²The converse is also true: If the factors do not price the cross section of asset returns, they represent uncompensated risks and are not suitable factors for portfolio construction.

³A factor-mimicking portfolio is a portfolio of assets constructed to mimic a factor (e.g., Roll and Srivastava 2018).

⁴We are not aware of an existing methodology to compute factor loadings using CMA covariances.

⁵In this analysis, we are not endorsing our factor definitions as optimal. Rather, we are showing that even when the definitions are simply motivated by economic intuition and common industry choices, they can still price the asset returns well across multiple CMA reports through time.

Using the CMA implied factor risk premiums and covariances, we perform conventional unconstrained mean–variance optimization to evaluate its usefulness in factor portfolio construction. We use mean–variance optimization absent of any methodological refinement and constraint to evaluate the robustness of its solutions. We find that the mean–variance factor weights are intuitive and stable over time. This finding suggests that changes to factor weights over time generally reflect changes in views embedded within the CMAs. Furthermore, the internal consistency between the relative mean–variance factor weights and the relative factor Sharpe ratios are more straightforward to spot. Over the same analysis period, the stability of the mean–variance solutions for factors is in stark contrast to that for assets, which varies significantly.

Armed with the mean–variance factor portfolio, the final step is to build an asset portfolio that respects this set of mean–variance factor weights. A recent and growing literature (e.g., Bergeron, Kritzman, and Sivitsky 2018; Aliaga-Diaz et al. 2020; Kolm and Ritter 2020; Konstantinov, Chorus, and Rebmann 2020) discusses how to bridge the gap between factor exposures and asset allocations. We contribute to this literature by providing a new methodology to build an asset portfolio that respects the desired factor exposures with an available closed-form solution.⁶ An empirical application of our methodology shows that our approach leads to portfolios that are generally practical and stable.

In summary, although CMA reports cannot easily be used directly in mean–variance optimization on individual asset classes, we demonstrate that they can be used for factor-based portfolio construction.⁷ Our comprehensive approach allows investors to use publicly available CMAs to (1) build a factor model, (2) determine the desired factor allocation, and (3) construct an asset portfolio that respects the desired factor exposures.

BUILDING A FACTOR MODEL

This section describes the step-by-step procedure for building the necessary components of a factor model. For a given set of factors, we describe the how to compute the assets' factor loadings and estimate the factor premiums from within the CMAs. We discuss the evaluation of our factor choice by examining the pricing errors in the section “Empirical Application”.

For a given CMA report, we define μ_a and Σ_a as the assets' expected excess return $N \times 1$ vector and the $N \times N$ covariance matrix, respectively. We assume that asset returns follow a linear factor structure. We define M macroeconomic factors and an additional private-specific factor, which represents a portfolio of specific risks from private assets orthogonal to the macroeconomic factors. The factor-mimicking portfolios for the macroeconomic factors are defined as portfolios of assets according to economic intuition and industry best practice. However, determining the mimicking portfolio for the private-specific factor requires several steps, which we discuss in this section.

The following methodology section consists of four subsections: (1) the computation of the macroeconomic factor loadings, (2) the computation of the private-specific factor-mimicking portfolio weights, (3) the combination of macroeconomic and private-specific factors, and (4) the estimation of factor risk premiums.

⁶To the best of our knowledge, there is no well-accepted theory that describes how to optimally translate factor exposures to asset portfolios.

⁷This is discussed by Jacobsen et al. (2019): CMAs are used by practitioners, but they do not influence actual portfolio allocations as much as the CMA providers hope.

Computing the Macroeconomic Factor Loadings

First, we calculate the assets' factor loadings to the macroeconomic factors. We define ω_{mf} as the $N \times M$ matrix that represents the user-defined mimicking portfolio weights for the M macroeconomic factors. We define β_{mf} as the loadings (betas) on the macroeconomic factors for the N assets. The standard approach to estimate β_{mf} is to use a time-series regression of the asset returns on the macroeconomic factor returns. However, the ordinary least square estimates can be formulated using covariance matrixes as

$$\beta_{mf} = \Sigma_a \omega_{mf} (\omega'_{mf} \Sigma_a \omega_{mf})^{-1} \tag{1}$$

where β_{mf} is a $N \times M$ matrix of factor loadings, and Σ_a is the asset covariance matrix, which can be calculated using the correlations and volatilities of assets from the CMA report.⁸ Each element (i, j) of β_{mf} defines the loading of asset i with respect to macroeconomic factor j . We provide the intuition behind Equation 1 in Appendix B.

Computing the Private-Specific Factor-Mimicking Portfolio Weights

Next, we construct the private-specific factor-mimicking portfolio. We define $\hat{\omega}_{pf}$ as an $N \times 1$ vector that represents the user-defined weights on a portfolio of private assets' specific risks orthogonal to the macroeconomic factors. Formally, we can compute the private-specific factor-mimicking portfolio weights ω_{pf} as

$$\omega_{pf} = \hat{\omega}_{pf} - \underbrace{\omega_{mf} \beta'_{mf} \hat{\omega}_{pf}}_{\substack{\text{Exposure to macroeconomic} \\ \text{factors of the private} \\ \text{assets' portfolio}}} \tag{2}$$

where $\hat{\omega}_{pf}$ is the portfolio of private asset classes, and $\omega_{mf} \beta'_{mf} \hat{\omega}_{pf}$ is the public asset portfolio that orthogonalizes the private asset portfolio with respect to the macroeconomic factors.

The aforementioned expression only pertains to the definition of the private-specific factor-mimicking portfolio that is orthogonal to the other macroeconomic factors. As the "Empirical Application" section shows, private assets' classes are exposed to both macroeconomic and the private-specific factors.

Combining Macroeconomic and Private-Specific Factors

We define $\omega_f := [\omega_{mf} \omega_{pf}]$ as the $N \times (M + 1)$ matrix of asset weights for both the macroeconomic (ω_{mf}) and private-specific (ω_{pf}) factor-mimicking portfolios. The factor loadings can be computed using an expression similar to Equation 1; however, we make a slight modification. We define $l := [l_{mf} l_{pf}]$, where l_{mf} is an $N \times M$ matrix of ones, and l_{pf} is an $N \times 1$ vector where an entry is equal to 1 for all private assets and 0 otherwise. The $N \times (M + 1)$ matrix of factor loadings β_f is computed as

$$\beta_f = [\Sigma_a \omega_f (\omega'_f \Sigma_a \omega_f)^{-1}] \circ l \tag{3}$$

The factor loading matrix β_f can be seen to have components β_{mf} and β_{pf} , where $\beta_f = [\beta_{mf} \beta_{pf}]$. β_{mf} is the $N \times M$ loading matrix for the macroeconomic factors, and β_{pf} is the $N \times 1$ loading vector for the private-specific factor. It is worth noting that β_{mf} computed from Equation 3 is equivalent to that computed using Equation 1. By applying

⁸As we show in the "Empirical Application" section, we use the volatilities and correlations from the Horizon Actuarial Capital Market Assumptions reports to compute the assets' covariance matrix Σ_a .

l in Equation 3, we are presetting public assets as not having private-specific factor exposures. Separately, using ω_p , the factor covariance matrix can be computed as

$$\Sigma_f = \omega_f' \Sigma_a \omega_f \quad (4)$$

Estimating the Factor Risk Premiums

We can compute the factor risk premiums in two different ways. The first way involves a simple computation of the factor-mimicking portfolios' expected excess returns

$$\mu_f = \omega_f' \mu_a \quad (5)$$

The second way involves a regression technique developed by Fama and MacBeth (1973).⁹ The Fama–MacBeth regression consists of two steps: (1) an estimation of factor loadings, which we compute with Equation 3, and (2) a cross-sectional regression of assets' expected returns on the estimated factor loadings β_f . This latter regression can be expressed formally as

$$\mu_{a,pub} = c_m + \beta_{mf,pub} \mu_{mf*} + \epsilon \quad (6)$$

$$\mu_{a,priv} - \beta_{mf,priv} \mu_{mf*} = c_p + \beta_{pf,priv} \mu_{pf*} + \eta \quad (7)$$

where μ_{mf*} and μ_{pf*} are the estimated risk premiums (and the regression coefficients) for the macroeconomic factors and the private-specific factor, respectively; $\mu_{a,pub}$ and $\mu_{a,priv}$ are vectors of CMAs' public and private asset returns, respectively; $\beta_{mf,pub}$ and $\beta_{mf,priv}$ are the matrixes of macroeconomic factor loadings for public and private asset classes, respectively; $\beta_{pf,priv}$ is the private factor loading for the private asset classes; c_m and c_p are regression intercepts; and ϵ and η are regression residuals. The preceding expressions imply that only the publicly traded assets are used to calculate the macroeconomic factor risk premiums.

FROM FACTORS TO ASSETS

The previous section provides the factor expected returns (μ_f) and risks (Σ_f). Investors can use this information to determine their desired factor allocation using their preferred allocation rule (e.g., mean–variance, maximum diversification, equal risk contributions, inverse volatility) and obtain a set of weights in the factor space. Given a desired (target) set of factor exposures defined as \bar{w}_F , the next step is to build an asset portfolio that respects this target factor mix. In this section, we provide an approach to build an asset portfolio that also explicitly accounts for the target factor exposures in the optimization.

The number of assets is—almost always—greater than the number of factors in most situations in practice, which implies that there is a large number of asset portfolios with the same set of factor exposures. Therefore, optimizing an asset portfolio that also respects the target factor exposures \bar{w}_F requires additional optimization constraints and/or objectives to make the portfolio selection unique. We contribute on this front by developing an alternative methodology to build an asset portfolio that

⁹The Fama–MacBeth regression method is a standard tool for estimating risk premiums in asset pricing research.

accounts for factor exposures with an available closed-form solution.¹⁰ Our approach is influenced by Greenberg, Babu, and Ang (2016) and Asl and Etula (2012).¹¹

In the following two subsections, we first present the formulation for the portfolio optimization and its solution. We then present the intuition behind the methodology.

