How Large are Pre-Default Costs of Financial Distress?

Estimates from a Dynamic Model

Abstract

We estimate the costs of financial distress prior to default (pre-default costs) separately from the loss incurred at default (the loss given default) using a dynamic trade-off model of capital structure. We show that pre-default costs account for a large fraction of total distress costs, approximately 64.1%. We demonstrate that the expected pre-default costs of financial distress vary significantly across industries with a range between 3.2% and 8.3%, and are higher for small firms relative to larger ones.

Keywords: Dynamic Capital Structure, Financial Distress, Structural Estimation

JEL Classification: G30, G32, G33

1 Introduction

How much value do firms lose because of financial distress? Davydenko et al. (2012) and Korteweg (2010) show that the average distress costs are approximately 15-30% for firms that are in or near default.[1] However, firms experience costs of financial distress prior to default (pre-default costs) as discussed in Titman (1984).[2] Elkamhi, Ericsson, and Parsons (2012) use a calibration exercise to show that pre-default costs can greatly increase the net present value of financial distress costs, and help match the observed leverage ratios. Despite the fact that pre-default costs have been shown to be important, there is limited empirical evidence on their magnitude. Some studies analyzed ex-post costs of financial distress documenting, for example, a drop in the sale price of used car when the carmaker's CDS spread increases (Hortaçsu et al., 2013), fire sales of aircrafts (Pulvino, 1998) and inability to respond to a competitor's entry in the casino business (Cookson, 2017). However, it has been proven to be difficult to find comparable empirical designs to quantify the ex-ante (expected) costs of financial distress for the average firm, and assess the cross-sectional variation amongst industries and firms with different characteristics. Our goal is to fill this gap.

Specifically, the purpose of this paper is to (1) analyze pre-default costs *jointly* with the loss given default (i.e. the expected loss at the time of default), (2) study the bias in the estimates of costs of financial distress if pre-default costs are omitted, (3) investigate how such costs vary amongst different industries and firm's characteristics, and (4) provide an alternative view to the selection bias discussed in Glover (2016). To answer the above questions, we first develop a dynamic trade-off model that accounts for pre-default costs and dynamic leverage.[3] We then fit our model to the data. To the best of our knowledge, this is the first study that jointly estimates both pre-default costs of financial distress and the loss given default.

Including pre-default costs of financial distress into a dynamic model of capital structure is

[1] Davydenko et al. (2012) estimate that the average costs of default are approximately 21.7% using a sample of 175 firms that defaulted between 1997 and 2010. Korteweg (2010) uses a structural estimation approach to recover the net benefits of leverage for 290 firms between 1994 and 2004, and finds that costs of financial distress are between 15% and 30%. Also, Andrade and Kaplan (1998) estimate that, in a sample of 31 leveraged buyout firms, the default costs are between 10-23%.

[2] For example, there is anecdotal evidence that customers might not buy the products of a highly levered firm because they fear that the firm will not be able to honour warranty, or managers might not focus on the core business because they have to deal with creditors when the firm is in distress. We refer to Hotchkiss et al. (2008) and Senbet and Wang (2012) for excellent reviews of the costs of financial distress.

[3] Our model is similar in spirit to Goldstein et al. (2001) and Elkamhi et al. (2012).

important for at least 3 reasons. First, pre-default costs have different implications for corporate policies and probabilities of default compared to the loss given default. For example, Hortaçsu et al. (2013) shows that car makers suffer from a loss of customers if they are perceived as financially distressed. Second, as shown by Chen, Hackbarth, and Strebulaev (2022), including pre-default costs of financial distress allows a trade-off model of capital structure to explain two apparently contradicting empirical puzzles: the negative relationship between probability of default and stock returns (Campbell et al., 2008) and the positive distress risk premium (Friewald et al., 2014). Third, as we discuss below, the inclusion of pre-default costs in trade-off models provides an alternative explanation on the selection bias discussed in Glover (2016).

Estimating the *ex-ante* pre-default costs of financial distress directly from the data is challenging because of the endogeneity of the data. When firms are experiencing pre-default costs of financial distress, they are likely to have changed also other characteristics compared to when they were "healthy". For example, automakers have experienced pre-default costs of financial distress during the 2008 crisis (Hortaçsu et al., 2013) and, during that period, other firms' characteristics might have been affected as well (e.g., their equity volatility was higher). If researchers were to estimate some automakers' firms characteristics during the financial crisis, their estimates would be measuring *ex-post* effects rather than *ex-ante* pre-default costs. Hence, estimates would be biased by the specific event that was affecting such firms at that time (e.g., financial crisis). To circumvent these estimation problems, we address the question using structural estimation.[4]

In our model, we solve for the value of contingent claims in a setting where the firm can experience pre-default costs. Pre-default costs are captured by letting the firm "leak" a percentage of its value and increase its riskiness during times of financial distress.[5] We estimate our model by fitting it to observed financing choices as well as actual default rates using Simulated Method of Moments (SMM). We show that the average pre-default cost expressed as a percentage of cash

[4]We stress here that our goal is to estimate of the *ex-ante* pre-default costs rather than *ex-post*. Estimating ex-post costs of financial distress can be achieved without using a structural estimation approach as shown by, for example, Hortaçsu et al. (2013)

[5]Elkamhi et al. (2012) calibrate a similar model to show the qualitative material effect of pre-default costs of financial distress on trade-off models of capital structure. Our aim in this paper is to estimate jointly the loss given default and pre-default costs of financial distress. The dynamic of leverage is modeled similarly to Goldstein et al. (2001), Chen (2010) and Chen et al. (2022).

flows lost during times of financial distress is approximately 7.4% per year. The estimated average loss given default is 26.3%, a value that is close to empirical evidence (Korteweg, 2010; Davydenko et al., 2012). We find that pre-default costs account 64.1% of the total costs of financial distress (loss given default plus pre-default).

Using the estimates from our model, we provide an alternative explanation for the selection bias discussed in Glover (2016), which states that defaulted firms have a significantly lower loss given default (LGD) than the average firm (i.e. LGD for defaulted firms is 25% vs. LGD of 45% for non-defaulted firms). Our results show that the timing of financial distress costs also causes a bias because firms that end up defaulting have already experienced pre-default costs of financial distress compared to non-defaulted ones. Therefore, measuring financial distress costs on a sample of defaulted firms would only measure the loss given default and would not capture the (large) pre-default costs that firms experienced before defaulting. Our alternative explanation does not invalidate the channel proposed by Glover (2016) but it highlights an additional mechanism that can co-exist with it.

Next, our findings shed light on the bias of estimating the loss given default in a model that ignores the costs of financial distress experienced prior to default. Understanding this bias has practical importance because of the widespread practice of calibrating trade-off models to gauge insights of firms' responses to a policy change or to evaluate the behaviour of credit spreads, leverage and probability of default.[6] To quantify this bias, we estimate the model again, but we fix the pre-default costs to zero. We then compare the results from this exercise to the ones from the model that includes pre-default costs. While the model with pre-default costs requires an average loss given default of 26.3%, the same model where we fix pre-default costs to zero needs an average loss given default of 55.0% in order to fit the data. Without pre-default costs, the model needs a loss at default which is approximately 2 times higher than in the model with pre-default costs. This difference suggests that omitting pre-default costs from trade-off models can lead to a significant bias when one calibrates them to the empirically observed values of loss at default.

[6]Examples of papers that use a calibration and/or estimation of structural models to study various phenomena include Huang and Huang (2012), Schaefer and Strebulaev (2008), Du, Elkamhi, and Ericsson (2018), Feldhütter and Schaefer (2018) and Bai, Goldstein, and Yang (2020).

In addition to measuring the average pre-default costs of financial distress, we also study cross-sectional variation of pre-default costs across industries and firms with different asset's tangibility. We find that there is a large cross-sectional variation in the estimates of pre-default costs and loss given default across industries. The estimate of pre-default costs ranges from 3.2% to 8.3% while the loss given default varies between 16.5% and 33.0%. We also estimate our model on sub-samples of firms split by the tangibility of assets. Consistent with the intuition that tangible firms should recover more than intangible firms in case of default (Elkamhi, Jacobs, and Pan, 2014), we find that firms with highly tangible assets have a loss given default that is considerably lower than intangible firms (i.e., 16.5% for firms with high tangibility vs. 35.2% for intangible firms). We also show that small firms have higher pre-default costs of financial distress relative to larger ones. This is consistent with the literature showing that small firms have less access to credit during bad times making them more likely to experience liquidity difficulties which would exacerbate financial distress (e.g., Campello et al., 2011).

Methodologically, this paper is close to the structural estimation literature that uses SMM to study various questions in corporate finance.[7] For example, Hennessy and Whited (2005) and Hennessy and Whited (2007) rely on this methodology to estimate a discrete-time dynamic capital structure model and show that it is able to generate several empirical facts (e.g., path-dependency of leverage and its relation with liquidity). Nikolov and Whited (2014) also use the same methodology to estimate a dynamic model and show that agency conflicts between managers and shareholders enhance the ability of the model to explain the dynamics of cash. We contribute to this literature by estimating the costs of financial distress incurred prior to default separately from the costs incurred at the time of default.

The rest of the paper is structured as follows. Section 2 develops a trade-off model of capital structure which allows for pre-default costs of financial distress. Section 3 presents the comparative statics of the model, while Section 4 presents the details of the identification and the structural estimation methodology. Section 5 discusses the results of the estimation and robustness tests. Section 6 concludes.

[7] See Strebulaev and Whited (2012) for a comprehensive review of papers that use a structural estimation approach in Corporate Finance.

2 Model

This section presents a dynamic trade-off model that incorporates costs of financial distress incurred before the firm defaults.

2.1 Cashflow Dynamics

Following Elkamhi et al. (2012), we relax the assumption that default costs are incurred exclusively as a lump sum when firms default. In our model, firms experience a deadweight loss at the moment they default (i.e. a proportion α of firm's assets is lost at default), and "leak" a fraction of their value when EBIT drops below the distress boundary X_D.[8] More specifically, a firm's EBIT is governed under the physical probability measure $\mathcal{Q}$ by the following process

$$\frac{dX_t}{X_t} = \begin{cases} \mu\, dt + \sigma_X^L dB_t & \text{for } X_B \leq X_D \leq X_t \\ (\mu - \gamma)dt + \sigma_X^H dB_t & \text{for } X_B < X_t < X_D \end{cases} \tag{1}$$

where μ is the firm specific (risk-neutral) expected growth rate of EBIT, γ is the constant rate at which the firm loses value when it is in financial distress (i.e. $X_B < X_t < X_D$), σ_X^L and σ_X^H are the volatilities when the firm is healthy (L stands for low volatility) and distressed (H stands for high volatility), X_D and X_B are the distress and default thresholds, respectively.

In Figure 1 we plot two EBIT paths using Equation (3). The two paths have been generated using the same random seeds and parameters except for the level of pre-default costs γ. The blue solid line depicts the EBIT path for a firm that does not suffer any pre-default costs (i.e. $\gamma = 0$). The red dashed line shows the EBIT path for a firm that suffers pre-default costs equal to a loss of 2% per year of the value of its assets during financial distress (i.e. $\gamma = 2\%$).

[Insert Figure 1 here]

The key intuition from Figure 1 is that pre-default costs strongly affect the probability of default. In Section 4.3 we confirm that this intuition holds even when we allow the firm to choose its optimal

[8] Equation (3) shows that pre-default costs are proportional to the value of X_t. Since the value of assets is a monotonically increasing function of EBIT X_t (as shown in Equation (9)) then pre-default are also proportional to the value of assets.

leverage given the parameters of the model. More specifically, we show that two firms that differ only in the parameter values for α and γ, and optimally choose identical leverage would exhibit different probabilities of default.

2.2 Pricing Kernel

Firms in our model are exposed to both systematic and idiosyncratic risks. We let the aggregate economy's operating cash flows (i.e. EBIT) follow a Geometric Brownian Motion

$$\frac{dX_{At}}{X_{At}} = \mu_A^{\mathcal{P}} dt + \sigma_A dB_t^{A,\mathcal{P}} \tag{2}$$

where $\mu_A^{\mathcal{P}}$ is the expected growth rate of the economy under the physical probability space, σ_A is the volatility parameter,and $dB_t^{A,\mathcal{P}}$ is a standard Brownian Motion. A common assumption in the literature (Leland, 1994; Abel, 2018) is that firms experience a deadweight loss at the moment they default. Creditors receive a fraction $1-\alpha$ of the continuation value of the firm in the event of default, so the total social cost is a fraction α of the continuation value, where $\alpha \in [0,1]$.

We do not include time-varying macroeconomic conditions in the estimation of our model which, in calibration exercises, have been shown to be an important component of this class of models to explain observed average leverage ratios and credit spreads (Bhamra et al., 2009; Chen, 2010; Bhamra et al., 2010; Chen et al., 2009; Elkamhi et al., 2020). Our choice is driven by the need to keep our model parsimonious in order to estimate it to the data. Furthermore, our aim is not to generate reasonable covariance between cash flows and pricing kernel and hence large default and risk premia. Our goal is to provide a robust estimate of pre-default costs of financial distress using a parsimonious model.

Following Glover (2016), Equations (1) and (2) imply that under the physical probability probability measure $\mathcal{P}$ by the process

$$\frac{dX_t}{X_t} = \begin{cases} \left(\mu + \beta(\mu_A^{\mathcal{P}} - r)\right) dt + \beta\sigma_A dB_t^{A,\mathcal{P}} + \sigma_F^L dB_t^F & \text{for } X_B \leq X_D \leq X_t \\ \underbrace{\left(\mu + \beta(\mu_A^{\mathcal{P}} - r)\right) dt}_{\substack{\text{Expected growth} \\ \text{rate without distress}}} + \underbrace{\beta\sigma_A dB_t^{A,\mathcal{P}} + \sigma_F^H dB_t^F}_{\substack{\text{Systematic and} \\ \text{Idiosyncratic shocks}}} \underbrace{-\gamma dt}_{\substack{\text{Pre-default} \\ \text{Costs}}} & \text{for } X_B < X_t < X_D \end{cases} \tag{3}$$

where β is the exposure to market risk, r is the constant risk-free rate, dB_t^F is a standard Brownian Motion (independent from $dB_t^{A,\mathcal{P}}$) that governs the idiosyncratic firm-specific shocks, σ_F^L and σ_F^H are the idiosyncratic volatility of the firm-specific shocks when the firm is healthy (L stands for low volatility) and distressed (H stands for high volatility), $\sigma_X^i = \sqrt{(\beta\sigma_A)^2 + (\sigma_F^i)^2}$ is the total volatility of the firm for $i \in \{L, H\}$.

We prove that under the risk neutral measure $\mathcal{Q}$, the firm's EBIT process is governed by Equation (1). Recall that the firm's EBIT under the physical probability space $\mathcal{P}$ follows the process defined in Equation (3). Let the (exogenous) pricing kernel be

$$\frac{d\xi_t}{\xi_t} = -rdt - \varphi dB_t^{A,\mathcal{P}} \tag{4}$$

where $\varphi = (\mu_A^{\mathcal{P}} - r)/\sigma_A$ is the market Sharpe ratio, and $B_t^{A,\mathcal{P}}$ is a standard Brownian Motion under the physical probability space.

We define the density process for the risk-neutral measure as

$$\nu_t = \mathbf{E}_t \left[\frac{d\mathcal{Q}}{d\mathcal{P}} \right]$$

Following Harrison and Kreps (1979), the density process and the pricing kernel are related as follows

$$\nu_t = \xi_t e^{\int_0^t r ds} = \xi_t e^{rt}$$

Applying Ito's lemma we have

$$d\nu_t = e^{rt} d\xi_t + \xi_t r e^{rt} dt \tag{5}$$

Substituting Equation (4) in Equation (5) and recalling that $\xi_t = \nu_t / e^{rt}$

$$d\nu_t = -\varphi \xi_t e^{rt} dB_t^{A,\mathcal{P}} \implies \frac{d\nu_t}{\nu_t} = -\varphi dB_t^{A,\mathcal{P}}$$

Applying the First Fundamental Theorem of Asset Pricing, we have

$$dB_t^{A,\mathcal{Q}} = dB_t^{A,\mathcal{P}} + \varphi dt \tag{6}$$

Substituting Equation (6) in Equation (3) we obtain that the firm's EBIT under the risk-neutral $\mathcal{Q}$ measure is governed by the process

$$\frac{dX_t}{X_t} = \begin{cases} \mu dt + \beta\sigma_A dB_t^{A,\mathcal{Q}} + \sigma_F^L dB_t^F & \text{for } X_B < X_D < X_t \\ (\mu - \gamma)dt + \beta\sigma_A dB_t^{A,\mathcal{Q}} + \sigma_F^H dB_t^F & \text{for } X_B < X_t < X_D \end{cases} \tag{7}$$

The total firm volatility is

$$\sigma_X^L = \sqrt{(\beta\sigma_A)^2 + (\sigma_F^L)^2} \text{ for } X_B < X_D < X_t$$
$$\sigma_X^H = \sqrt{(\beta\sigma_A)^2 + (\sigma_F^H)^2} \text{ for } X_B < X_t < X_D$$

therefore we can re-write firm's EBIT process under the risk-neutral measure $\mathcal{Q}$ more compactly as follows

$$\frac{dX_t}{X_t} = \begin{cases} \mu dt + \sigma_X^L dB_t & \text{for } X_B < X_D \leq X_t \\ (\mu - \gamma)dt + \sigma_X^H dB_t & \text{for } X_B < X_t < X_D \end{cases} \tag{8}$$

where

$$dB_t = \begin{cases} \frac{\beta\sigma_A}{\sigma_X^L} dB_t^{A,\mathcal{Q}} + \frac{\sigma_F^L}{\sigma_X^L} dB_t^F & \text{for } X_B < X_D \leq X_t \\ \frac{\beta\sigma_A}{\sigma_X^H} dB_t^{A,\mathcal{Q}} + \frac{\sigma_F^H}{\sigma_X^H} dB_t^F & \text{for } X_B < X_t < X_D \end{cases}$$

Equation (8) is exactly the firm's EBIT process under the risk neutral measure $\mathcal{Q}$ described in Equation (1).

Earnings are taxed at a constant rate τ^c. Therefore, firms have an incentive to issue debt to benefit from its tax-shield. As in Leland (1994), we ensure a time-homogeneous setting by assuming that firms issue an infinitely lived debt which pays a continuous flow of coupons C. To allow for dynamic leverage, debt is callable and issued at par. Following Goldstein et al. (2001), firms can adjust their capital structure upwards by incurring a proportional cost λ but they cannot reduce their debt downward. The firm's initial debt structure remains fixed until either the firm goes default or calls its debt at par and restructures with newly issued debt. The proceeds from debt issuance are distributed proportionally to shareholders[9]. The personal tax rate on dividends is τ^e

[9] This is a standard assumption when our goal is not to examine the dynamics of cash while it would be restrictive

and coupon payments' tax rate is τ^d. We define $\tau = 1 - (1 - \tau^c)(1 - \tau^e)$ as the effective tax rate including both corporate and personal taxes. All investors face the same tax rates.

The value of (after-tax) unlevered assets is given by

$$V(X_t) = \mathbb{E}^{\mathbb{Q}}\left[\int_t^{\infty} (1-\tau) X_{is} e^{-rs} ds\right] = (1-\tau)\frac{X_t}{r-\mu} \tag{9}$$

2.3 Pricing of Debt and Equity

Before discussing optimal capital structure decisions, we compute the value of debt and equity for fixed levels of coupon (C), distress (X_D) and default (X_B) thresholds as well as the restructuring boundary (X_U). The value of EBIT at the time when the firm makes its decision if X_0. When the firm's EBIT reaches X_U, the firm retires its previously issued debt at par and it issues new one. We provide the solutions below and refer to Appendix A for a full explanation of the model.

2.3.1 Net Income

We start by computing the value of a claim on net income, which represents the cash flows continuously accruing to shareholders at each time t, $(1-\tau)(X_t - C)$. The value of net income is equal to the expected net present value of cash flows accrued to shareholders over the entire life of the firm which we can write as follows

$$\mathbf{NI}(X_t, C) = \begin{cases} \mathbf{n}(X_t, C) + \mathbf{p}_{ND}^U(X_t) \cdot \mathbf{NI}(X_U, C_U) & \text{for } X_D \leq X_t \leq X_U \\ \underbrace{\mathbf{n}(X_t, C)}_{\substack{\text{Net Income} \\ \text{over 1 cycle}}} + \underbrace{\mathbf{p}_{DS}^U(X_t) \cdot \mathbf{NI}(X_U, C_U)}_{\substack{\text{NPV of net income} \\ \text{for future cycles}}} & \text{for } X_B < X_t < X_D \end{cases} \tag{10}$$

where X_U is the restructuring boundary that defines when the firm issues new debt and retires the existing one and C_U is the new coupon paid by the firm after having restructured its debt. The scaling property implies that $\mathbf{NI}(X_U, C_U) = \rho\mathbf{NI}(X_0, C)$ and $C_U = \rho C$ where $\rho = X_U/X_0$. All other variables are defined in Appendix A.

when studying precautionary saving and cash dynamics as shown, for example, in Hennessy and Whited (2005) and Nikolov and Whited (2014).

2.3.2 Debt

The value of a claim on the coupons that the firm will pay over its entire life, which we denote this claim as $\mathbf{TD}(X_t, C)$, is equal to

$$\mathbf{TD}(X_t, C) = \begin{cases} \mathbf{d}(X_t, C) + \mathbf{p}_{ND}^{U}(X_t) \cdot \mathbf{TD}(X_U, C_U) & \text{for } X_D \leq X_t \leq X_U \\ \mathbf{d}(X_t, C) + \mathbf{p}_{DS}^{U}(X_t) \cdot \mathbf{TD}(X_U, C_U) & \text{for } X_B < X_t < X_D \end{cases} \tag{11}$$

where X_U is the restructuring boundary and C_U is the new coupon paid by the firm after having restructured its debt. The scaling property implies that $\mathbf{TD}(X_U, C_U) = \rho \mathbf{TD}(X_0, C)$ where $\rho = X_U/X_0$. All other variables are defined in Appendix A.