Factor-Targeted Asset Allocation Methodology

In our methodology, an investor would trade off between matching the target factor exposures and matching the target asset weights (i.e., portfolio weights based on a chosen optimization method) by optimizing the following objective function:

$$\arg \min_w \underbrace{\gamma (w'\beta_f - \bar{w}'_F)(w'\beta_f - \bar{w}'_F)'}_{\text{Deviations from Target Factor Exposures}} + (1 - \gamma) \underbrace{(w - \bar{w}_A)'(w - \bar{w}_A)}_{\text{Deviations from Target Asset Weights}} \tag{8}$$

$\gamma \in (0, 1)$ is a user-defined parameter that controls the relative importance between the target factor exposures (\bar{w}_F) and the target asset weights (\bar{w}_A). β_f is the matrix of factor beta loadings, and $w'\beta_f$ is the factor exposure implied by the asset weights w from the optimization. Solving Equation 8 yields the following solution:

$$w = [\gamma\beta_f\beta'_f + (1 - \gamma)I_{N \times N}]^{-1}(\gamma\beta_f\bar{w}_F + (1 - \gamma)\bar{w}_A) \tag{9}$$

where $I_{N \times N}$ is an $N \times N$ identity matrix. In the section “Empirical Application”, we elect \bar{w}_F to be the mean–variance tangency factor portfolio and \bar{w}_A to be the inverse volatility asset portfolio. This means that Equation 9 can be further rewritten as

$$w = [\gamma\beta_f\beta'_f + (1 - \gamma)I_{N \times N}]^{-1} \left(\gamma\beta_f \frac{\Sigma_f^{-1}\mu_f}{|\mathbf{1}'_M \Sigma_f^{-1} \mu_f|} + (1 - \gamma) \frac{D^{-1}\mathbf{1}_N}{|\mathbf{1}'_N D^{-1} \mathbf{1}_N|} \right) \tag{10}$$

where $\mathbf{1}'_M$ and $\mathbf{1}'_N$ are vectors of ones with lengths equal to the number of factors (M) and the number of assets (N), respectively, and D is a diagonal matrix with the asset volatilities along its diagonal. Equation 10 represents the closed-form solution used in the “Empirical Application” section.

Intuition behind Our Methodology

Our method is similar in spirit to shrinkage estimators pioneered by Stein (1956). Shrinkage estimators produce an estimate by shrinking the original raw estimate toward a common value. In our methodology, \bar{w}_F serves as that raw estimate, and \bar{w}_A serves as the common value. The intuition for selecting the mean–variance factor and the inverse volatility asset portfolios for Equation 10 are discussed next.

The goal of portfolio optimization is to maximize return for a given level of risk. By applying mean–variance on factors, our methodology looks for the factor mix that harnesses the risk premiums with the highest factor Sharpe ratio for the given μ_f and Σ_f .¹² Therefore, by matching the portfolio factor exposures to the mean–variance

¹⁰Our approach does not require a set of constraints and can be solved in closed form. However, investors wanting to add constraints can modify the optimization described in Equation 8 to allow for any type of constraints. Depending on the type of constraints, closed-form solutions might not be available but the optimization can be performed numerically.

¹¹We are not advocating that our approach supersedes other methodologies in the literature. Rather, we are contributing an additional methodology to the literature.

¹²Obviously, the optimality depends on the reliability of μ_f and Σ_f and the factor identification itself. In the “Empirical Application” section, we show that our overall methodology can determine whether factors are priced. If they are, we can compute reliable and stable mean–variance factor weights.

factor weights, the first term in Equation 8 achieves the mean–variance tangency (i.e., highest Sharpe ratio) portfolio in the factor space. Because the number of assets is greater than the number of factors, many combinations of assets can match the mean–variance factor weights. Therefore, we need an additional criterion to obtain a unique portfolio. With the second term in Equation 8, we elect to focus on a risk-based criterion. We choose the inverse volatility approach over other risk-based approaches (e.g., the global minimum variance portfolio) because (1) it avoids negative allocations to asset classes (negative weights could be undesirable for asset owners), (2) it enforces risk diversification across asset class line items, and (3) it is marginally affected by estimation error because it relies solely on volatility estimates, which are considerably more reliable than correlation or expected returns estimates.¹³

Although it is true that the performance of the inverse volatility portfolio is highly dependent on how investors define the asset class line item, this deficiency is compensated by the first term of the optimization in Equation 8 because it forces the portfolio underlying factor exposures to respect the target factor mix. Thus, our approach balances between having the most risk-diversified asset portfolio and achieving a particular factor exposure mix. This balancing act avoids a risk-diversified portfolio that inadvertently concentrates on a particular factor (i.e., economic growth).¹⁴ Specifically, among all the asset portfolios that have the same factor exposures that are close to the desired factor mix, our approach chooses the one that is closest to the inverse volatility portfolio, thus inheriting its practical properties (i.e., avoiding large negative positions and diversifying based on the volatility of asset classes). Overall, we choose the inverse volatility portfolio because \bar{w}_A should produce practical and robust portfolio weights.

EMPIRICAL APPLICATION

We apply our methodology to the CMA surveys published by Horizon Actuarial Services (HAS) to demonstrate how a factor model can be built from the CMAs in practice. Using the factor model, we build the mean–variance tangency factor portfolio and then construct asset portfolios using our factor-to-asset portfolio construction approach described in the previous section.

HAS provides on their website a history of annual CMA surveys that consolidate CMAs from many participating investment advisors.¹⁵ We use these surveys because they represent market consensus on assets forecasts. We use the 10-year horizon assumptions from the annual surveys published between 2013 and 2020 for our analysis. Before describing the factor-mimicking portfolios, we emphasize that our methodology does not necessarily require investors to come up with estimates of priced factors with external knowledge. Instead, our methodology aims at uncovering the priced factors that are consistent within the CMA reports by computing the internally consistent risk premiums and evaluating the associated pricing errors in expected returns. In other words, we provide a methodology to check whether the candidate

¹³This footnote provides more clarity on the meaning of *line item*. For example, suppose investors are optimizing on 10 asset classes of their choice. This would involve 10 expected return forecasts and a 10-by-10 covariance matrix. Each asset class would be considered a line item in this list of asset classes. What constitutes a line item is arbitrary based on investors' preferences. For the asset allocation of a multi-asset fund, the line items are asset classes. For an active equity manager, the line items are single stocks.

¹⁴As an example, an inverse volatility portfolio of nine equity indexes and only one bond index is likely concentrated with the economic growth factor. Using our approach, this is unlikely to be the outcome if the target factor portfolio has a balanced mix.

¹⁵<https://www.horizonactuarial.com/blog/2020-survey-of-capital-market-assumptions>.

EXHIBIT 2

Factor-Mimicking Portfolio Definition: ω_{mf} and $\hat{\omega}_{pf}$

Asset	ω_{mf}			$\hat{\omega}_{pf}$
	Economic Growth	Real Rate	Inflation	Private Specific
US Equity-Large Cap	0.25			
Non-US Equity-Developed	0.20			
Non-US Equity-Emerging	0.10			
US Corporate Bonds-Core			-1.60	
US Corporate Bonds-Long Duration				
US Corporate Bonds-High Yield				
Non-US Debt-Developed				
Non-US Debt-Emerging				
TIPS (Inflation-Protected)		1.65	1.60	
Commodities	0.10		0.30	
Hedge Funds				
Real Estate				0.40
Infrastructure				0.40
Private Equity				0.40

NOTES: This exhibit reports the definition of the three macroeconomic factors (ω_{mf}) and the private-specific factor ($\hat{\omega}_{pf}$), which are used in Equations 1 and 2. The column $\hat{\omega}_{pf}$ shows the weights used to calculate the private-specific factor orthogonal to the macroeconomic factors in Equation 2.

factors would explain the cross section of the CMA returns. Once satisfied with the results, investors can use those factors for their portfolio allocation exercises.

We build the factor-mimicking portfolios as follows. The economic growth factor is mimicked by a portfolio of equities and commodities.¹⁶ The real rate factor is mimicked by inflation-linked bonds.¹⁷ The inflation factor is mimicked by a portfolio of commodities and a breakeven inflation exposure.¹⁸ Finally, the private-specific factor is defined as a portfolio that is exposed to the risks of private assets (real estate, infrastructure, and private equity) orthogonalized by the macroeconomic factors.¹⁹

Our factor-mimicking portfolio weights are guided by common industry choices. Admittedly, although the mimicking portfolio weights might look arbitrary, we show that this set of factor-mimicking portfolios prices the cross section of asset class returns reasonably well.²⁰ One of the benefits of defining factors exogenously—if they work generally well across different models—is that it allows for consistency between the factor models implemented across different platforms or systems in practice (i.e., different asset allocation or risk management systems in an investment organization).

Exhibit 2 reports the asset universe used in this analysis as well as the weights of the factor-mimicking portfolios, ω_{mf} and $\hat{\omega}_{pf}$. The factor-mimicking portfolio weights

¹⁶These two asset classes are commonly used in the industry to explain the tie between financial assets and economic growth.

¹⁷Owing to the limitation of the assets within the CMA surveys, we can only use US inflation-linked bonds rather than a multicountry basket of inflation-linked bonds.

¹⁸The breakeven inflation exposure is represented by a long position in inflation-linked bonds and a short position in nominal bonds.

¹⁹This is captured by building an equally weighted basket of real estate, infrastructure, and private equity and shorting a basket of public assets that completely offsets the macroeconomic factor exposures of the private assets. This is done using Equation 2.

²⁰As a robustness check, we show in Appendix C that we obtain similar results using an alternative definition of factor-mimicking portfolios that involves more asset classes and is less sparse.

EXHIBIT 3

Factor Covariance Matrixes

Year	σ_{EG}	σ_{RR}	σ_{IF}	σ_{PF}	$\rho_{EG,RR}$	$\rho_{EG,IF}$	$\rho_{RR,IF}$
2013	11.5%	9.7%	10.3%	11.1%	11.8%	24.6%	53.8%
2014	11.1%	10.4%	10.3%	10.8%	14.3%	26.2%	56.4%
2015	11.0%	10.4%	9.9%	9.6%	13.5%	27.0%	50.5%
2016	10.9%	10.7%	10.9%	9.8%	11.2%	25.2%	49.5%
2017	10.5%	10.4%	10.3%	9.9%	15.8%	31.5%	53.2%
2018	10.4%	10.3%	10.2%	9.6%	13.8%	27.8%	50.4%
2019	10.2%	10.1%	9.9%	10.0%	11.6%	24.2%	50.6%
2020	10.2%	10.0%	10.0%	10.8%	12.0%	24.8%	49.8%
Average	10.7%	10.3%	10.2%	10.2%	13.0%	26.4%	51.8%
Std. Dev.	0.5%	0.3%	0.3%	0.6%	1.6%	2.4%	2.4%

NOTES: This exhibit reports the elements of the factor covariance matrixes using capital market assumptions from HAS for the period 2013–2020 as described in Equation 4. The columns σ_{EG} , σ_{RR} , σ_{IF} , and σ_{PF} denote the volatility of the economic growth, real rate, inflation, and private-specific factors, respectively. Correlations between the economic growth (EG), real rate (RR), and inflation (IF) factors are reported in columns $\rho_{EG,RR}$, $\rho_{EG,IF}$, and $\rho_{RR,IF}$, and they are calculated using the covariance matrix defined in Equation 4. Correlations between the private-specific factor and the three macroeconomic factors are zero by construction, and they are omitted from this exhibit. The bottom two rows show the averages and standard deviations of the volatilities and correlations over this time period.

are scaled such that all factors have approximately 10% annualized volatilities over our analysis period. This is done to allow for easier interpretation and comparison.