2.3.3 Adjustment Costs

After having issued debt, the total value of the adjustment costs is equal to the expected adjustment costs that the firm will incur over its entire life which we can write as follows

$$\mathbf{AC}(X_t, C) = \begin{cases} \mathbf{p}_{ND}^{U}(X_t)\rho\mathbf{AC}(X_0, C) & \text{for } X_D \leq X_t < X_U \\ \mathbf{p}_{DS}^{U}(X_t) \underbrace{\rho\mathbf{AC}(X_0, C)}_{\substack{\mathbf{AC}(X_U, C_U) \\ \text{NPV of future} \\ \text{adjustment costs}}} & \text{for } X_B < X_t < X_D \end{cases} \tag{12}$$

2.3.4 Firm and Equity Value

At any time t, the levered asset value of the firm, $\mathbf{v}(X_t, C)$, is the sum of the present value of cash flows to shareholders plus cash flows to all debtholders minus the net present value of the adjustment costs. It is given by

$$\mathbf{v}(X_t, C) = \underbrace{\mathbf{NI}(X_t, C)}_{\substack{\text{NPV of claim} \\ \text{on net income}}} + \underbrace{\mathbf{TD}(X_t, C)}_{\substack{\text{NPV of claim} \\ \text{on total debt}}} - \underbrace{\mathbf{AC}(X_t, C)}_{\substack{\text{Adjustment} \\ \text{costs}}} \tag{13}$$

Equity is a residual claim and its value is the difference between the total value of the firm

$\mathbf{v}(X_t, C)$ and the value of current debt $\mathbf{D}(X_t, C)$:

$$\mathbf{E}(X_t, C) = \mathbf{v}(X_t, C) - \mathbf{D}(X_t, C) \tag{14}$$

2.4 Optimal Policies

We assume that financing decisions are made by shareholders, and for simplicity we abstract from conflicts of interests between firms' managers and shareholders. When the firm decides its capital structure, it faces a trade-off between the tax benefits of debt and the expected costs of financial distress (both pre-default costs and deadweight loss at the time of default). Since proceeds from debt issuance are distributed proportionally to shareholders and the debt is fairly priced due to complete and arbitrage-free markets, shareholders' objective is to maximize the value of the firm.

Shareholders choose the optimal default threshold X_B by maximizing the value of equity. This is equivalent to applying the smooth-pasting condition as in Leland (1994) to find the optimal default threshold X_B such that

$$\left.\frac{\partial \mathbf{E}(x, C)}{\partial x}\right|_{x=X_B} = 0 \tag{15}$$

The total value of the firm $\mathbf{v}(\cdot)$ is a function of the amount of debt that is issued which affects the coupon level C, and the restructuring boundary X_U which affects how often the firm restructures its debt. At time 0 shareholders choose the coupon C and the restructuring boundary X_U to maximize the ex-ante value of the firm. Formally, shareholders solve the following problem

$$\max_{C, X_U} \mathbf{v}(\cdot) \quad \text{subject to Equation (15)} \tag{16}$$

The above problem can be solved using standard numerical procedures. For any given set of parameters, solving the problem in Equation (16) yields the optimal coupon C^*, the optimal restructuring boundary X_U^* and the optimal default threshold X_B^*.

2.5 Distress Boundary

The role of the distress boundary (X_D) is to act as a trigger start for pre-default costs of financial distress. Following Titman and Tsyplakov (2007) and Chen, Hackbarth, and Strebulaev (2022), we define the onset of financial distress to be when X_t drops below the coupon C paid on outstanding debt. As shown by previous research, this choice is reasonable. As an example, one of the most used covenants on loans is the minimum interest coverage ratio which imposes a minimum ratio of interest expenses over operating cash flows that firms need to maintain. Covenants on loans are such that the average interest coverage ratio for firms in the U.S. is 2.5 (Greenwald, 2019) while our assumption implies that firms start experiencing distress costs when their interest coverage ratio drops below one.[10] Therefore, our choice of distress boundary is coherent with the assumption that firms start experiencing distress costs when they are already in violation of the interest coverage ratio covenant, and it is reasonable to assume that a firm experiences some financial distress costs when they violate a covenant.[11]

Last, as discussed in Chen, Hackbarth, and Strebulaev (2022), linking the distress threshold to the required coupon payments leads to an endogenous distress boundary because the optimal coupon is itself an endogenous decision that the firm makes as shown in Equation (16). The higher the coupon chosen by the firm, not only the firm will have a higher leverage but it will also start experiencing financial distress earlier. This has strong implications for leverage and default probabilities which we discuss in the next section.

3 Comparative Statics

We now examine the predictions of our model for financing decisions and provide a first look at the importance of pre-default costs in capital structure choice. In Table 1 we report the comparative statics describing the effects of the main parameters of the model on: (i) target leverage which is the optimal leverage chosen by the firm at the time of issuing debt; (ii) the leverage at restructuring, which is informative of the restructuring boundary X_U; (iii) the leverage at the time the firm

[10]The average BBB firm has an interest coverage ratio of 4.2 (Kaplan and Zingales, 2000).

[11]For example, Chava and Roberts (2008) show a sharp decline in investment following a covenant violation, and Nini, Smith, and Sufi (2012) document a reduction in acquisitions and a decrease in payouts to shareholders.

becomes distressed, which is informative of the distress threshold X_D; (iv) the recovery rate for debtholders at the time of default.

We set the base case parameters as follows. For parameters that are not estimated, we use the same calibrated values as discussed in Section 4.2. For the parameters that we estimated in Section 4, we set the growth rate and idiosyncratic volatility of cash flows to $\mu = 1.8\%$, $\sigma_F^L = 23.1\%$, and $\sigma_F^H = 32\%$.[12] The exposure to market risk parameter is set to $\beta = 0.94$. This calibration implies a total volatility for the firm's EBIT –when not in distress – $\sqrt{\beta^2\sigma_A^2 + (\sigma_F^L)^2} = 22.53\%$. The loss given default parameter is $\alpha = 26.3\%$, the pre-default costs are set to $\gamma = 7.4\%$. The proportional adjustment costs is set to $\lambda = 1.0\%$ which is in line with empirical evidence (Kim, Palia, and Saunders, 2008). Following Titman and Tsyplakov (2007) and Chen, Hackbarth, and Strebulaev (2022), we set the distress boundary X_D equal to the value of coupon C (see Section 2.5 for a discussion). We normalize the initial value of operating cash flows $X_0 = \$5.0$. We choose this calibration of our model to match the estimated values that we discuss in Section 5.

Table 1 shows that an increase in the loss given default α lowers the target leverage, the restructuring boundary and the distress threshold. Consistent with Goldstein et al. (2001), an increase in α also lowers the recovery rate for debtholders. An increase in pre-default costs γ has qualitatively a similar effect on target leverage, restructuring boundary and distress trigger. However, we show in Section 4.3 that γ has a strong effect on total factor productivity, which allows us to estimate it separately from α. Also, γ affects the recovery value only through the change in optimal leverage and lower default threshold. The intuition underlying this result is as follows. A lower optimal leverage implies a lower default threshold which lowers the recovery value. An increase in α also leads to a lower leverage and a lower default threshold. However, in addition to this effect, an increase in α also directly lowers the recovery value $(1 - \alpha)$ for debtholders at the time of default.

[Insert Table 1 here]

The rest of Table 1 shows the effect on leverage and recovery rate of EBIT growth rate μ and volatility σ_F, firm's beta β, and adjustment costs λ. The results are consistent with those previously

[12] All rates and volatilities reported in this paper are annualized. A detailed description of the data is provided in Section 4.1.

reported in the literature (see, for example, Strebulaev (2007)).

4 Identification and Structural Estimation

We estimate the model parameters using Simulated Method of Moments (SMM). Before presenting the results of our estimations in Section 5, we discuss the empirical data in Section 4.1, we discuss the calibration of parameters in Section 4.2 and then proceed to discuss the identification of our model in Section 4.3. A detailed description of the SMM methodology is provided in Appendix C.

4.1 Empirical Data

We collect financial statements from Compustat Fundamentals Quarterly (library "compd", file "fundq"). Following the literature (see, for example, Hennessy and Whited (2005, 2007)), we drop financial firms (SIC codes 6000 - 6999), utilities (4900 - 4999), and public administration firms (9000 - 9999). We gather firms' equity returns from the Center for Research in Security Prices (CRSP). We define a firm's quasi-market leverage at time t as the book debt at time t (i.e., total assets minus book equity) divided by the sum of book debt and market value of equity ($prcc_f \times csho$). Since our model is developed based on operating cash flows X_t, we define the return on assets (Operating ROA) as the ratio between operating profits and lagged total assets. A detailed description of the variables is provided in Table 2. For the Total Factor Productivity (TFP) we follow Imrohoroglu and Tuzel (2014).

Observations with missing total assets, quasi-market leverage, common shares outstanding, closing price or equity returns are excluded. Also, we keep firms with total value of assets of at least \$10 million and a market value of equity of at least \$5 million. We winsorize all variables at the 1% level to avoid the influence of outliers. After applying the aforementioned selection criteria, we obtain a panel data set with $351,444$ firm-year observations between 1960 and 2020 at the yearly frequency.

[Insert Table 2 here]

Table 3 provides the descriptive statistics for our sample, which are representative of generic samples from Compustat.

[Insert Table 3 here]

4.2 Parameters' Calibration

For the calibration of the aggregate economy, we use the quarterly aggregate earnings series from NIPA (Series "Net value added of corporate business: Net operating surplus", Table 1.14, Line 8). We calculate the quarterly returns and calibrate the parameters of the aggregate economy, $\mu_A^{\mathcal{P}} = 3.71\%$ and $\sigma_A = 8.68\%$, to the mean and standard deviation of such returns.

We set the base case parameters as follows. Using the estimates in Graham (1999, 2000), we set the personal tax rate on dividends $\tau^e = 11.6\%$ and the tax rate on interest income $\tau^d = 29.3\%$. The corporate tax rate is set to the maximum marginal tax rate $\tau^c = 35.0\%$. The risk-free rate is $r = 4.39\%$ and it is calibrated to the 3-Month Treasury Bill Secondary Market Rate. The proportional adjustment costs is set to $\lambda = 1.0\%$ as in Morellec et al. (2012). We normalize the initial value of operating cash flows $X_0 = 5.0$ and set the distress boundary X_D equal to the value of coupon C.

[Insert Table 4 here]

4.3 Identification

We estimate 6 parameters: the parameter γ that represents the pre-default costs of financial distress, the loss given default α, the expected (risk neutral) growth rate of EBIT μ, the idiosyncratic EBIT volatility during non-distress σ_F^L, the idiosyncratic EBIT volatility during pre-distress σ_F^H, and the exposure to market risk β. The selection of moments used in the SMM estimation is important to ensure that the parameters of interest are identified. Therefore, it is important to choose moments that are a priori informative about the unknown structural parameters. Intuitively, a moment is informative about an unknown parameter if that moment is sensitive to changes in the parameter.

We use eight empirical moments for the identification of our model: the mean and volatility of operating ROA, the average excess equity returns, the quasi-market leverage, the average and volatility of the growth rate of Total Factor Productivity (ΔTFP) during times of distress and non-distress. We estimate the rest of the model parameters (for example, the risk-free rate r)

separately and set them equal to the values of the Base Scenario which are set to the values described in Section 3. For each scenario, we simulate the model 5,000 times for a time period of 150 years. We keep only the last 80 quarters of data to remove the effect of the initial conditions. For each simulation, we calculate model moments and then we average across all simulations.

Every moment is affected by all parameters of our model. However, some parameters are more important than others for a particular moment. In Table 5 we describe which moment is most important to identify each parameter using the elasticities of the model moments with respect to each parameter. The elasticity of moment m with respect to parameters p is defined as $\frac{dm/m}{dp/p}$. The elasticities are calculated at the estimated parameter values discussed in Section 5.