The following sections are organized as follows. First, we discuss the factor-mimicking portfolios and the assets' loadings with respect to the factors. Second, we show the factor risk premiums computed using Equations 5 and 6. Third, we examine the performance of mean–variance optimization using factors. Finally, we discuss the performance of our factor-to-asset portfolio construction methodology.

CMA Implied Factor Risk Model

Exhibit 3 shows the factor volatilities and pairwise correlations computed using the covariance matrix specified in Equation 4 for each year between 2013 and 2020. The results show that both the factor volatilities and correlations computed using the CMAs are stable over time. For example, the average volatility for the economic growth factor is 10.7% with a standard deviation of just 0.5%; the average correlation between the economic growth factor and real rate factor is 13% with a standard deviation of just 1.6%. These results are expected because these risk assumptions are long term in nature and should not vary much year over year.

We calculate the factor loadings according to Equation 3 for each year between 2013 and 2020. Panels A, B, and C of Exhibit 4 present the loadings with respect to the economic growth factor, real rate factor, and inflation factor, respectively. The rightmost two columns of each panel report the time-series average and standard deviation of the loadings for each asset class. The factor loadings shown in Exhibit 4 are consistent with the expectations of the financial industry (e.g., Podolsky, Johnson, and Jennings 2012) and with the predictions from theory. For example, consistent with the model of Bansal and Shaliastovich (2013), our results show that equities are positively correlated with economic growth shocks and negatively correlated with inflation shocks. Bonds are negatively affected by positive shocks to inflation, which is well documented in the literature (e.g., Ibbotson and Sinquefeld 1976). Panel D presents the private-specific factor loadings for real estate, infrastructure,

EXHIBIT 4

CMA Implied Factor Loadings

Assets\Years	2013	2014	2015	2016	2017	2018	2019	2020	Avg	Std
Panel A: Economic Growth Loadings										
US Equity-Large Cap	1.53	1.52	1.52	1.50	1.53	1.52	1.51	1.52	1.52	0.01
Non-US Equity-Developed	1.71	1.72	1.71	1.73	1.73	1.74	1.71	1.71	1.72	0.01
Non-US Equity-Emerging	2.13	2.10	2.13	2.13	2.11	2.09	2.12	2.10	2.11	0.01
US Corporate Bonds-Core	0.12	0.12	0.12	0.12	0.12	0.12	0.13	0.13	0.12	0.00
US Corporate Bonds-Long Duration	0.36	0.26	0.24	0.20	0.20	0.19	0.21	0.20	0.23	0.05
US Corporate Bonds-High Yield	0.74	0.71	0.72	0.71	0.70	0.66	0.67	0.66	0.70	0.03
Non-US Debt-Developed	0.21	0.21	0.18	0.16	0.18	0.17	0.22	0.16	0.19	0.03
Non-US Debt-Emerging	0.68	0.62	0.67	0.69	0.74	0.70	0.68	0.62	0.67	0.04
TIPS (Inflation-Protected)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Commodities	0.62	0.65	0.64	0.66	0.61	0.63	0.68	0.67	0.65	0.02
Hedge Funds	0.57	0.54	0.55	0.54	0.53	0.55	0.57	0.54	0.55	0.02
Real Estate	0.33	0.46	0.47	0.50	0.58	0.58	0.72	0.89	0.56	0.17
Infrastructure	0.85	0.69	0.67	0.75	0.78	0.80	0.77	0.84	0.77	0.06
Private Equity	1.69	1.69	1.77	1.68	1.62	1.59	1.65	1.57	1.66	0.06
Panel B: Real Rate Loadings										
US Equity-Large Cap	0.09	0.06	0.04	0.02	0.02	0.00	0.02	0.03	0.03	0.02
Non-US Equity-Developed	0.00	0.03	0.04	0.04	0.04	0.04	0.05	0.05	0.04	0.02
Non-US Equity-Emerging	0.01	0.10	0.06	0.04	0.11	0.11	0.12	0.09	0.08	0.04
US Corporate Bonds-Core	0.56	0.55	0.56	0.57	0.56	0.57	0.56	0.56	0.56	0.01
US Corporate Bonds-Long Duration	1.10	1.03	0.97	0.93	0.92	0.92	0.88	0.88	0.95	0.08
US Corporate Bonds-High Yield	0.41	0.33	0.30	0.31	0.30	0.31	0.32	0.28	0.32	0.04
Non-US Debt-Developed	0.47	0.45	0.45	0.44	0.42	0.41	0.41	0.38	0.43	0.03
Non-US Debt-Emerging	0.59	0.45	0.45	0.45	0.49	0.47	0.45	0.44	0.47	0.05
TIPS (Inflation-Protected)	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.61	0.00
Commodities	-0.22	-0.30	-0.24	-0.19	-0.25	-0.19	-0.27	-0.27	-0.24	0.04
Hedge Funds	0.09	0.04	0.01	0.02	0.04	0.02	0.05	0.02	0.04	0.03
Real Estate	-0.04	0.03	0.10	0.09	0.05	0.06	0.15	0.28	0.09	0.09
Infrastructure	0.24	0.31	0.19	0.19	0.21	0.21	0.18	0.23	0.22	0.04
Private Equity	-0.18	-0.26	-0.33	-0.22	-0.16	-0.12	-0.17	-0.16	-0.20	0.06
Panel C: Inflation Loadings										
US Equity-Large Cap	-0.36	-0.34	-0.41	-0.33	-0.32	-0.27	-0.27	-0.27	-0.32	0.05
Non-US Equity-Developed	-0.16	-0.15	-0.15	-0.19	-0.19	-0.19	-0.21	-0.20	-0.18	0.02
Non-US Equity-Emerging	0.01	-0.03	0.10	0.08	0.02	-0.03	-0.06	-0.08	0.00	0.06
US Corporate Bonds-Core	-0.40	-0.41	-0.40	-0.42	-0.41	-0.42	-0.41	-0.41	-0.41	0.01
US Corporate Bonds-Long Duration	-0.85	-0.80	-0.74	-0.68	-0.72	-0.70	-0.73	-0.72	-0.74	0.05
US Corporate Bonds-High Yield	-0.28	-0.30	-0.26	-0.22	-0.22	-0.18	-0.25	-0.20	-0.24	0.04
Non-US Debt-Developed	-0.20	-0.24	-0.20	-0.19	-0.22	-0.17	-0.20	-0.17	-0.20	0.02
Non-US Debt-Emerging	-0.40	-0.31	-0.23	-0.20	-0.27	-0.24	-0.27	-0.24	-0.27	0.06
TIPS (Inflation-Protected)	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Commodities	1.20	1.17	1.21	1.12	1.16	1.09	1.16	1.15	1.16	0.04
Hedge Funds	0.00	0.04	0.04	0.01	-0.01	0.00	-0.02	-0.02	0.01	0.03
Real Estate	0.15	0.01	0.01	0.07	0.01	-0.02	-0.03	-0.21	0.00	0.10
Infrastructure	-0.08	-0.23	-0.18	-0.14	-0.17	-0.12	-0.18	-0.09	-0.15	0.05
Private Equity	-0.10	-0.03	-0.26	-0.12	-0.10	-0.02	-0.04	-0.03	-0.09	0.07
Panel D: Private-Specific Loadings										
Real Estate	0.62	0.72	0.86	0.94	0.84	0.82	0.88	0.86	0.82	0.10
Infrastructure	0.57	0.58	0.56	0.53	0.66	0.65	0.70	0.63	0.61	0.06
Private Equity	1.30	1.19	1.08	1.03	1.00	1.03	0.92	1.02	1.07	0.12

NOTES: This exhibit reports the factor loadings of the individual asset classes with respect to the four factors. For each year, we calculate the factor loadings according to Equation 3. Panels A, B, C, and D present the loadings to the economic growth, real rate, inflation, and private-specific factors, respectively. The rightmost two columns present the time-series averages and standard deviations of the factor loadings.

EXHIBIT 5

Macroeconomic Factor Risk Premiums

Year	Economic Growth		Real Rate		Inflation		Intercept
	μ_f	μ_{mf^*}	μ_f	μ_{mf^*}	μ_f	μ_{mf^*}	c_m
2013	3.6%	3.2%	1.0%	0.1%	0.3%	0.2%	0.7%
2014	3.1%	2.9%	1.6%	1.2%	0.3%	0.2%	0.4%
2015	3.1%	2.9%	1.4%	0.8%	0.0%	0.1%	0.5%
2016	2.9%	3.0%	1.1%	1.2%	-0.4%	-0.1%	0.2%
2017	2.7%	2.8%	1.0%	1.3%	-0.1%	0.3%	-0.1%
2018	2.4%	2.6%	0.7%	1.1%	-0.3%	0.3%	-0.3%
2019	2.3%	2.5%	0.7%	1.1%	-0.4%	0.0%	-0.2%
2020	3.0%	3.0%	0.7%	0.6%	-0.5%	-0.3%	0.1%
Average	2.9%	2.9%	1.0%	0.9%	-0.1%	0.1%	0.2%
Std. Dev.	0.4%	0.2%	0.3%	0.4%	0.3%	0.2%	0.3%

NOTES: This exhibit reports the risk premiums for the economic growth, real rate, and inflation factors calculated using two different methods. For each macroeconomic factor, we present (1) μ_f , which is the vector of excess returns calculated using the definition of factor-mimicking portfolios with Equation 5, and (2) μ_{mf^*} , which is the vector of excess returns implied by the Fama–MacBeth cross-sectional regression with Equation 6. The intercept values of the Fama–MacBeth regression are reported in the column c_m . Hedge funds, real estate, infrastructure, and private equity are not used in the calculation of the macroeconomic factor risk premiums.

and private equity. By examining the time-series standard deviations, Exhibit 4 also shows that the factor loadings are stable through time.