The idiosyncratic volatility σ_F^L is directly related to the volatility of ΔTFP during non-distress. This can be seen from the diffusion parameter of the process described in Equation (3) and from the strong effect that σ_F^L has on the volatility of ΔTFP during non-distress with an elasticity of 0.75 as shown in Table 5. Similarly, σ_F^H is related to the volatility of ΔTFP during times of distress as shown by the elasticity of 0.834. The parameters μ and β are identified mainly through their effects on the average Operating ROA, excess equity returns, as well as the average ΔTFP in times of non-distress and distress. Intuitively, both β and μ affect the profitability of the firm but they have different effects on the four moments which allow us to estimate both parameters. The parameter $beta$ also affects the volatility of the firm through the exposure to the market volatility. To confirm that we are able to separately identify σ_F^L, σ_F^H and β, in Figure 2 we plot excess equity returns and the volatility of ΔTFP in non-distress and distress times as a function of β, σ_F^L and σ_F^H. This figure shows that β is positively related only to equity returns (see Panel A) and has a much more muted effect on the volatility of ΔTFP (see panels D and E).

[Insert Table 5 and Figure 2 here]

Disentangling pre-default costs of financial distress from the loss given default requires us to separately identify the parameters α and γ. In Table 5, we show that both α and γ negatively affect leverage which is consistent with other models in the literature (e.g. Elkamhi et al., 2012). However, γ strongly affects the average ΔTFP in times of distress with a sensitivity of -1.337 while α has almost no effect on such moment. Therefore, the combination of leverage and average ΔTFP in times of distress should provide enough information to identify α and γ because the two parameters have different effects on the simultaneous behaviour of these two moments.

Figure 3 provides additional evidence for the identification of α and γ. In Panel A, we show how leverage and average ΔTFP in times of distress are affected by α. Specifically, we simulate the model 5,000 times for a horizon of 150 years. We keep only the last 80 quarters of data to remove the effect of the initial conditions. For each simulation, we calculate the average Quasi-Market Leverage and average ΔTFP in times of distress. We then average across all simulations. We plot Quasi-Market Leverage (left y-axis) and the average ΔTFP in times of distress (right y-axis) as a function of α for a fixed level of $\gamma = 7.4\%$, and the other model parameters are set to the Base Case scenario described in Table 1. Panel A shows that increasing α leads to lower leverage (blue solid line) and while average ΔTFP in times of distress does not change much (i.e., the red dashed line is flat). In Panel B we repeat the same exercise but we vary γ for a fixed level of $\alpha = 26.3\%$. Panel B shows that increasing γ decreases leverage (blue solid line) as well as average ΔTFP in times of distress (red dashed line). This is different from the implication that α had on these two moments (Panel A of Figure 3).

[Insert Figure 3 here]

5 SMM Estimation Results

We first present the results of the estimation of our model using SMM on the entire sample under the assumption that financial distress starts when the operating cash flows of the firm are lower than the required coupon payments. Next, we present the results for sub-samples split by industries and tangibility. We then show the results of the estimation under alternative definitions of financial distress and calculate the net present value of expected financial distress costs.

5.1 Estimations for the entire sample

We estimate two different specifications of our model. We label them (1) *Model With Pre-Default Costs*, and (2) *Model Without Pre-Default Costs*. The *Model With Pre-Default Costs* estimates 6 parameters using the moments described in details in Section 4.3. This model includes both parameter γ (pre-default costs) and parameter α (loss given default).

Table 6 presents the estimated parameters for the estimation of the *Model With Pre-Default Costs*. The estimated parameter β (exposure of the firm's cash flows to market risk) is approximately equal to 0.94 . The parameter μ (risk-neutral expected growth rate of the firm) is 1.8% and the volatility of the expected earnings' growth rate during non-distress periods σ_F^L is estimated at 23.1% while σ_F^H (volatility during distress) is estimated at 32%.

The two main parameters of interest for our study are γ and α. We find that $\gamma = 7.4\%$ and $\alpha = 26.3\%$. Our estimate of γ implies that, on average, when firms are in financial distress they are expected to lose approximately 7.4% of their value per year. This value is large and, as we explain further below, is such that pre-default costs of financial distress constitute a large proportion of the total costs of financial distress.

In addition, our estimated value corroborates the findings of Hortaçsu et al. (2013) related to the automotive sector. The authors find that during the 2008 financial crisis General Motors would have lost approximately 6% of its operating margins if it were to become distressed.[13] Our estimate of γ is also consistent with the empirical evidence from the casino industry in Cookson (2017). The author studies a specific type of pre-default costs, namely the inability of highly levered casinos to respond to the entry of a new competitor. His evidence suggest that such pre-default costs can account for approximately 5% of firm value.

The estimated parameter α implies that firms are expected to lose 26.3% at the time of default. Such a value is consistent with empirical values estimated by Davydenko et al. (2012) and Korteweg (2010) that estimate the average costs to be approximately 15-30% of firm value. Glover (2016) shows that firms that have lower expected default costs use more leverage and, consequently, they end up defaulting more often. Even if the average distress costs are higher for the average firm in the economy, when calculated for the sub-sample of defaulted firms distress costs are approximately 25%, a value consistent with empirical evidence (Andrade and Kaplan, 1998; Davydenko et al., 2012).

[13]They define General Motors to be distressed if it experiences a 3,000 CDS point increase relative to Ford. Such a loss in operating margins could imply a loss in firm value of up to 10%. Please see Hortaçsu et al. (2013) for more details.

Our results provide an alternative explanation that can co-exist with the one in Glover (2016). In addition to the argument proposed by Glover, studying a sample of defaulted firms might lead to a bias in the estimate of distress costs because defaulted firms lose only the loss given default α and they have already experienced (large) pre-default costs of financial distress. Therefore, a selection bias still exists but the source can be in the timing of when distress costs are experienced. Indeed, our model shows that even if our firms are all equal ex-ante, there would be a bias in the measurement of financial distress costs if they are measured only at default. Only the loss given default would be captured while the (large) pre-default costs would be omitted.

Next, we address the bias caused by the omission of pre-default costs of financial distress. The standard procedure to calibrate trade-off models consists of using the empirical estimates of tax benefits (Graham, 2000) and default costs (Andrade and Kaplan, 1998; Davydenko et al., 2012). However, in order to calculate empirical estimates of default costs, previous studies estimate the change in firm value that happens around the time of default, when the firm has (potentially) already incurred pre-default costs. Therefore, applying such estimates to firms that are not close to default would underestimate the effective costs of financial distress (Elkamhi et al., 2012). To measure the magnitude of this underestimation, we estimate a modified version of our model which we label *Model Without Pre-Default Costs*. In this specification, we exogenously set $\gamma = 0$ so the costs of financial distress are experienced only at the time of default when the firm is expected to lose a portion α of its value. We fit this specification to the data using the same eight moments that we used to estimate the *Model With Pre-Default Costs*. Comparing the estimated α in the *Model Without Pre-Default Costs* with the one from the *Model With Pre-Default Costs* provides an estimate of the factor by which empirical estimates should be multiplied.

Table 6 shows that that the estimated loss given default α in the *Model Without Pre-Default Costs* is 55.0%. This value is more than twice as large as the 26.3% that we had estimated with the *Model With Pre-Default Costs*. This suggests that there is a large bias. Researchers should not only be aware of such bias but they should also be using a higher value of loss given default than the empirical estimates (Davydenko et al., 2012) in the calibration of their models if they omit pre-default costs.

[Insert Table 6 and Table 7 here]

Table 7 shows the model fit. We report the empirical moments (column "Data") as well as the simulated moments from the model. The t-statistics for the difference between empirical and simulated moments are reported in parenthesis. The *Model With Pre-Default Costs* fits the data well. The simulated quasi-market leverage and average ΔTFP during distress are 40.2% and -3.9%, respectively. As confirmed by the low t-statics, these values are not statistically different from their empirical counterparts which are 39.7% for leverage and -5.0% for the average ΔTFP during distress. Table 7 also confirms that the other simulated moments are close to their their empirical counterparts, even if some are statistically different. Furthermore, the J-test cannot reject the model at 1% level as shown by the J-statistic which is 0.208.

The *Model Without Pre-Default Costs* also fits the data well but, unlike the *Model With Pre-Default Costs*, it requires the use of a loss at default α that is much higher than what empirical findings suggest to be reasonable. As is the case for the Model with Pre-Default Costs, even for the Model Without Pre-Default Costs, the J-test cannot reject the model at 1% level (J-statistic is 0.110).

5.2 Splits by Tangibility, Size and Industries

It is known that firms with more pledgeable (tangible) assets take on more leverage (Almeida and Campello, 2007) and they are also able to recover more in case of default (Elkamhi et al., 2014). Therefore, we split our sample based on the tangibility of their assets to evaluate how loss at default and pre-default costs change with the tangibility of assets. Intuitively, for a given level of leverage, we should expect firms with more tangible assets to recover more than intangible firms since creditors will have physical assets to claim. Therefore the loss given default should be lower. Using Compustat, we define tangibility as "Property Plant and Equipment - Total (Gross)" divided by Total Assets. For every year, we group firms into "High Tangibility" (above median) and "Low Tangibility" (below median).

Table 8 shows the model fit for the estimation of the *Model With Pre-Default Costs* on the subsample of firms split by tangibility. We present the estimated parameters and their standard errors (in parentheses) based on the splits by tangibility in Panel A. The loss at default α for high and low tangibility are 16.5% and 35.2%, respectively. This large difference in loss given default between the two sub-samples confirms the intuition that more tangible firms are expected to recover more (i.e. have a lower loss at default) than intangible firms. In addition, our estimation shows that there is a clear difference not only for the loss given default but also for the pre-default costs' parameter γ. The parameter γ is equal to 8.3% for highly tangible firms and 2.6% for low tangibility firms, thus showing that highly tangible firms are expected to suffer more from pre-default costs of financial distress. This result challenges the literature documenting that distressed firms with low tangible (high intangible) capital lose more human capital than high tangible (low intangible) capital (Babina, 2020; Gortmaker et al., 2019; Baghai et al., 2017).[14] Panel B of Table 8 shows the empirical moments, the simulated moments, and the t-statistics for the difference between empirical and simulated moments. The model fits both low and high tangibility firms well since none of the t-statistics display a statistically significant difference between empirical and simulated moments.

Table 9 shows the model fit for the estimation of the *Model With Pre-Default Costs* on the sub-sample of firms split by firm size. We measure firm size as the market value of the firm which is given by the market value of equity plus the book value of debt. Alternative definitions for firm size are possible such as using the accounting value of total assets. We elect to use the firm's market value to capture changes in the size of the firm as dictated by the market rather than simply by accounting variables. Specifically, for each year, we sort firms based on their firm market value and then classify them as Big as they are above median and small otherwise. We present the estimated parameters and their standard errors (in parentheses) based on the splits by firm size in Panel A of Table 9. The loss given default α for Big and Small firms are 31.7% and 24.2%, respectively. The parameter γ is equal to 2.6% for Big firms and 7.4% for Small firms. The difference in pre-default costs of financial distress is rather large while the difference in loss at default is relatively very small. This suggests that small firms have overall higher total total costs of financial distress and they lose more than big firms when they are financially distressed. This is consistent with the literature

[14]Babina (2020) shows that employees in sectors with more intangible assets are considerably more likely to leave their (distressed) firms and start new companies. Gortmaker et al. (2019) and Baghai, Silva, Thell, and Vig (2017) show that talented employees are more likely to leave the firm after a deterioration in a firm's credit quality.

showing that smaller firms pay higher markups over LIBOR during financial crises and have less access to credit lines which would lead to liquidity problems in case of distress (e.g., Campello, Giambona, Graham, and Harvey, 2011).

[Insert Table 8 and Table 9 here]

To further evaluate how pre-default costs vary cross-sectionally, we estimate our model on samples splits by industry. We define an industry according to the 2-digit SIC code, which results in 8 different industries. We present the parameter estimates in Table 10 while Table 11 displays the model fit for each industry. There is a large variation in the estimated parameters across industries. The pre-default costs vary from a minimum of 3.2% to a maximum of 8.3%. Industries such as Mining, Retail Trade and Services show considerably higher estimated values of pre-default costs γ compared to Construction and Wholesale Trade. The loss at default α also displays large cross-sectional variation. The estimated loss given default varies from 16.5% to a maximum of 33.0%.

Our results show that the average α and γ by industry are not only varying when considered independently but there is also a cross-sectional variation in the combined levels. That is, for some industries such as Construction the estimates of both α and γ are below average. For some others (e.g. Transportation) the estimate of α is above average while the estimate of γ is below average.

[Insert Table 10 and Table 11 here]

Overall, the model is able to fit the various industries quite well.

5.3 Pre-default costs and loss at default as a percentage of total distress costs

It is interesting to understand how large pre-default costs of financial distress are with respect to the loss at default. To provide such estimate we can calculate the net present value of pre-default costs of financial distress as well as the net present value of the loss at default and compare them.Specifically, our intuition is as follows. First, we calculate the price of a claim –which we label PDC– that pays a fraction γ of firm's EBIT (i.e., $\gamma \times X_t$) when the firm is in distress (i.e., $X_t < X_D$). Second we calculate the price of a claim –which we label LGD– that pays the recovery value of the firm in case of default (i.e., $((1-\alpha) \times V(X_B))$. The total NPV of financial distress is equal to $LGD + PDC$.[15]

Table 12 shows the value of pre-default costs and loss at default as a percentage of total distress costs. Specifically, the column $\% \, PDC$ presents the percentage of pre-default costs as a fraction of the total distress costs (loss given default plus pre-default costs); the column $\% \, LGD$ presents the same percentage for the loss given default. The percentage of pre-default costs as a fraction of the total is 68.5%. This result shows that the net present value of pre-default costs of financial distress are large compared to the loss given default.

6 Conclusion

This paper quantifies the expected costs of financial distress that firms experience prior to default (pre-default costs) separately from the loss at default. We develop a dynamic model of capital structure which includes pre-default costs and we fit it to the data. We find that pre-default costs are large and on average equal to 7.4% of firm value per year during times of financial distress. We show that there is a large cross sectional variation of pre-default costs across industries, from 3.2% to 8.3%. Also, our estimates show that firms with more tangible assets have a lower loss given default and higher pre-default costs than intangible firms.

Our study has implications for academics using calibrated dynamic capital structure models to gauge insights of the firms' responses to a policy change or to evaluate the behaviour of leverage. We also show an alternative explanation to the selection bias discussed in Glover (2016) based on the timing of financial distress. While Glover (2016) demonstrate that there exists a selection bias because firms are ex-ante different and that defaulted firms have lower distress costs than the average firm, we show that measuring distress costs from defaulted firms would lead to a bias because we would not capture the (large) pre-default costs experienced prior to default. Our rationale does not invalidate the argument in Glover (2016) but can co-exists with it. Furthermore, we document that omitting pre-default costs leads to a large bias in the estimates of loss at default when using dynamic models of capital structure. While the average loss given default in our model with pre-default costs of financial distress is 26.3%, the same model where we exogenously fix

[15] We provide the expressions to value pre-default costs and loss given default in Appendix D.

pre-default costs to zero needs a loss given default of 55.0%.

Lastly, we do not "micro-found" the parameter that measures pre-default costs of financial distress (γ) therefore our results are silent on the consequences that pre-default costs of financial distress have on investment, dividend payouts, customer/supplier relations, etc. These are all important questions that highlight a number of new avenues that could be explored in future studies.

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Figures

Figure 1
Model Intuition

This figure shows the impact that pre-default costs have on the path of EBIT. EBIT follows a geometric brownian motion as described in Equation (3):

$$\frac{dX_t}{X_t} = \begin{cases} \left(\mu + \beta(\mu_A^{\mathcal{P}} - r)\right) dt + \beta\sigma_A dB_t^{A,\mathcal{P}} + \sigma_F^L dB_t^F & \text{for } X_B \leq X_D \leq X_t \\ \underbrace{\left(\mu + \beta(\mu_A^{\mathcal{P}} - r)\right) dt}_{\substack{\text{Expected growth} \\ \text{rate without distress}}} + \underbrace{\beta\sigma_A dB_t^{A,\mathcal{P}} + \sigma_F^H dB_t^F}_{\substack{\text{Systematic and} \\ \text{Idiosyncratic shocks}}} \underbrace{-\gamma dt}_{\substack{\text{Pre-default} \\ \text{Costs}}} & \text{for } X_B < X_t < X_D \end{cases}$$

The figure shows two scenarios. The blue solid line shows the path of the firm's EBIT if it had no pre-default costs of financial distress ($\gamma = 0$) while the red dashed line shows the scenario when the firm's has pre-default costs ($\gamma = 2\%$). The rest of the parameters are the same in the two scenarios. Both paths start from X_0 and they evolve according to the law of motion described above. When X_t is below X_D, the firm experiences pre-default costs. When X_t reaches X_B the firm defaults.

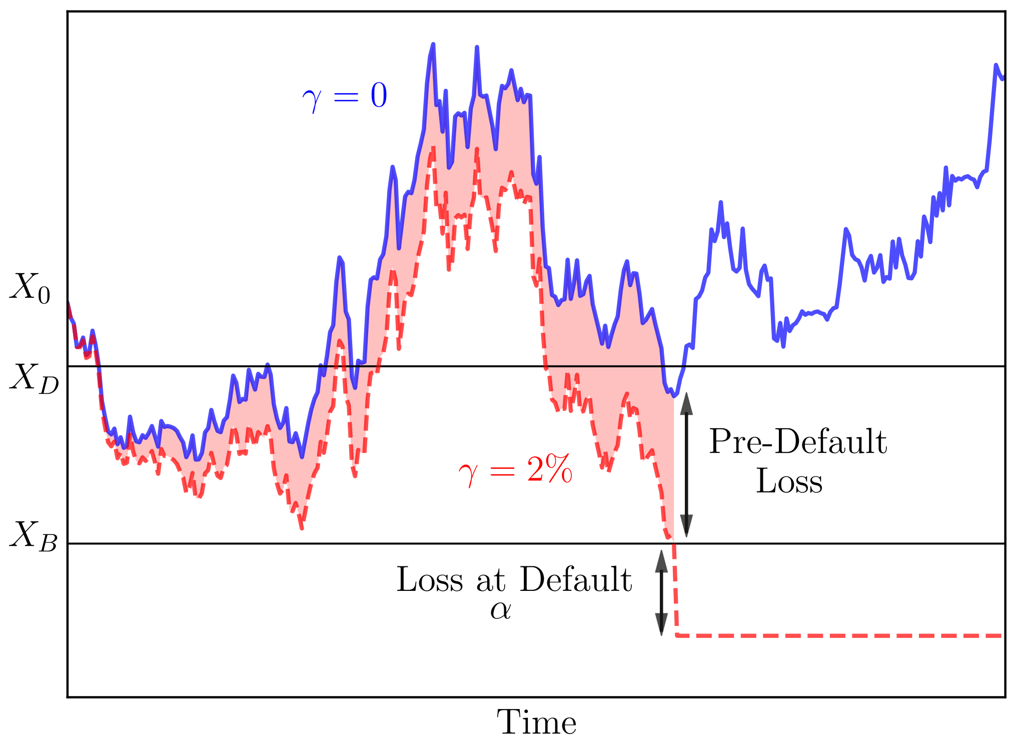


Figure 2

Identification β, σ_F^L, and σ_F^H

This figure depicts the relation between the firm's exposure to market risk β and Equity Return (Panel B), the relation between the idiosyncratic volatility of non-distress EBIT growth σ_F^L and volatility of TFP growth during non-distress period (Panel B), and the relation between the idiosyncratic volatility of distress EBIT growth σ_F^H and volatility of TFP growth during distress period (Panel C). The data used to create these figures have been generate as follows. We simulate the model 5,000 times for a time period of 125 years. We keep only the last 60 years of data to remove the effect of the initial conditions, and we calculate the Variance of Equity and the Equity Return. The model parameters are set to the base case scenario described in Table 1. The model parameters are set to the base case scenario described in Table 1.

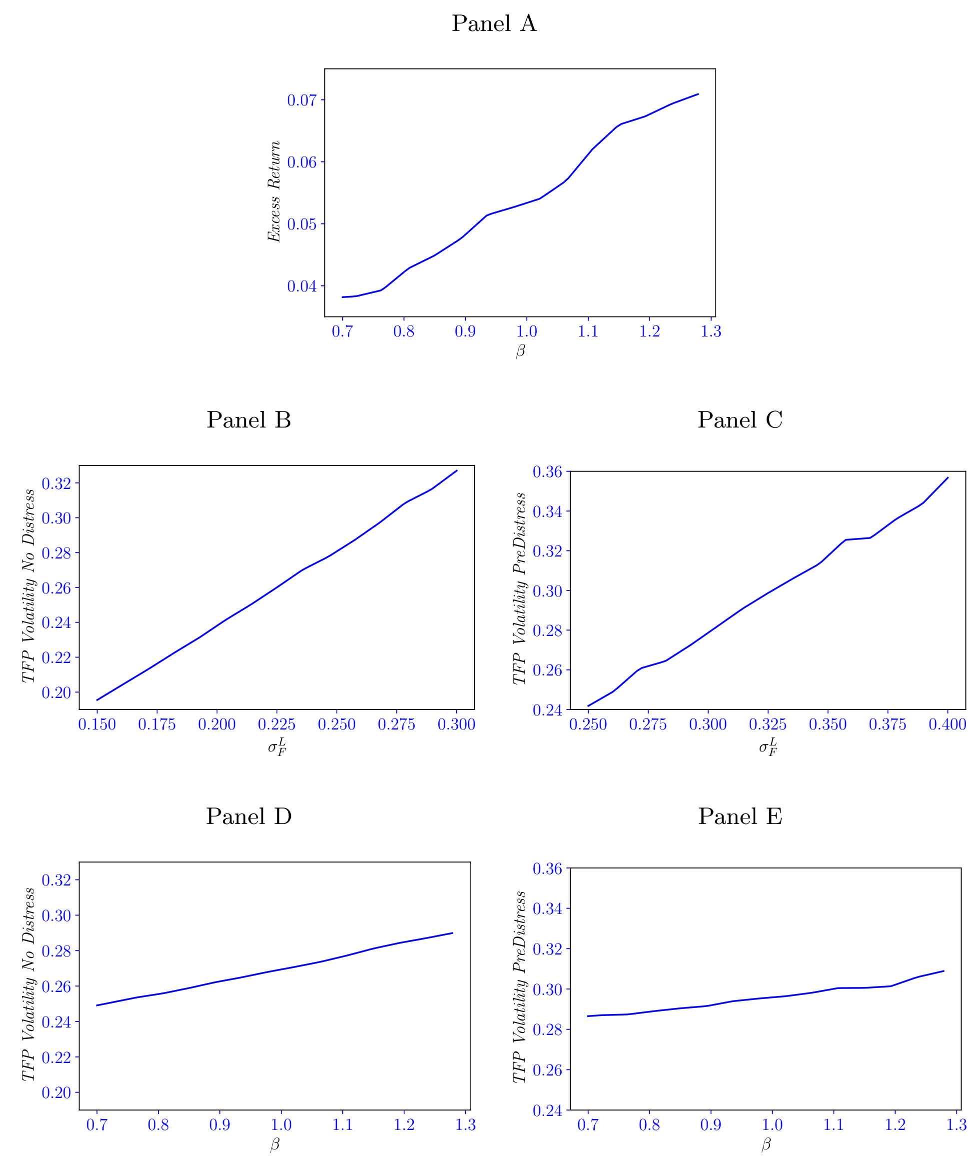
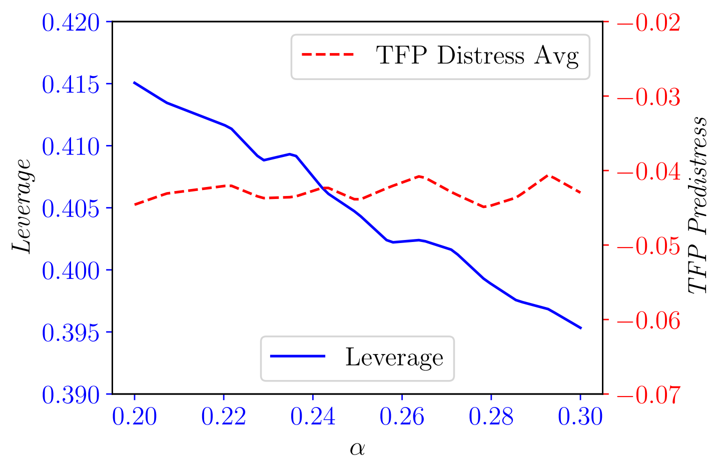