CMA Implied Factor Risk Premiums

In this section, we study the implied risk premiums for our factors. For each macroeconomic factor, we report two measures of factor risk premiums: (1) the excess returns μ_f calculated as the expected returns of the factor-mimicking portfolios using Equation 5, and (2) μ_{mf^*} , the estimated risk premiums by running a Fama–MacBeth cross-sectional regression using Equation 6. Exhibit 5 shows that for all three macroeconomic factors, μ_f and μ_{mf^*} are similar on average. This result suggests that these factors can price and explain the expected returns in CMAs. Our findings are encouraging because the factor-mimicking portfolios are built with an intent to represent macroeconomic risk drivers and are not specifically designed to generate risk premiums to match μ_f .

To further investigate the observation that the estimated risk premiums are similar to the factor portfolio excess returns μ_f , we examine the pricing errors. Exhibit 6 shows the pricing errors calculated for each year from 2013 to 2020. We define a pricing error as the absolute difference between an asset's excess return from CMAs and the excess return predicted using either of the two measures of factor risk premiums reported in Exhibit 5 and the factor loadings presented in Exhibit 4.

The pricing errors are generally small relative to the average excess return for the various asset classes. For example, the average CMA excess return of US equity large cap is 4.42%, whereas the average pricing errors are 0.10% and 0.13% with standard deviations of 0.06% and 0.08% for the two measures of risk premiums, respectively. The other asset classes all display pricing errors below 1.00%.

For the private-specific factor, we follow a different procedure. The private-specific factor-mimicking portfolio excess returns are significant (an average of 2.8% per year between 2013 and 2020). However, there are only three assets (real estate, infrastructure, and private equity) to reliably conduct the two-step cross-sectional

EXHIBIT 6

Asset Pricing Errors

Assets\Years	2013	2014	2015	2016	2017	2018	2019	2020	Average	Std. Dev.	Average μ_a
Panel A: Using Excess Returns from Cross-Sectional Regression (Equation 6)											
US Equity-Large Cap	0.08%	0.04%	0.16%	0.16%	0.03%	0.06%	0.26%	0.23%	0.13%	0.08%	4.42%
Non-US Equity-Developed	0.18%	0.21%	0.23%	0.37%	0.08%	0.06%	0.00%	0.14%	0.16%	0.11%	4.96%
Non-US Equity-Emerging	0.27%	0.14%	0.17%	0.16%	0.29%	0.18%	0.17%	0.26%	0.21%	0.06%	6.10%
US Corporate Bonds-Core	0.04%	0.07%	0.06%	0.03%	0.18%	0.33%	0.22%	0.05%	0.12%	0.10%	1.08%
US Corporate Bonds-Long Duration	0.16%	0.00%	0.18%	0.21%	0.04%	0.22%	0.38%	0.38%	0.20%	0.13%	1.42%
US Corporate Bonds-High Yield	0.70%	0.49%	0.77%	1.10%	0.60%	0.55%	0.62%	0.97%	0.73%	0.20%	3.15%
Non-US Debt-Developed	0.65%	0.78%	1.00%	0.88%	0.95%	0.87%	0.90%	1.09%	0.89%	0.12%	0.17%
Non-US Debt-Emerging	0.55%	0.76%	0.96%	0.86%	0.52%	0.50%	0.93%	1.23%	0.79%	0.24%	3.28%
TIPS (Inflation-Protected)	0.14%	0.12%	0.08%	0.23%	0.07%	0.00%	0.02%	0.11%	0.10%	0.07%	0.62%
Commodities	0.11%	0.10%	0.03%	0.01%	0.10%	0.02%	0.03%	0.05%	0.06%	0.04%	1.94%
Panel B: Using Excess Returns from Factor-Mimicking Portfolios (Equation 5)											
US Equity-Large Cap	0.09%	0.05%	0.05%	0.08%	0.05%	0.16%	0.24%	0.11%	0.10%	0.06%	4.42%
Non-US Equity-Developed	0.19%	0.17%	0.15%	0.23%	0.08%	0.05%	0.09%	0.01%	0.12%	0.07%	4.96%
Non-US Equity-Emerging	0.47%	0.22%	0.17%	0.08%	0.15%	0.04%	0.10%	0.09%	0.17%	0.13%	6.10%
US Corporate Bonds-Core	0.12%	0.08%	0.11%	0.11%	0.08%	0.09%	0.06%	0.07%	0.09%	0.02%	1.08%
US Corporate Bonds-Long Duration	0.54%	0.08%	0.43%	0.20%	0.14%	0.46%	0.55%	0.44%	0.36%	0.17%	1.42%
US Corporate Bonds-High Yield	0.73%	0.61%	0.88%	1.23%	0.56%	0.45%	0.56%	1.07%	0.76%	0.26%	3.15%
Non-US Debt-Developed	0.46%	0.61%	0.87%	0.73%	1.00%	1.03%	0.99%	1.00%	0.84%	0.20%	0.17%
Non-US Debt-Emerging	0.45%	0.85%	0.99%	1.01%	0.52%	0.45%	0.91%	1.31%	0.81%	0.29%	3.28%
TIPS (Inflation-Protected)	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.62%
Commodities	0.64%	0.43%	0.60%	0.59%	0.43%	0.46%	0.33%	0.38%	0.48%	0.11%	1.94%

NOTES: This exhibit reports the absolute pricing errors between the surveys' excess returns and excess returns computed using two different methods: (1) Panel A uses the forecasted returns computed using Equation 6, and (2) Panel B uses the excess returns calculated from the factor-mimicking portfolios with Equation 5. The absolute pricing errors as well as their averages and standard deviations are reported for all publicly traded assets. The rightmost column provides the averages of the surveys' excess returns to allow for an assessment of the relative magnitude of the errors.

regression given the CMA surveys. Thus, we perform another small-scale study using data from JP Morgan's 2020 CMA report, which involves more private assets.²¹ We find evidence that all four of our factors are priced by using the same factor-mimicking portfolio weights and conducting the same examinations.²²

Our findings provide visible evidence that our factors can explain the cross section of expected returns in capital market assumptions by and large. This suggests that the forecasters surveyed by HAS likely follow—deliberately or not—a factor structure in developing their return assumptions, which span a dimensional space that can also be spanned by our factors. This finding suggests that investors can derive a priced factor model out of CMAs, as we demonstrated. In the following section, we evaluate the usefulness of these factors in portfolio construction.

²¹The JP Morgan CMAs report is different from the HAS assumptions. The JP Morgan report is available at <https://am.jpmorgan.com/ca/en/asset-management/institutional/insights/portfolio-insights/lcma/>.

²²In untabulated results, we apply our methodology to all public asset classes and the seven private asset classes reported in JP Morgan's 2020 LTCMAs report: Private Equity, US Core Real Estate, US Value-Added Real Estate, European ex-UK Core Real Estate, Asia Pacific Core Real Estate, Global Infrastructure Equity, and Global Infrastructure Debt. We find that all four factors—including the private specific—are priced well using the cross-sectional regressions on asset class returns with the same factor-mimicking portfolios.

Using Mean–Variance Optimization on Assets versus Factors

As we highlighted in the introduction, mean–variance asset portfolios are often sensitive to expected return inputs. In this section, we examine whether this is the case for factors as well by comparing the reasonableness and stability of the mean–variance weights for assets and factors derived from CMA reports.

Panel A of Exhibit 7 shows the mean–variance tangency (maximum Sharpe ratio) asset portfolio weights from 2013 to 2020 using HAS’s CMA surveys.²³ We use the mean–variance tangency portfolio for this analysis because many institutional investors can use leverage to move the capital market line up and down to target the desired risk level while building the best possible portfolio. Panel A of Exhibit 7 clearly shows that these mean–variance portfolios have large positive and negative weights. In addition, weights for several assets are volatile through time. For instance, the allocations to US large-cap equity and non-US corporate bonds (non-US debt developed) range from –32% to –4% and from –74% to –10% over the 2013–2020 period, respectively. Overall, their weights have large standard deviations of 10% and 22%, respectively, over this same period.

In Panel B, we repeat the same exercise for our four factors. We use Equations 4 and 5 as inputs to calculate the mean–variance tangency factor weights for the same 2013–2020 period.²⁴ The results in Panel B are in stark contrast to those in Panel A. Factor weights are reasonable and stable over time: The weights for the economic growth, real rate, and inflation factors have averages of 53%, 28%, and –32%, respectively. But more importantly, they are stable and have standard deviations of only 6%, 3%, and 5%, respectively.

Our results from Panel A and B suggest that although the CMA reports from HAS are not practical when used directly in asset mean–variance optimization, they are practical for factor portfolio construction. Specifically, if one can estimate reasonably well—as shown earlier—the hidden factor structure behind the CMA reports, then a mean–variance factor optimization not only is justified theoretically, but it also leads to practical and stable weights that can be used in practice by investment managers. The final step is to leverage this insight to build an asset portfolio.

From Factor Exposures to Asset Portfolio in Practice

To build the asset portfolio using the mean–variance factor weights with Equation 10, we set \bar{w}_F to be the factor weights shown in Panel B and set \bar{w}_A to be the inverse volatility portfolio weights. Panel C of Exhibit 7 shows the portfolio weights using our factor-targeted asset allocation approach computed with the closed-form solution provided in Equation 10.

We show that our approach leads to much more stable weights when compared to the mean–variance asset weights (Panel A). Specifically, the asset weights computed by our approach do not exhibit large short positions. Comparing the standard deviations of weights across Panels A and C provides a stark contrast in stability. For instance, the weight standard deviations range from 2% to as high as 23% for the mean–variance asset portfolios. Using our approach, the asset weight standard deviations range from 1% to 3%.

In Panel C of Exhibit 7, we use $\gamma = 0.7$ to illustrate a specific trade-off between the mean–variance factor and the inverse volatility asset weights. In Exhibit 8, we

²³The mean–variance tangency asset weights are calculated as $(\Sigma_a^{-1}\mu_a)/|\mathbf{1}'\Sigma_a^{-1}\mu_a|$, where $\mathbf{1}$ is a vector of ones with the same dimension as μ_a . This is the maximum Sharpe ratio portfolio with weights normalized to one.

²⁴The mean–variance tangency factor weights are calculated as $(\Sigma_r^{-1}\mu_r)/|\mathbf{1}'\Sigma_r^{-1}\mu_r|$, where $\mathbf{1}$ is a vector of ones with the same dimension as μ_r . The weights sum to one.