Figure 3

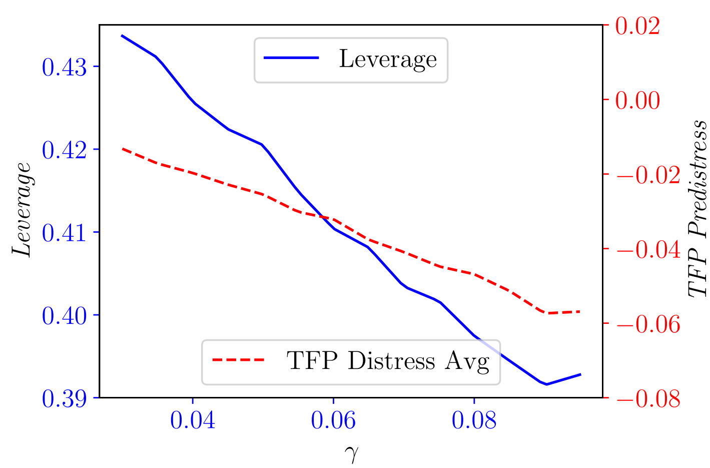
Identification of α and γ

This figure depicts the relation between the default-loss α, the pre-default costs γ, and the moments of leverage and pre-distress TFP growth. The data used to create these figures have been generate as follows. We simulate the model 5,000 times for a time period of 125 years. We keep only the last 60 years of data to remove the effect of the initial conditions. For each simulation, we calculate the Quasi-Market Leverage and pre-distress TFP growth. We then average across all simulations. Panel A plots the Quasi-Market Leverage (left y-axis) and the pre-distress TFP growth (right y-axis) as a function of α for a fixed level of $\gamma = 7.4\%$. Panel B plots Quasi-Market Leverage (left y-axis) and pre-distress TFP growth (right y-axis) as a function of γ for a fixed level of $\alpha = 26.3\%$. The other model parameters are set to the base case scenario described in Table 1.

Panel A



Panel B



Tables

Table 1
Comparative Statics

This table presents the comparative statics of the model with regards to financing decisions. We set the base case parameters as follows. The personal tax rate on dividends $\tau^e = 11.6\%$, the tax rate on interest income $\tau^d = 29.3\%$, the corporate tax rate $\tau^c = 35.0\%$, the aggregate earnings growth rate and volatility are $\mu_A = 3.71\%$ and $\sigma_A = 8.68\%$, the risk-free rate is $r = 4.39\%$, the growth rate and volatility of cash flows are $\mu = 1.8\%$, $\sigma_F^L = 23.1\%$, and $\sigma_F^H = 32\%$, the exposure to market risk parameter is set to $\beta = 0.94$, the loss given default parameter is $\alpha = 26.3\%$, the pre-default costs are set to $\gamma = 7.4\%$, the proportional adjustment costs is set to $\lambda = 1.0\%$. We normalize the initial value of operating cash flows $X_0 = 5.0$ and set the distress boundary X_D equal to the value of coupon C.

		Quasi-Market Leverage (%) at			
Scenario	X_B/X_0	Distress	Target	Restructuring	Recovery Rate (%)
Base	0.092	41.814	29.592	14.571	36.342
Pre-Default Costs (Base: $\gamma = 7.4\%$)					
$\gamma = 10\%$	0.079	40.930	30.188	6.047	31.088
$\gamma = 3\%$	0.132	44.486	34.112	16.856	44.899
Costs at Default (Base: $\alpha = 26.3\%$)					
$\alpha = 29\%$	0.090	41.796	29.234	14.382	34.860
$\alpha = 23\%$	0.094	41.827	30.055	14.795	38.125
Expected Growth Rate of EBIT (Base: $\mu = 1.8\%$)					
$\mu = 1.9\%$	0.090	40.485	29.088	13.944	36.415
$\mu = 1.5\%$	0.100	44.874	32.003	15.857	36.687
Idiosyncratic Volatility (Base: $\sigma_F^L = 23.1\%$)					
$\sigma_F^L = 25\%$	0.090	41.185	28.491	13.610	37.048
$\sigma_F^L = 20\%$	0.096	43.026	31.792	16.408	35.162
Idiosyncratic Volatility (Base: $\sigma_F^H = 32\%$)					
$\sigma_F^H = 35\%$	0.088	41.047	29.667	14.673	34.645
$\sigma_F^H = 30\%$	0.094	42.344	29.536	14.503	37.521
Exposure to market risk (Base: $\beta = 0.94$)					
$\beta = 1.35$	0.088	41.015	28.688	13.850	36.259
$\beta = 0.5$	0.094	42.389	30.271	15.127	36.388

Table 2

Variables Definition

This table presents a description of the empirical variables.

Variables	Definition
Compustat	
Book equity	Stockholders Equity Total (SEQ) + Deferred Taxes and Investment Tax Credit (TXDITC) - Preferred/ Preference Stock (Capital) Total (PSTK). If (PSTK) is missing then we use Preferred Stock Redemption Value (PSTKRV); if (PSTKRV) is missing then we use Preferred Stock Liquidating Value (PSTKL).
Book debt	Assets total (AT) - Book Equity
Market value of equity	Common Shares Outstanding (CSHO) $\times$ Price Close Annual Fiscal Year (PRCC_F)
Quasi-market leverage	Book debt/(Assets total (AT) - Book equity + Market value of equity)
Operating profit	Operating Income Before Depreciation (OIBDP) if OIBDP missing, Sales (SALE) - Operating Expenses (XOPR) if SALE - XOPR missing, Revenues (REVT) - Operating Expenses (XOPR)
Operating ROA	Operating Profit / Lagged Total Asset (AT)
TFP	As defined in Imrohoroglu and Tuzel (2014)
Interest coverage ratio	Interest expense (XINT)/Operating profit
CRSP and FRED	
Risk free rate	3-Month Treasury Bill Secondary Market Rate
Excess return	Equity Returns - Risk-Free Rate

Table 3

Descriptive Statistics

This table presents descriptive statistics for the variables used in the estimation. The sample is based on financial statements from annual Compustat Industrial Files and returns from CRSP. Table 2 provides a detailed definition of the moments.

	Mean	St.Dev	25th	Median	75th
Total assets (billions)	2.039	10.428	0.055	0.184	0.797
Operating ROA	0.163	0.082	0.109	0.152	0.204
Quasi-market leverage	0.397	0.216	0.220	0.382	0.555
Excess return	0.059	0.235	-0.026	0.079	0.169
TFP growth (nodistress)	0.054	0.213	-0.024	0.030	0.116
TFP growth (predistress)	-0.050	0.264	-0.167	-0.023	0.054

Table 4

Parameters' Calibration

This table presents the parameters that are calibrated. A detailed description of the calibration methodology is provided in Section 4.2.

Parameter	Description	Calibrated Value
μ_a	Average growth rate of aggregate economy	3.71%
σ_A	Volatility of aggregate economy	8.68%
τ^e	Personal tax rate on dividends	11.6%
τ^d	Personal tax rate on net income	29.3%
τ^c	Corporate tax rate	35%
r	Risk-free rate	4.39%
λ	Proportional adjustment costs	1%

Table 5

Elasticity of Moments to Parameters

This table shows the elasticity of model-implied moments (in columns) with respect to model parameters (in rows). The elasticity of moment m with respect to parameters p is defined as $\frac{dm/m}{dp/p}$. The elasticities are calculated at the estimated parameter values from Table 6. Parameter α is the parameter that captures the expected loss given default, β is the firm's exposure to market risk, γ is the parameter that captures pre-default costs (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet),μ is the risk-neutral expected growht rate of EBIT, σ_F^L is the idiosyncratic EBIT volatility during non-distress, and σ_F^H is the idiosyncratic EBIT volatility during distress. A description of the moments is provided in Table 2.

	Moments							
	Operating ROA avg	Operating ROA vol	Quasi-market leverage	Excess returns	ΔTFP avg distress	ΔTFP vol distress	ΔTFP avg non-distress	ΔTFP vol non-distress
α	0.000	-0.017	-0.079	-0.850	-0.069	0.001	-0.302	-0.028
β	0.023	0.183	-0.059	0.425	0.900	0.035	0.765	0.260
γ	-0.011	-0.028	-0.120	-1.187	-1.337	0.014	-0.583	-0.023
μ	-0.678	-0.647	-0.287	0.231	1.573	0.109	0.309	0.034
σ_F^L	-0.008	0.567	0.052	0.368	-0.046	0.111	-0.081	0.750
σ_F^H	0.012	0.317	-0.212	0.844	-1.080	0.834	0.104	0.010

Table 6

Parameter Estimates

This table reports the structural parameters estimated via Simulated Method of Moments (SMM). Standard errors are in parentheses. The parameter γ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), α is the parameter that captures the expected loss given default, β is the exposure of the firm's cash flows to market risk, μ captures a firm fixed effect for the expected earnings' growth rate, σ_F^L is a firm fixed effect for the volatility of the expected earnings' growth rate during non-distress, and σ_F^H is a firm fixed effect for the volatility of the expected earnings' growth rate during distress. We estimate three different specifications of our model: *Model With Pre-Default Costs* includes both the parameter γ to capture pre-default costs of financial distress and the parameter α that captures the expected loss given default and *Model Without Pre-Default Costs* exogenously sets $\gamma = 0$ so the costs of financial distress are experienced only at the time of default when the firm is expected to lose a portion α of its value. Standard errors are clustered by firm.