EXHIBIT 7**Portfolio Weights under Optimization**

Assets\Years	2013	2014	2015	2016	2017	2018	2019	2020	Avg	Std
Panel A: Unconstrained Mean-Variance Optimization on Assets										
US Equity-Large Cap	-0.04	-0.05	-0.12	-0.18	-0.17	-0.32	-0.30	-0.25	-0.18	0.10
Non-US Equity-Developed	0.05	0.03	0.01	-0.04	0.06	0.07	0.04	0.12	0.04	0.04
Non-US Equity-Emerging	-0.09	-0.11	-0.07	-0.05	-0.07	-0.06	-0.07	-0.11	-0.08	0.02
US Corporate Bonds-Core	0.59	0.50	0.78	0.80	0.81	1.09	1.14	1.13	0.85	0.23
US Corporate Bonds-Long Duration	-0.16	-0.15	-0.22	-0.26	-0.08	-0.19	-0.33	-0.32	-0.21	0.08
US Corporate Bonds-High Yield	-0.02	-0.05	-0.02	0.12	-0.02	-0.04	-0.06	0.08	0.00	0.06
Non-US Debt-Developed	-0.10	-0.13	-0.23	-0.21	-0.49	-0.74	-0.46	-0.61	-0.37	0.22
Non-US Debt-Emerging	0.13	0.20	0.16	0.20	0.22	0.20	0.28	0.34	0.22	0.06
TIPS (Inflation-Protected)	-0.15	-0.02	-0.09	-0.18	-0.17	-0.17	-0.18	-0.26	-0.15	0.07
Commodities	-0.03	-0.05	-0.05	-0.07	-0.06	-0.08	-0.08	-0.09	-0.06	0.02
Hedge Funds	0.47	0.44	0.44	0.44	0.43	0.58	0.49	0.52	0.48	0.05
Real Estate	0.27	0.15	0.17	0.16	0.20	0.19	0.06	0.06	0.16	0.07
Infrastructure	0.07	0.22	0.19	0.17	0.24	0.32	0.28	0.26	0.22	0.07
Private Equity	0.00	0.01	0.05	0.09	0.12	0.15	0.20	0.12	0.09	0.06
Panel B: Unconstrained Mean-Variance Optimization on Factors										
Economic Growth	0.52	0.46	0.47	0.51	0.54	0.54	0.55	0.65	0.53	0.06
Real Rate	0.26	0.35	0.29	0.28	0.26	0.23	0.28	0.26	0.28	0.03
Inflation	-0.22	-0.29	-0.30	-0.34	-0.33	-0.34	-0.37	-0.40	-0.32	0.05
Private-Specific	0.45	0.47	0.54	0.54	0.53	0.57	0.55	0.48	0.52	0.04
Panel C: Portfolio Optimization Using Our Factor-to-Asset Methodology with $\gamma = 0.7$										
US Equity-Large Cap	0.02	0.00	0.00	0.03	0.02	0.03	0.02	0.04	0.02	0.01
Non-US Equity-Developed	0.00	-0.02	-0.02	0.00	0.00	0.00	0.00	0.02	0.00	0.01
Non-US Equity-Emerging	-0.03	-0.05	-0.06	-0.06	-0.05	-0.05	-0.06	-0.03	-0.05	0.01
US Corporate Bonds-Core	0.15	0.16	0.15	0.14	0.14	0.13	0.14	0.13	0.14	0.01
US Corporate Bonds-Long Duration	0.05	0.09	0.08	0.09	0.07	0.07	0.08	0.07	0.07	0.01
US Corporate Bonds-High Yield	0.04	0.05	0.05	0.05	0.05	0.04	0.06	0.06	0.05	0.01
Non-US Debt-Developed	0.08	0.10	0.10	0.10	0.09	0.09	0.08	0.09	0.09	0.01
Non-US Debt-Emerging	0.04	0.06	0.04	0.04	0.03	0.03	0.04	0.04	0.04	0.01
TIPS (Inflation-Protected)	0.11	0.12	0.12	0.09	0.08	0.06	0.08	0.06	0.09	0.02
Commodities	-0.03	-0.04	-0.04	-0.09	-0.07	-0.11	0.10	-0.11	-0.07	0.03
Hedge Funds	0.07	0.07	0.07	0.07	0.08	0.08	0.07	0.08	0.07	0.01
Real Estate	0.14	0.16	0.20	0.20	0.19	0.20	0.19	0.17	0.18	0.02
Infrastructure	0.10	0.14	0.15	0.13	0.15	0.15	0.17	0.12	0.14	0.02
Private Equity	0.18	0.17	0.19	0.19	0.19	0.21	0.18	0.19	0.19	0.01

NOTES: Panel A reports the mean-variance tangency asset weights under unconstrained mean-variance optimization. The rightmost two columns show the average and standard deviation of the weights for each asset over the 2013–2020 time period. Panel B shows the mean-variance tangency portfolio weights for the four factors. Panel C shows the asset weights when we apply our factor-to-asset portfolio construction methodology using Panel B as the target factor weights. The factors' excess returns are calculated according to Equation 5, and the factor covariance matrixes are calculated according to Equation 4. The rightmost two columns report the averages and standard deviations of the factor weights over the time period.

report the change in portfolio weights as a function of γ . Panel A of Exhibit 8 reports the portfolio weights using Equation 10, and Panels B and C report the implied factor exposures and Sharpe ratios, respectively, for the portfolio weights in Panel A. We use the expected returns and covariances from the 2020 HAS CMA report for this exhibit.

When γ approaches zero, the asset weights converge to the inverse volatility portfolio, in which weights are proportional to the inverse of asset volatilities. When γ approaches one, the implied portfolio factor weights converge to the target

EXHIBIT 8**Effect of Parameter γ on Our Factor-to-Asset Methodology****Panel A: Asset Weights for Various γ**

Asset\(γ	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
US Equity-Large Cap	0.05	0.05	0.05	0.05	0.04	0.04	0.04	0.04	0.04	0.05	0.05
Non-US Equity-Developed	0.04	0.04	0.04	0.03	0.03	0.03	0.02	0.02	0.02	0.02	0.02
Non-US Equity-Emerging	0.03	0.03	0.02	0.01	0.00	-0.01	-0.02	-0.03	-0.03	-0.04	-0.04
US Corporate Bonds-Core	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.13	0.13	0.12	0.12
US Corporate Bonds-Long Duration	0.08	0.08	0.08	0.08	0.08	0.08	0.08	0.07	0.07	0.06	0.06
US Corporate Bonds-High Yield	0.08	0.08	0.08	0.07	0.07	0.07	0.06	0.06	0.05	0.05	0.05
Non-US Debt-Developed	0.11	0.11	0.11	0.10	0.10	0.10	0.09	0.09	0.08	0.07	0.07
Non-US Debt-Emerging	0.07	0.07	0.07	0.06	0.06	0.05	0.04	0.04	0.03	0.02	0.02
TIPS (Inflation-Protected)	0.13	0.12	0.11	0.10	0.09	0.08	0.07	0.06	0.04	0.02	0.01
Commodities	0.04	0.02	0.00	-0.03	-0.05	-0.07	-0.09	-0.11	-0.14	-0.16	-0.18
Hedge Funds	0.10	0.10	0.09	0.09	0.09	0.09	0.08	0.08	0.08	0.08	0.08
Real Estate	0.05	0.08	0.10	0.11	0.13	0.14	0.16	0.17	0.17	0.18	0.19
Infrastructure	0.06	0.07	0.09	0.10	0.10	0.11	0.12	0.12	0.13	0.13	0.13
Private Equity	0.04	0.07	0.09	0.12	0.14	0.15	0.17	0.19	0.21	0.22	0.23

Panel B: Factor Exposure Implied by the Asset Weights

Factor\(γ	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
Economic Growth	0.60	0.67	0.68	0.68	0.68	0.67	0.67	0.67	0.66	0.66	0.65
Real Rate	0.34	0.35	0.36	0.35	0.35	0.34	0.33	0.31	0.30	0.28	0.26
Inflation	-0.16	-0.20	-0.23	-0.25	-0.27	-0.30	-0.32	-0.34	-0.36	-0.38	-0.40
Private-Specific	0.12	0.18	0.23	0.28	0.31	0.35	0.38	0.41	0.44	0.46	0.48

Panel C: Sharpe Ratios

γ	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	0.99
Sharpe Ratio	36.8%	38.4%	39.4%	40.2%	40.7%	41.0%	41.1%	41.0%	40.8%	40.4%	40.0%

NOTES: This exhibit shows the effect that the parameter γ has on the asset weights, factor exposures, and Sharpe ratios with our factor-to-asset methodology. For any given γ , Panel A shows the asset weights calculated according to our factor-to-asset methodology (i.e., Equation 9). Panels B and C show, respectively, the factor exposures and the Sharpe ratios implied by the asset weights shown in Panel A. In this exhibit, we use expected returns and risks for the year 2020 reported in the HAS report.

factor exposures. It is worth directly comparing the portfolio weights computed using the 2020 CMA report shown in Panel A of Exhibit 7 with the weights obtained using $\gamma=0.99$ (Panel A of Exhibit 8). In both cases, the factor exposures are identical (growth: 0.65; real rate: 0.26; inflation: -0.40; private specific: 0.48), but the assets weights are dramatically different; with little doubt, most investors would find the portfolios shown in Panel A of Exhibit 8 to be more practical.

Exhibit 9 compares the Sharpe ratios between the mean-variance asset portfolios and portfolios constructed using our factor-targeted asset allocation approach. The Sharpe ratios are calculated using two sets of assets' excess returns: (1) the averages of the CMA's excess returns over the 2013-2020 period, which we label μ_1 in Exhibit 9, and (2) the average of the excess returns computed using Equations 6 and 7 over the 2013-2020 period, which we label μ_2 in Exhibit 9.