Parameter	Description	Model With Pre-Default Costs	Model Without Pre-Default Costs
γ	Pre-default costs	0.074	
		(0.000)	
α	Loss at-default	0.263	0.550
		(0.046)	(0.030)
β	Exposure to market risk	0.940	0.752
		(0.006)	(0.004)
μ	Idiosyncratic component	0.018	0.022
		(0.000)	(0.000)
σ_F^L	Volatility of EBIT during non distress	0.231	0.262
		(0.001)	(0.000)
σ_F^H	Volatility of EBIT during distress	0.320	0.336
		(0.001)	(0.001)

Table 7

Simulated Moments Estimation

The estimation is conducted via Simulated Method of Moments (SMM) which searches for the model parameters that minimize the distance between the empirical moments and the moments calculated from a simulated panel of firms. This table shows the empirical moments (column Data) as well as the simulated moments. The t-statistics for the difference between empirical and simulated moments are reported in parenthesis. We estimate three different specifications of our model: *Model With Pre-Default Costs* includes both the parameter γ to capture pre-default costs of financial distress and the parameter α that captures the expected loss given default and *Model Without Pre-Default Costs* exogenously sets $\gamma = 0$ so the costs of financial distress are experienced only at the time of default when the firm is expected to lose a portion α of its value.

	Data	Model With Pre-Default Costs	Model Without Pre-Default Costs
Operating ROA avg	0.163	0.186	0.160
		(0.283)	(0.040)
Operating ROA vol	0.061	0.071	0.066
		(0.269)	(0.122)
Quasi-market leverage	0.397	0.402	0.388
		(0.026)	(0.040)
Excess returns	0.059	0.049	0.099
		(0.043)	(0.165)
ΔTFP avg distress	-0.050	-0.039	0.006
		(0.037)	(0.190)
ΔTFP vol distress	0.321	0.298	0.309
		(0.103)	(0.054)
ΔTFP avg non-distress	0.054	0.056	0.065
		(0.006)	(0.045)
ΔTFP vol non-distress	0.304	0.265	0.283
		(0.213)	(0.114)
J-test		0.208	0.110

Table 8

Simulated Moments Estimation: Splits by Tangibility

This table describes the results for firms split by the tangibility. We define tangibility as "Property Plant and Equipment - Total (Net)" over Total Assets. Panel A reports the structural parameters estimated via Simulated Method of Moments (SMM). Standard errors are in parentheses. The parameter γ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), α is the parameter that captures the expected loss given default, β is the exposure of the firm's cash flows to market risk, μ captures a firm fixed effect for the expected earnings' growth rate, σ_F^L is a firm fixed effect for the volatility of the expected earnings' growth rate during non-distress, and σ_F^H is a firm fixed effect for the volatility of the expected earnings' growth rate during distress. Standard errors are clustered by firm. Panel B shows the empirical moments, the simulated moments, and the t-statistics for the difference between empirical and simulated moments. A detailed description of the moments is provided in Table 2.

Panel A: Parameter Estimates by Tangibility

	γ	α	β	μ	σ_F^L	σ_F^H
Low tang	0.026	0.352	0.780	0.019	0.241	0.379
	(0.002)	(0.096)	(0.016)	(0.000)	(0.000)	(0.004)
High tang	0.083	0.165	0.800	0.015	0.192	0.312
	(0.000)	(0.043)	(0.007)	(0.000)	(0.000)	(0.001)

Panel B: Model Fit by Tangibility

		Operating ROA avg	Operating ROA vol	Quasi-mkt leverage	Excess returns	ΔTFP avg predistress	ΔTFP vol predistress	ΔTFP avg nodistress	ΔTFP vol nodistress
L tang	Data	0.163	0.061	0.362	0.146	-0.047	0.337	0.063	0.314
	Model	0.180	0.074	0.404	0.089	-0.016	0.347	0.062	0.264
	t-stat	0.205	0.318	0.199	0.237	0.106	0.047	0.002	0.269
H tang	Data	0.166	0.056	0.449	0.048	-0.049	0.296	0.049	0.283
	Model	0.207	0.069	0.436	0.029	-0.052	0.283	0.046	0.219
	t-stat	0.512	0.342	0.060	0.080	0.012	0.057	0.013	0.346

Table 9

Simulated Moments Estimation: Splits by Firm Size

This table describes the results for firms split by the firm size. Firm size is defined as the market value of the firm (market value of equity plus book-value of debt). Panel A reports the structural parameters estimated via Simulated Method of Moments (SMM). Standard errors are in parentheses.The parameter γ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), α is the parameter that captures the expected loss given default, β is the exposure of the firm's cash flows to market risk, μ captures a firm fixed effect for the expected earnings' growth rate, σ_F^L is a firm fixed effect for the volatility of the expected earnings' growth rate during non-distress, and σ_F^H is a firm fixed effect for the volatility of the expected earnings' growth rate during distress. Standard errors are clustered by firm. Panel B shows the empirical moments, the simulated moments, and the t-statistics for the difference between empirical and simulated moments. A detailed description of the moments is provided in Table 2.

Panel A: Parameter Estimates by Firm Size

	γ	α	β	μ	σ_F^L	σ_F^H
Small	0.074	0.242	0.320	0.017	0.282	0.312
	0.001	0.053	0.018	0.000	0.001	0.001
Big	0.026	0.317	0.880	0.017	0.208	0.328
	0.001	0.073	0.010	0.000	0.001	0.001

Panel B: Model Fit by Firm Size

		Operating ROA avg	Operating ROA vol	Quasi-market leverage	Excess returns	ΔTFP avg distress	ΔTFP vol distress	ΔTFP avg non-distress	ΔTFP vol non-distress
Small	Data	0.162	0.060	0.410	0.091	-0.068	0.331	0.046	0.319
	Model	0.191	0.076	0.456	0.023	-0.061	0.273	0.030	0.280
	t-stat	0.367	0.427	0.217	0.279	0.024	0.254	0.072	0.208
Big	Data	0.174	0.056	0.353	0.052	-0.016	0.297	0.066	0.272
	Model	0.194	0.070	0.429	0.078	-0.010	0.305	0.060	0.241
	t-stat	0.251	0.352	0.358	0.109	0.020	0.033	0.024	0.168

Table 10

Parameter Estimates: Industry Splits

This table reports the structural parameters estimated via Simulated Method of Moments (SMM) for various industries. Standard errors are in parentheses. The parameter γ captures pre-default costs of financial distress (i.e. constant rate at which the firm loses value when it is in financial distress but has not defaulted yet), α is the parameter that captures the expected loss given default, β is the exposure of the firm's cash flows to market risk, μ captures a firm fixed effect for the expected earnings' growth rate, σ_F^L is a firm fixed effect for the volatility of the expected earnings' growth rate during non-distress, and σ_F^H is a firm fixed effect for the volatility of the expected earnings' growth rate during distress. Standard errors are clustered by firm.

	γ	α	β	μ	σ_F^L	σ_F^H
Agriculture, Forestry, & Fishing	0.082	0.180	0.525	0.022	0.263	0.520
	(0.002)	(0.012)	(0.010)	(0.000)	(0.000)	(0.004)
Construction	0.037	0.165	0.470	0.020	0.231	0.520
	(0.000)	(0.038)	(0.006)	(0.000)	(0.000)	(0.001)
Manufacturing	0.065	0.330	1.293	0.020	0.231	0.320
	(0.001)	(0.109)	(0.007)	(0.000)	(0.001)	(0.003)
Mining	0.064	0.285	0.880	0.019	0.294	0.370
	(0.001)	(0.054)	(0.007)	(0.000)	(0.001)	(0.001)
Retail Trade	0.083	0.170	0.880	0.015	0.200	0.320
	(0.001)	(0.030)	(0.010)	(0.000)	(0.001)	(0.002)
Services	0.064	0.330	0.840	0.020	0.263	0.320
	(0.001)	(0.050)	(0.005)	(0.000)	(0.001)	(0.001)
Transportation	0.048	0.285	0.640	0.019	0.200	0.320
	(0.001)	(0.037)	(0.008)	(0.000)	(0.000)	(0.001)
Wholesale Trade	0.032	0.235	0.480	0.017	0.231	0.320
	(0.000)	(0.028)	(0.005)	(0.000)	(0.000)	(0.001)
Average	0.059	0.248	0.751	0.019	0.239	0.376

Table 11

Simulated Moments Estimation: Industry Splits

This table shows the empirical moments and the simulated moments for different industries. The t-statistics for the difference between empirical and simulated moments are also reported. A detailed description of the moments is provided in Table 2.

		Operating ROA avg	Operating ROA vol	Quasi-market leverage	Excess returns	ΔTFP avg distress	ΔTFP vol distress	ΔTFP avg non-distress	ΔTFP vol non-distress
Agriculture, Forestry,	Data	0.141	0.047	0.411	0.082	-0.053	0.344	0.081	0.340
& Fishing	Model	0.157	0.058	0.272	0.040	0.000	0.201	0.051	0.282
	t-stat	0.203	0.265	0.648	0.174	0.180	0.625	0.132	0.310
Construction	Data	0.121	0.055	0.564	0.104	-0.027	0.424	0.071	0.356
	Model	0.171	0.082	0.397	0.081	-0.017	0.490	0.068	0.244
	t-stat	0.627	0.700	0.780	0.097	0.035	0.293	0.014	0.604
Manufacturing	Data	0.168	0.064	0.375	0.131	-0.071	0.331	0.065	0.321
	Model	0.173	0.071	0.371	0.077	-0.028	0.311	0.074	0.293
	t-stat	0.062	0.180	0.017	0.222	0.146	0.087	0.035	0.151
Mining	Data	0.177	0.074	0.419	0.069	-0.060	0.377	0.092	0.342
	Model	0.179	0.083	0.398	0.070	-0.041	0.339	0.062	0.320
	t-stat	0.031	0.221	0.100	0.005	0.062	0.168	0.127	0.118
Retail Trade	Data	0.163	0.053	0.455	0.093	-0.069	0.300	0.034	0.268
	Model	0.208	0.073	0.432	0.037	-0.058	0.286	0.051	0.233
	t-stat	0.557	0.521	0.107	0.230	0.037	0.062	0.075	0.190
Services	Data	0.162	0.061	0.339	0.155	-0.025	0.301	0.027	0.291
	Model	0.171	0.070	0.398	0.058	-0.037	0.293	0.055	0.287
	t-stat	0.104	0.223	0.276	0.402	0.041	0.032	0.118	0.020
Transportation	Data	0.153	0.051	0.498	0.121	0.032	0.276	0.041	0.228
	Model	0.178	0.060	0.412	0.046	-0.030	0.287	0.048	0.219
	t-stat	0.318	0.223	0.400	0.307	0.209	0.046	0.032	0.052
Wholesale Trade	Data	0.138	0.049	0.512	0.122	-0.054	0.326	0.062	0.304
	Model	0.194	0.071	0.463	0.069	-0.027	0.286	0.047	0.239
	t-stat	0.698	0.557	0.229	0.219	0.092	0.176	0.063	0.352

Table 12

Composition of Financial Distress Costs

This table presents a decomposition of total financial distress costs into pre-default costs and loss given default. The column $\%\,PDC$ presents the percentage of pre-default costs over the total distress costs (loss given default plus pre-default costs); the column $\%\,LGD$ presents the same percentage for the loss given default. We provide the expressions to value pre-default costs and loss given default in Appendix D.