The results show that our approach generates similar Sharpe ratios across the different years and, importantly, across different return assumptions (μ_1 and μ_2), suggesting that the Sharpe ratios generated from our approach are more robust against changes to expected return inputs than mean-variance portfolios would be. Conversely, the mean-variance portfolio Sharpe ratios exhibit more variability across years, but more critically, they exhibit relatively large differences across the two sets

EXHIBIT 9

Sharpe Ratio Comparisons

Panel A: 2013–2020 Average Assets' Excess Returns

Assets	μ_1	μ_2
US Equity-Large Cap	4.42%	4.52%
Non-US Equity-Developed	4.96%	5.12%
Non-US Equity-Emerging	6.10%	6.31%
US Corporate Bonds-Core	1.08%	0.98%
US Corporate Bonds-Long Duration	1.42%	1.62%
US Corporate Bonds-High Yield	3.15%	2.42%
Non-US Debt-Developed	0.17%	1.06%
Non-US Debt-Emerging	3.28%	2.49%
TIPS (Inflation-Protected)	0.62%	0.72%
Commodities	1.94%	1.90%
Hedge Funds	3.18%	1.77%
Real Estate	4.01%	4.03%
Infrastructure	4.73%	4.72%
Private Equity	7.00%	6.89%

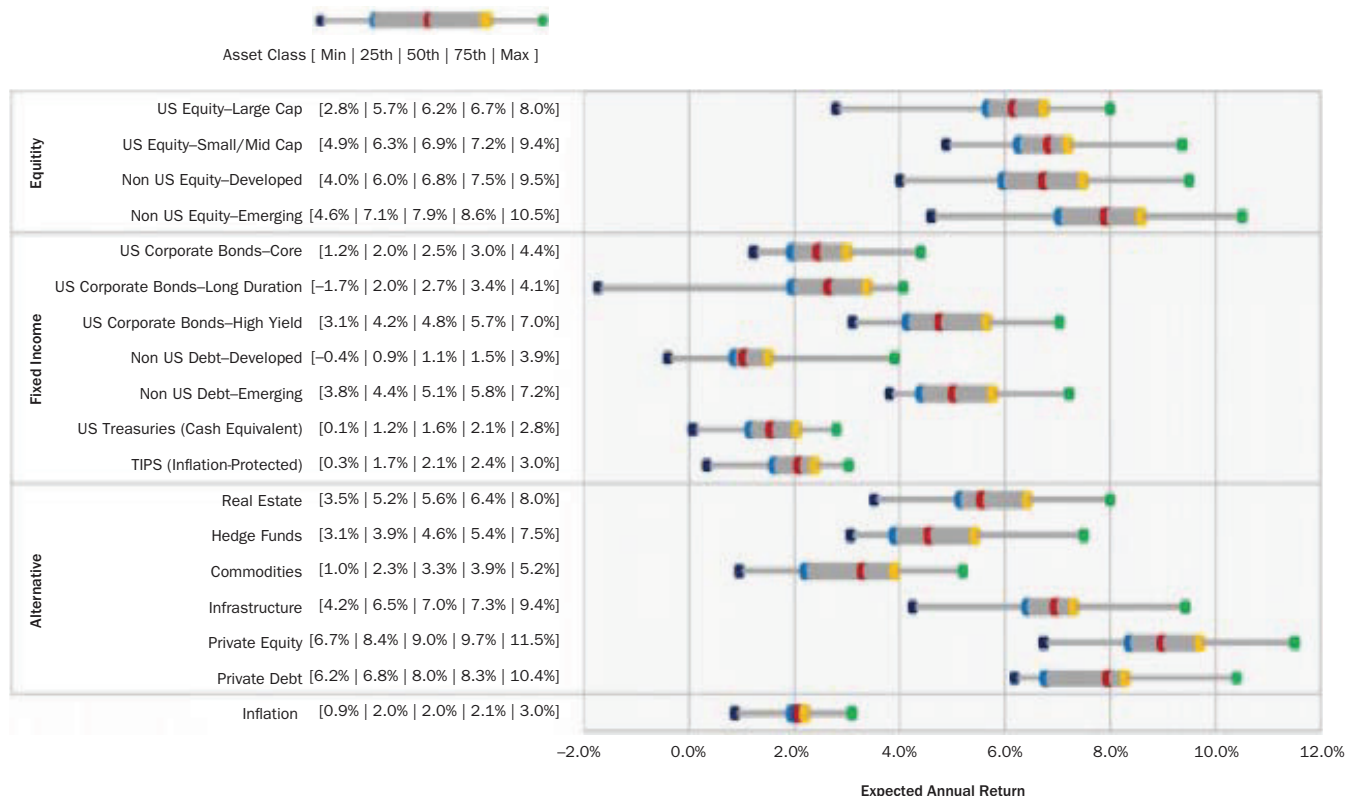
Panel B: Portfolio Sharpe Ratios (SR) Using Different Assets' Excess Returns

Portfolio Weights Based on CMA Report Year	Asset Mean–Variance			Our Factor-to-Asset Approach		
	SR(μ_1)	SR(μ_2)	Abs. Difference	SR(μ_1)	SR(μ_2)	Abs. Difference
2013	47.3%	32.6%	14.7%	40.0%	38.7%	1.3%
2014	49.6%	34.5%	15.1%	40.3%	38.9%	1.4%
2015	51.1%	33.6%	17.5%	40.4%	39.1%	1.3%
2016	50.4%	32.2%	18.3%	40.2%	38.9%	1.3%
2017	50.8%	34.9%	15.8%	40.6%	39.3%	1.3%
2018	50.0%	31.3%	18.7%	40.2%	38.9%	1.2%
2019	49.2%	30.8%	18.4%	40.7%	39.3%	1.3%
2020	49.5%	29.3%	20.2%	39.8%	38.4%	1.4%
Average	49.7%	32.4%	17.3%	40.3%	38.9%	1.3%
Std. Dev.	1.2%	1.9%	1.9%	0.3%	0.3%	0.1%

NOTES: Panel A reports the average CMA assets' excess returns over the 2013–2020 period. Column μ_1 contains the excess returns computed from the asset returns and the risk-free rate in CMA reports directly. Column μ_2 contains the excess returns computed using the cross-sectional regression (Equation 6). Panel B contains the portfolio Sharpe ratios obtained using two different methodologies: (1) the asset mean–variance portfolios shown in Panel A of Exhibit 7 and (2) the portfolios obtained from our factor-to-asset methodology shown in Panel C of Exhibit 7. For each of these two methods and each year, we compute the portfolio Sharpe ratios using μ_1 (column SR(μ_1)) and μ_2 (column SR(μ_2)). The absolute difference between the Sharpe ratios computed using μ_1 and μ_2 is provided in the abs. difference column. The bottom two rows show the averages and standard deviations for the Sharpe ratios and the absolute differences. The assets' covariance matrix used for Panel B is the average of the covariance matrixes from 2013 to 2020.

of expected return assumptions, highlighting the high sensitivity of mean–variance portfolio Sharpe ratios to expected return inputs. This point is particularly important because CMA forecasts across different providers often have wide ranges (see Exhibit 10), and this underscores the significance of using a robust portfolio allocation method in practice because any one of these CMA forecasts can be used.

EXHIBIT 10
Distribution of CMA Return Forecasts from Different Providers



NOTES: This exhibit shows the distribution of the 2020 CMA return forecasts with a 10-year horizon from different providers. For each asset class, the exhibit shows the dispersion of the return forecasts from different providers. This survey is available at https://www.horizonactuarial.com/uploads/3/0/4/9/30499196/rpt_cma_survey_2020_v0716.pdf.

Overall, the results demonstrate that our factor-targeted asset allocation approach generates practical weights that respect the desired target factor allocation when applied on capital market assumptions.

CONCLUSION

CMA reports often discuss the economic environment in terms of economic growth, interest rate, inflation, and a handful of other macro variables. Our results suggest that when forecasters develop their asset class assumptions, they embed—knowingly or not—a factor structure in determining the drivers of returns.

We show that four factors—economic growth, real rate, inflation, and private specific—are priced well in the cross section of CMA assets’ expected returns. We demonstrate that although using the CMA forecasts of individual asset classes directly in mean–variance optimization leads to undesirable portfolio weights, the mean–variance factor portfolios are stable and in line with industry intuition. Subsequently, to make use of this factor portfolio, we provide a portfolio construction methodology to build an asset portfolio that respects the desired factor mix. Our methodology not only respects the desired factor exposures but also generates practical weights.

In summary, our research provides a comprehensive methodology to (1) build a factor model for a given set of factors using CMAs, (2) validate that this set of factors can price the cross section of CMAs' expected returns, and (3) build an asset portfolio that respects the desired factor allocation.

APPENDIX A

SIMULATION: DISTRIBUTION OF WEIGHTS USING CMAS

We provide a description of the simulation methodology used to create Exhibit 1. We use the distribution of 10-year geometric returns from the 2020 HAS CMA report. The report provides, for each asset class, the minimum and maximum and the 25th, 50th, and 75th percentiles of expected returns from the survey participants (see Exhibit 10). We also use the standard deviations and correlations from the report for our analysis. Our goal is to draw the expected returns from a distribution consistent with the expected return percentiles described earlier and calculate the mean–variance optimal weights. We perform 1,000 simulations and plot the distribution of the weights in Exhibit 1. We keep the covariance matrix fixed for our simulation.

Little is known about the distribution of the expected returns; therefore, we proceed as follows. We assume that the expected returns follow a joint multivariate normal distribution with a covariance matrix computed from the standard deviations and correlations from the CMA report. For each asset class, the mean of the distribution is equal to the 50th percentile expected return. The standard deviation is calibrated such that there is a 0.5% probability of being below the minimum or above the maximum value. We also need to make an assumption on the correlation between these expected returns. In generating Exhibit 1, we assume a pairwise correlation of 0.9 for all assets' expected returns. This means that in a single simulation (representing a single set of forecasts), the expected returns for all assets are likely higher or lower than their respective survey medians together. We run several robustness tests and find that our results are robust to different specifications of the covariance matrix in Exhibit A1.

Having defined the distribution of expected returns, the simulation is straightforward. We run 1,000 simulations by drawing expected returns from our distribution of expected returns. For each simulation, we calculate the mean–variance optimal weights. For each asset class, we plot the distribution of the 1,000 weights using a boxplot as shown in Exhibit 1. To ensure that the optimization resembles a real-world application, we do not allow for short sales for any asset class.

APPENDIX B

INTUITION BEHIND EQUATION 1

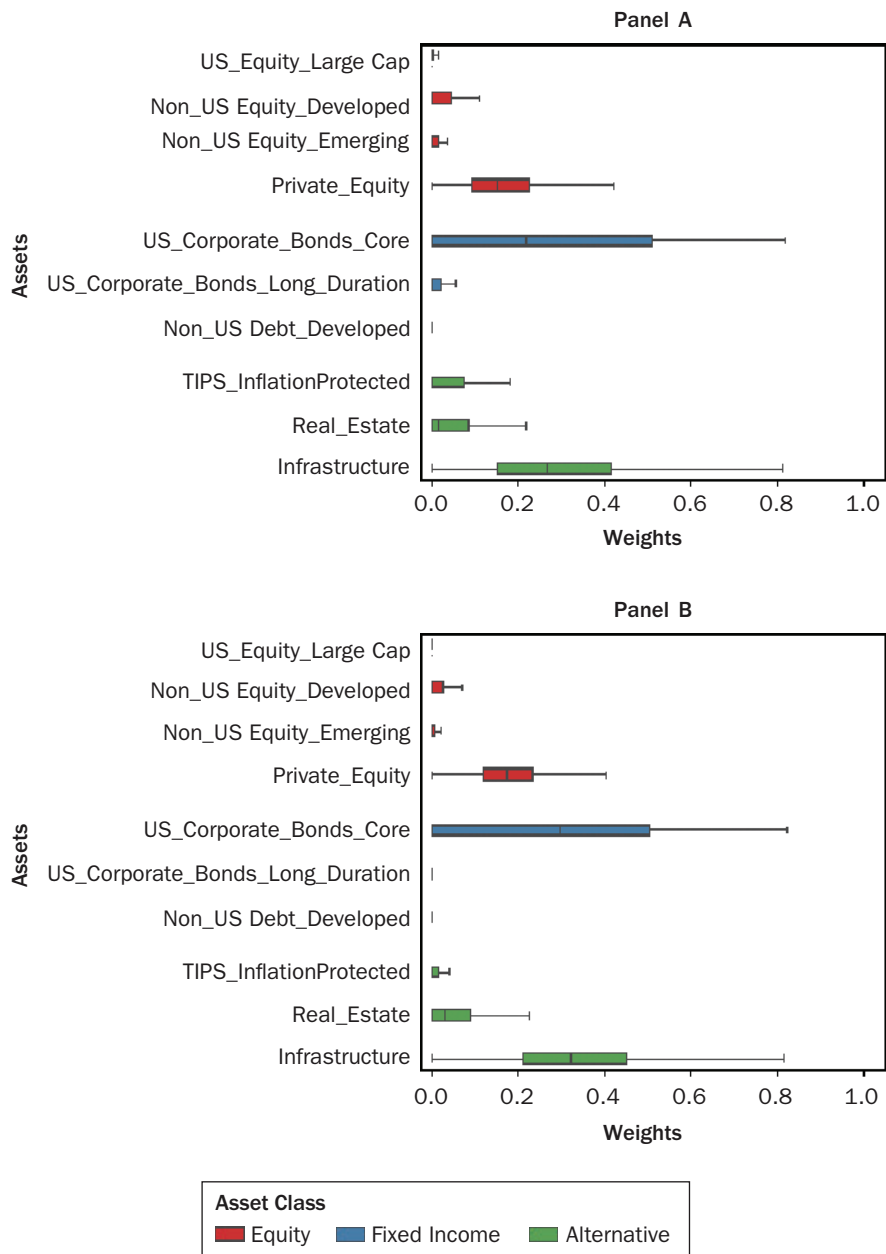
This appendix clarifies the intuition behind Equation 1. Let us define R_A as an $N \times T$ matrix of returns for N assets, where T is the number of observations in the time series.²⁵ We assume that there are M factors defined through mimicking portfolios according to the matrix of weights ω ($N \times M$); that is, $R_F = \omega'R_A$ is a $M \times T$ matrix of factor returns. We can estimate the matrix of factor loadings β by running a regression of asset returns on factor returns: $R_A = \beta R_F + \epsilon$, where β is an $N \times M$ matrix of factor loadings and ϵ is a vector of errors. Using matrix algebra, it is possible to estimate β as follows:

$$\hat{\beta} = R_A R_F' [R_F R_F']^{-1} = \frac{1}{T-1} R_A R_A' \omega \left[\omega' \left(\frac{1}{T-1} R_A R_A' \right) \omega \right]^{-1} \quad (\text{B1})$$

²⁵ For convenience, we omit the mean of the asset return series to simplify this explanation.

EXHIBIT A1

Distribution of Weights Using CMAs under Different Assumptions



NOTES: This exhibit shows the distribution of weights allocated to various asset classes when investors use expected returns and covariances provided in the HAS report together with mean-variance optimization. The distribution of weights is generated using 1,000 simulations. We describe the simulation methodology in Appendix A. Panel A shows the results when we assume that the correlation between expected returns in the CMA survey is zero, and Panel B shows the results when we assume that the correlations between expected returns in the CMA survey are the same as the correlation matrix. For each asset class, the median weight is shown by a vertical line segment, and the colored box shows the interquartile range (25%–75% percentiles). The black lines that extend from the colored box cover 99% of the distribution.

where $\hat{\beta}$ is the estimator of β , and the second equality follows from substituting $R_F = \omega'R_A$ and multiplying and dividing by $\frac{1}{T-1}$. Noting that $\frac{1}{T-1}R_AR_A'$ is the sample-based estimator of the covariance matrix of asset returns—which we define as $\hat{\Sigma}$ —Equation B1 simplifies to $\hat{\beta} = \hat{\Sigma}^{-1}\omega(\omega'\hat{\Sigma}\omega)^{-1}$. This expression shows that our definition of factor loadings in Equation 1 is none other than an estimate of those factor loadings using a time-series regression.

APPENDIX C

ANALYSIS USING AN ALTERNATIVE DEFINITION FOR THE MACROECONOMIC FACTORS (ω_{mf})

The aim of this appendix is to show that the results generated using our factor-mimicking portfolios defined in Exhibit 2 are similar to those generated using an alternative definition for macroeconomic factors (ω_{mf}) that involves more assets (i.e., a less sparse definition). Similar to the definition shown in Exhibit 2, this alternative factor definition reflects similar economic intuitions. More specifically, we proceed as follows.

The economic growth factor is mimicked by a portfolio of equities (both developed and emerging markets), high-yield bonds, and emerging-market bonds. For example, equities are known to be exposed to economic growth (Vassalou 2003; Amenc et al. 2019), and high-yield domestic and emerging market bonds intrinsically have an equity-like component (Fridson 1994; Bulow, Summers, and Summers 1990). As shown in Exhibit C1, in the definition of the economic growth factor, we include US large cap, non-US developed, and emerging equities with weights that are approximately proportional to their market capitalization. We also add high-yield domestic and emerging market bonds with approximately the same weights, and we short US corporate bonds such that the basket of long US high-yield versus short US corporate bonds reflects a credit spread position, which is known to be correlated with economic growth.

The real rate factor is likely to be the easiest one to define. We use inflation-protected bonds because real interest rates are the key drivers for their short-term performance, as shown in the literature (see, for example, Campbell, Pflueger, and Viceira 2020).²⁶

For the inflation factor, we aim to create a portfolio that replicates breakeven inflation, and we augment it with commodities. For the breakeven component, we follow the literature (e.g., Martellini, Milhau, and Tarelli 2014) and use a portfolio that is long inflation-protected bonds and short nominal bonds so that the impact of discount rate changes from the inflation-protected and nominal bonds offset each other. Although our choice of inflation-protected bonds is limited to US TIPS because it is the only inflation-linked asset class provided in the CMA reports, we use a portfolio of nominal bonds for the short leg of the breakeven portfolio. Specifically, we allocate weights between US and developed markets in terms of their volatility risks, with proportions of approximately two-thirds and one-third, respectively.²⁷ Commodities are also included in the inflation factor because they are commonly considered inflation assets by many institutional investors and pension funds (Conover et al. 2010; Dempster and Artigas 2010). The weights are also chosen such that both the breakeven portfolio and commodities are important contributors to the volatility risk of the inflation factor.

²⁶The inflation-adjusted coupon payments from inflation-protected bonds affect the long-term performance only and are inconsequential to short-term returns.

²⁷We also take into account the duration of the various asset classes. Because US long-duration bonds have approximately twice the duration of the other asset classes, their weight in Exhibit C1 is half the weight of US core bonds.

Exhibit C1 reports the factor definition according to the description provided earlier. We leave the private-specific factor weights unchanged from Exhibit 2. In Exhibit C2, we analyze the pricing errors obtained from the definition of factor-mimicking portfolios provided in Exhibit C1. Panels A-1 and A-2 show the pricing errors computed using the previously described (less sparse) factors. Comparing them with Panels A and B from Exhibit 6, respectively, shows that the pricing errors are similar between the two sets of factor definitions. Panel B of Exhibit C2 shows the mean–variance factor weights using the previously described factors. Comparing them with Panel B from Exhibit 7 shows that the mean–variance tangency factor weights are similar between the two sets of factor definitions.

EXHIBIT C1

Alternative Factor-Mimicking Portfolios' Definition

Asset	ω_{mf}			$\hat{\omega}_{pf}$
	Economic Growth	Real Rate	Inflation	Private Specific
US Equity-Large Cap	0.25			
Non-US Equity-Developed	0.20			
Non-US Equity-Emerging	0.05			
US Corporate Bonds-Core	-0.10		-0.50	
US Corporate Bonds-Long Duration			-0.25	
US Corporate Bonds-High Yield	0.10			
Non-US Debt-Developed			-0.50	
Non-US Debt-Emerging	0.10			
TIPS (Inflation-Protected)		1.65	1.50	
Commodities			0.30	
Hedge Funds				
Real Estate				0.40
Infrastructure				0.40
Private Equity				0.40

NOTES: This exhibit reports the definition of the three macroeconomic factors (ω_{mf}) and the private-specific factor ($\hat{\omega}_{pf}$) described in Appendix C. The column $\hat{\omega}_{pf}$ shows the weights used to calculate the private-specific factor orthogonal to the macroeconomic factors.

EXHIBIT C2

Asset Pricing Errors and Mean–Variance Factor Weights

Assets\Years	2013	2014	2015	2016	2017	2018	2019	2020	Average	Std. Dev.
Panel A-1: Asset Pricing Errors: Using Excess Returns from Cross-Sectional Regression (Equation 6)										
US Equity-Large Cap	0.10%	0.03%	0.04%	0.07%	0.07%	0.03%	0.22%	0.18%	0.09%	0.07%
Non-US Equity-Developed	0.14%	0.26%	0.26%	0.39%	0.08%	0.05%	0.01%	0.16%	0.17%	0.12%
Non-US Equity-Emerging	0.26%	0.12%	0.20%	0.18%	0.27%	0.15%	0.18%	0.27%	0.20%	0.05%
US Corporate Bonds-Core	0.06%	0.09%	0.19%	0.21%	0.31%	0.49%	0.38%	0.27%	0.25%	0.13%
US Corporate Bonds-Long Duration	0.02%	0.17%	0.03%	0.02%	0.17%	0.04%	0.17%	0.15%	0.10%	0.07%
US Corporate Bonds-High Yield	0.51%	0.42%	0.70%	1.03%	0.53%	0.47%	0.59%	0.91%	0.65%	0.20%

(continued)

EXHIBIT C2 (continued)**Asset Pricing Errors and Mean-Variance Factor Weights**

Assets\Years	2013	2014	2015	2016	2017	2018	2019	2020	Average	Std. Dev.
Non-US Debt-Developed	0.55%	0.79%	1.01%	0.96%	0.99%	0.89%	0.94%	1.12%	0.91%	0.16%
Non-US Debt-Emerging	0.44%	0.70%	0.83%	0.74%	0.43%	0.40%	0.82%	1.06%	0.68%	0.22%
TIPS (Inflation-Protected)	0.23%	0.24%	0.17%	0.34%	0.16%	0.12%	0.13%	0.25%	0.20%	0.07%
Commodities	0.04%	0.01%	0.07%	0.07%	0.01%	0.07%	0.16%	0.12%	0.07%	0.05%
Panel A-2: Asset Pricing Errors: Using Excess Returns from Factor-Mimicking Portfolios (Equation 5)										
US Equity-Large Cap	0.08%	0.03%	0.05%	0.16%	0.04%	0.12%	0.33%	0.33%	0.14%	0.12%
Non-US Equity-Developed	0.27%	0.37%	0.40%	0.49%	0.17%	0.05%	0.06%	0.30%	0.26%	0.15%
Non-US Equity-Emerging	0.62%	0.37%	0.60%	0.34%	0.37%	0.05%	0.15%	0.46%	0.37%	0.19%
US Corporate Bonds-Core	0.41%	0.36%	0.49%	0.50%	0.46%	0.54%	0.46%	0.51%	0.46%	0.05%
US Corporate Bonds-Long Duration	0.02%	0.41%	0.24%	0.38%	0.48%	0.26%	0.15%	0.27%	0.28%	0.14%
US Corporate Bonds-High Yield	0.60%	0.53%	0.82%	1.14%	0.54%	0.46%	0.57%	0.95%	0.70%	0.23%
Non-US Debt-Developed	0.28%	0.53%	0.72%	0.68%	0.84%	0.86%	0.83%	0.89%	0.70%	0.19%
Non-US Debt-Emerging	0.47%	0.83%	0.89%	0.91%	0.53%	0.51%	0.91%	1.20%	0.78%	0.24%
TIPS (Inflation-Protected)	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%
Commodities	0.22%	0.06%	0.19%	0.02%	0.24%	0.31%	0.50%	0.42%	0.24%	0.16%
Panel B: Unconstrained Mean-Variance Optimization Weights on Factors										
Economic Growth	0.47	0.43	0.42	0.48	0.47	0.46	0.48	0.59	0.47	0.05
Real Rate	0.13	0.23	0.17	0.18	0.12	0.07	0.11	0.10	0.14	0.05
Inflation	0.00	-0.09	-0.04	-0.12	-0.05	-0.01	-0.03	-0.07	-0.05	0.04
Private-specific	0.39	0.42	0.46	0.46	0.46	0.48	0.44	0.38	0.44	0.03

NOTES: Panels A-1 and A-2 report the absolute pricing errors between the surveys' excess returns and excess returns computed using (1) the cross-sectional regression approach (Equation 6) and (2) the factor-mimicking portfolio returns (Equation 5). Panel B reports the mean-variance tangency factor weights. This exhibit uses the definition of the factor mimicking portfolios from Exhibit C1.

ACKNOWLEDGMENT

The authors would like to thank Michael Wissell for his support on innovation and research and Eric Ng for his helpful suggestions. The authors also thank the Referee for the constructive comments and recommendations.

REFERENCES

- Aliaga-Diaz, R., G. Renzi-Ricci, A. Daga, and H. Ahluwalia. 2019. "Vanguard Asset Allocation Model: An Investment Solution for Active-Passive-Factor Portfolios." 2019, <https://institutional.vanguard.com/iam/pdf/ISGVAAM.pdf>.
- . 2020. "Portfolio Optimization with Active, Passive, and Factors: Removing the Ad Hoc Step." *The Journal of Portfolio Management* 46 (4): 39–51.
- Amenc, N., M. Esakia, F. Goltz, and B. Luyten. 2019. "Macroeconomic Risks in Equity Factor Investing." *The Journal of Portfolio Management* 45 (6): 39–60.

- Amihud, Y., A. Hameed, W. Kang, and H. Zhang. 2015. "The Illiquidity Premium: International Evidence." *Journal of Financial Economics* 117 (2): 350–368.
- Ang, A. *Asset Management: A Systematic Approach to Factor Investing*. Oxford, UK: Oxford University Press, 2014.
- Asl, F. M., and E. Etula. 2012. "Advancing Strategic Asset Allocation in a Multi-Factor World." *The Journal of Portfolio Management* 39 (1): 59–66.
- Bansal, R., and I. Shaliastovich. 2013. "A Long-Run Risks Explanation of Predictability Puzzles in Bond and Currency Markets." *The Review of Financial Studies* 26 (1): 1–33.
- Bass, R., S. Gladstone, and A. Ang. 2017. "Total Portfolio Factor, Not Just Asset, Allocation." *The Journal of Portfolio Management* 43 (5): 38–53.
- Bender, J., J. Le Sun, and R. Thomas. 2018. "Asset Allocation vs. Factor Allocation—Can We Build a Unified Method?" *The Journal of Portfolio Management* 45 (2): 9–22.
- Bergeron, A., M. Kritzman, and G. Sivitsky. 2018. "Asset Allocation and Factor Investing: An Integrated Approach." *The Journal of Portfolio Management* 44 (4): 32–38.
- Blyth, S., M. C. Szigety, and J. Xia. 2016. "Flexible Indeterminate Factor-Based Asset Allocation." *The Journal of Portfolio Management* 42 (5): 79–93.
- Bulow, J. I., L. H. Summers, and V. P. Summers. *Distinguishing Debt from Equity in the Junk Bond Era*. Harvard Institute of Economic Research, Harvard University, 1990.
- Campbell, J. Y., C. Pflueger, and L. M. Viceira. 2020. "Macroeconomic Drivers of Bond and Equity Risks." *Journal of Political Economy* 128 (8): 3148–3185.
- Conover, C. M., G. R. Jensen, R. R. Johnson, and J. M. Mercer. 2010. "Is Now the Time to Add Commodities to Your Portfolio?" *The Journal of Investing* 19 (3): 10–19.
- Cornell, B. 2010. "Economic Growth and Equity Investing." *Financial Analysts Journal* 66 (1): 54–64.
- DeMiguel, V., L. Garlappi, and R. Uppal. 2009. "Optimal versus Naive Diversification: How Inefficient Is the 1/N Portfolio Strategy?" *The Review of Financial Studies* 22 (5): 1915–1953.
- Dempster, N., and J. C. Artigas. 2010. "Gold: Inflation Hedge and Long-Term Strategic Asset." *The Journal of Wealth Management* 13 (2): 69–75.
- Elkamhi, R., J. S. Lee, and M. Salerno. "Portfolio Tilts Using Views on Macroeconomic Regimes." Working paper, Pacific Center for Asset Management, 2020.
- Fama, E. F. 1981. "Stock Returns, Real Activity, Inflation, and Money." *The American Economic Review* 71 (4): 545–565.
- Fama, E. F., and J. D. MacBeth. 1973. "Risk, Return, and Equilibrium: Empirical Tests." *Journal of Political Economy* 81 (3): 607–636.
- Fridson, M. S. 1994. "Do High-Yield Bonds Have an Equity Component?" *Financial Management* 23 (2): 82–84.
- Greenberg, D., A. Babu, and A. Ang. 2016. "Factors to Assets: Mapping Factor Exposures to Asset Allocations." *The Journal of Portfolio Management* 42 (5): 18–27.
- Ibbotson, R. G., and R. A. Sinquefeld. 1976. "Stocks, Bonds, Bills, and Inflation: Year-by-Year Historical Returns (1926–1974)." *The Journal of Business* 49 (1): 11–47.
- Jacobsen, B., W. Lee, E. Cheng, F. Cooke, and C. Ma. 2019. "Avoiding the CMA-Portfolio Disconnect." 2019, <https://www.wellsfargoassetmanagement.com/assets/public/pdf/insights/investing/avoiding-the-cma-portfolio-disconnect.pdf>.

Jagannathan, R., and T. Ma. 2003. "Risk Reduction in Large Portfolios: Why Imposing the Wrong Constraints Helps." *The Journal of Finance* 58 (4): 1651–1683.

Kolm, P. N., and G. Ritter. 2020. "Factor Investing with Black–Litterman–Bayes: Incorporating Factor Views and Priors in Portfolio Construction." *The Journal of Portfolio Management* 47 (2): 113–126.

Konstantinov, G., A. Chorus, and J. Rebmann. 2020. "A Network and Machine Learning Approach to Factor, Asset, and Blended Allocation." *The Journal of Portfolio Management* 46 (6): 54–71.

Lawler, B., B. Mossman, P. Nolan, and A. Ang. 2020. "Factors and Advisor Portfolios." *The Journal of Wealth Management* 22 (4): 37–61.

Lee, B.-S. 1992. "Causal Relations among Stock Returns, Interest Rates, Real Activity, and Inflation." *The Journal of Finance* 47 (4): 1591–1603.

Lorenzen, K. A., and S. F. Järner. "Factor Investing the ATP Way." 2017, <https://www.atp.dk/en/dokument/factor-investing-atp-way>.

Markowitz, H. 1952. "Portfolio Selection." *The Journal of Finance* 7 (1): 77–91.

Martellini, L., V. Milhau, and A. Tarelli. 2014. "Hedging Inflation-Linked Liabilities without Inflation-Linked Instruments through Long/Short Investments in Nominal Bonds." *The Journal of Fixed Income* 24 (3): 5–29.

Merton, R. C. 1980. "On Estimating the Expected Return on the Market: An Exploratory Investigation." *Journal of Financial Economics* 8 (4): 323–361.

Michaud, R. O. 1989. "The Markowitz Optimization Enigma: Is 'Optimized' Optimal?" *Financial Analysts Journal* 45 (1): 31–42.

Moskowitz, T. J., and A. Vissing-Jørgensen. 2002. "The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?" *American Economic Review* 92 (4): 745–778.

Podolsky, P., R. Johnson, and O. Jennings "The All Weather Story." White paper, Bridgewater Associates, 2012.

Roll, R., and A. Srivastava. 2018. "Mimicking Portfolios." *The Journal of Portfolio Management* 44 (5): 21–35.

Stein, C. "Inadmissibility of the Usual Estimator for the Mean of a Multivariate Normal Distribution." Stanford University, Stanford, California, 1956.

Vassalou, M. 2003. "News Related to Future GDP Growth as a Risk Factor in Equity Returns." *Journal of Financial Economics* 68 (1): 47–73.

Disclaimer

This article is for informational purposes only and should not be construed as legal, tax, investment, financial, or other advice. The views and opinions expressed here are those of the authors alone and do not necessarily reflect the views of their employers and their affiliates.

To order reprints of this article, please contact David Rowe at d.rowe@pageantmedia.com or 646-891-2157.