	$\%\,PDC$	$\%\,LGD$	Total
$X_D = 1 \times C$	64.1%	35.9%	100%

Appendix A Model details

We compute the value of debt and equity for fixed levels of coupon (C), distress (X_D) and default (X_B) thresholds as well as the restructuring boundary (X_U). The value of EBIT at the time when the firm makes its decision if X_0. When the firm's EBIT reaches X_U, the firm retires its previously issued debt at par and it issues new one.

A.1 Net Income

We start by computing the value of a claim on net income, which represents the cash flows continuously accruing to shareholders at each time t, $(1-\tau)(X_t - C)$. We define a refinancing cycle as the period of time over which the firm's capital structure does not change. That is, the period of time when X_t, remains between X_U and X_B. Formally, let $T = \min\{T^U, T^B\}$ where $T^U = \inf\{t > 0 : X_t \geq X_U \text{ and}, \forall\, s < t\,, X_s > X_B\}$ is the first time EBIT reaches the restructuring boundary conditional on not having hit the default threshold, and $T^B = \inf\{t > 0 : X_t \leq X_B$ and $\forall\, s < t\,, X_s < X_U\}$ is the first time X_t reaches the default threshold conditional on not having hit the restructuring boundary. The refinancing cycle is defined as the period of time until time T, $\{t : 0 < t < T\}$.

Consider a claim over net income over one refinancing cycle.[16] This claim depends on the level of EBIT, X_t, and the coupon paid on the firm's outstanding debt. Intuitively, this claim is simply the expected net present value of the cash flows accrued to shareholders between time t and T (the first time the firm changes its capital structure either by defaulting or restructuring its debt). Its value is

$$\mathbf{n}(X_t, C) = \begin{cases} \mathbb{E}^{\mathcal{Q}}\left[\int_t^T (1-\tau)(X_s - C)e^{-r(s-t)}ds \mid X_D \leq X_t \leq X_U\right] \\ \mathbb{E}^{\mathcal{Q}}\left[\int_t^T (1-\tau)(X_s - C)e^{-r(s-t)}ds \mid X_B < X_t < X_D\right] \end{cases} \tag{A.1}$$

Equation (A.1) considers two cases: for $X_D \leq X_t \leq X_U$, the firm does not incur pre-default costs and its EBIT X_t is governed by the process described in the top expression in Equation (3); for $X_B < X_t < X_D$, the firm experiences pre-default costs and it loses a fraction γ of its EBIT per period of time as described in the bottom expression in Equation (3).

To simplify the notation, we denote the state when the firm is not distressed, $X_D \leq X_t \leq X_U$, as ND (No Distress state) and the state when it is distressed, $X_B < X_t < X_D$, as DS (Distressed State). Let $\mathbf{p}^U_{ND}(X_t)$ and $\mathbf{p}^U_{DS}(X_t)$ be the present value of \$1 to be received at the time of restructuring, contingent on restructuring occurring before default when the state is ND and DS, respectively. Similarly, let $\mathbf{p}^B_{ND}(X_t)$ and $\mathbf{p}^B_{DS}(X_t)$ be the present value of \$1 to be received at the time of default, contingent on default occurring before restructuring when the state is ND and DS, respectively. It follows that the solutions to Equation (A.1) can be written as follows

$$\mathbf{n}(X_t, C) = \begin{cases} (1-\tau)\left[\frac{X_t}{r-\mu} - \frac{C}{r} - \mathbf{p}^U_{ND}(X_t)\left(\frac{X_U}{r-\mu} - \frac{C}{r}\right) - \mathbf{p}^B_{ND}(X_t)\left(\frac{X_B}{r-\mu} - \frac{C}{r}\right)\right] \\ (1-\tau)\left[\frac{X_t}{r-\mu} - \frac{C}{r} - \mathbf{p}^U_{DS}(X_t)\left(\frac{X_U}{r-\mu} - \frac{C}{r}\right) - \mathbf{p}^B_{DS}(X_t)\left(\frac{X_B}{r-\mu} - \frac{C}{r}\right)\right] \end{cases} \tag{A.2}$$

where the expressions for $\mathbf{p}^U_{ND}(X_t)$, $\mathbf{p}^B_{ND}(X_t)$, $\mathbf{p}^U_{DS}(X_t)$, and $\mathbf{p}^B_{DS}(X_t)$ are provided in Appendix B.

To calculate the value of a claim on net income over all refinancing cycles we need an intermediate result. We need to show that, at the time of restructuring T_U, all claims are scaled up by the same proportion $\rho = X_U/X_0$. This feature of the model is known as the scaling property, and it is widely

[16] We follow a notation similar to Morellec et al. (2012).

used in this class of models.[17]

The value of net income over all refinancing cycles is equal to the expected net present value of cash flows accrued to shareholders over the entire life of the firm which we can write as follows

$$\mathbf{NI}(X_t, C) = \begin{cases} \mathbf{n}(X_t, C) + \mathbf{p}^U_{ND}(X_t) \cdot \mathbf{NI}(X_U, C_U) & \text{for } X_D \leq X_t \leq X_U \\ \underbrace{\mathbf{n}(X_t, C)}_{\substack{\text{Net Income} \\ \text{over 1 cycle}}} + \underbrace{\mathbf{p}^U_{DS}(X_t) \cdot \mathbf{NI}(X_U, C_U)}_{\substack{\text{NPV of net income} \\ \text{for future cycles}}} & \text{for } X_B < X_t < X_D \end{cases} \tag{A.3}$$

where X_U is the restructuring boundary that defines when the firm issues new debt and retires the existing one and C_U is the new coupon paid by the firm after having restructured its debt. The scaling property implies that $\mathbf{NI}(X_U, C_U) = \rho \mathbf{NI}(X_0, C)$ and $C_U = \rho C$ where $\rho = X_U / X_0$. At the time of debt issuance, the value of net income over all refinancing cycles simplifies to

$$\mathbf{NI}(X_0, C) = \begin{cases} \frac{\mathbf{n}(X_0, C)}{1 - \rho \cdot \mathbf{p}^U_{ND}(X_0)} & \text{for } X_D \leq X_0 \leq X_U \\ \frac{\mathbf{n}(X_0, C)}{1 - \rho \cdot \mathbf{p}^U_{DS}(X_0)} & \text{for } X_B < X_0 < X_D \end{cases} \tag{A.4}$$

A.2 Debt

The value of a claim on coupon payments over one refinancing cycle includes the expected present value of all coupons to be received until the firm either restructures its debt or goes default plus the recovery value in the event of default. Accounting for taxes, we can write the value of this claim as follows

$$\mathbf{d}(X_t, C) = \begin{cases} \mathbb{E}^{\mathcal{Q}} \left[\int_t^T (1 - \tau^d) C e^{-r(s-t)} ds + (1 - \alpha) \cdot V(X_B) \cdot e^{-rT_B} \mid X_D \leq X_t \leq X_U \right] \\ \mathbb{E}^{\mathcal{Q}} \left[\underbrace{\int_t^T (1 - \tau^d) C e^{-r(s-t)} ds}_{\substack{\text{NPV of coupons} \\ \text{over 1 cycle}}} + \underbrace{(1 - \alpha) \cdot V(X_B) \cdot e^{-rT_B}}_{\substack{\text{NPV of unlevered} \\ \text{assets at default}}} \mid X_B < X_t < X_D \right] \end{cases} \tag{A.5}$$

where α represents the default costs which are proportional to the value of the unlevered assets. Similar to the analysis for net income, Equation (A.5) contains two cases: the expression above evaluates the value of debt over one refinancing cycle when the firm is not in financial distress ($X_D \leq X_t \leq X_U$, see top expression in Equation (3)) while the one below provides the value of debt when the firm is experiencing pre-default costs ($X_B < X_t < X_D$, see bottom expression in Equation (3)).

We can write the solution to Equation (A.5) as follows

$$\mathbf{d}(X_t, C) = \begin{cases} \left(1 - \mathbf{p}^U_{ND}(X_t) - \mathbf{p}^B_{ND}(X_t)\right) \frac{(1-\tau^d)C}{r} + \mathbf{p}^B_{ND}(X_t)(1 - \alpha) \cdot V(X_B) \\ \left(1 - \mathbf{p}^U_{DS}(X_t) - \mathbf{p}^B_{DS}(X_t)\right) \frac{(1-\tau^d)C}{r} + \mathbf{p}^B_{DS}(X_t)(1 - \alpha) \cdot V(X_B) \end{cases} \tag{A.6}$$

Debt is retired at par when the firm restructures its debt. Assuming that the last time the firm issued debt was at time $t = 0$, the value of the outstanding debt, $\mathbf{D}(X_t, C)$, is equal to the value of debt over one refinancing cycle $\mathbf{d}(X_t, C)$ (provided in Equation (A.6)) plus the expected net present value of the repayment when the firm retires the debt at par. Since debt is issued at par, the face value of debt is equal to $\mathbf{D}(X_0, C)$. We can write the value of the outstanding debt

[17] See, for example, Goldstein et al. (2001) and Morellec et al. (2012). A formal proof that shows the validity of the scaling property in this type of models can be found in Goldstein et al. (2001), pages 509-511.

as follows

$$\mathbf{D}(X_t, C) = \begin{cases} \mathbf{d}(X_t, C) + \mathbf{p}_{ND}^U(X_t) \cdot \mathbf{D}(X_0, C) & \text{for } X_D \leq X_t \leq X_U \\ \mathbf{d}(X_t, C) + \underbrace{\mathbf{p}_{DS}^U(X_t) \cdot \mathbf{D}(X_0, C)}_{\substack{\text{NPV of repayment when} \\ \text{debt is retired at par}}} & \text{for } X_B < X_t < X_D \end{cases} \tag{A.7}$$

Equation (A.7) holds for any $X_t \in [X_B, X_U]$ therefore at the time of issuance, when the value of EBIT is X_0, the value of debt is

$$\mathbf{D}(X_0, C) = \begin{cases} \frac{\mathbf{d}(X_0)}{1-\mathbf{p}_{ND}^U(X_0)} & \text{for } X_D \leq X_0 \leq X_U \\ \frac{\mathbf{d}(X_0)}{1-\mathbf{p}_{DS}^U(X_0)} & \text{for } X_B < X_0 < X_D \end{cases} \tag{A.8}$$

The value of the outstanding debt reflects the value of debt for current debtholders but it does not include the value of debt for future debt issues. To account for new debt issued in the future, we calculate the value of a claim on the coupons that the firm will pay over its entire life (i.e. including the increased coupons following debt restructurings). We denote this claim as $\mathbf{TD}(X_t, C)$. We can write its value as follows

$$\mathbf{TD}(X_t, C) = \begin{cases} \mathbf{d}(X_t, C) + \mathbf{p}_{ND}^U(X_t) \cdot \mathbf{TD}(X_U, C_U) & \text{for } X_D \leq X_t \leq X_U \\ \mathbf{d}(X_t, C) + \mathbf{p}_{DS}^U(X_t) \cdot \mathbf{TD}(X_U, C_U) & \text{for } X_B < X_t < X_D \end{cases} \tag{A.9}$$

where X_U is the restructuring boundary and C_U is the new coupon paid by the firm after having restructured its debt. The scaling property implies that $\mathbf{TD}(X_U, C_U) = \rho\mathbf{TD}(X_0, C)$ where $\rho = X_U/X_0$. It follows that the value of total debt at X_0 is

$$\mathbf{TD}(X_0) = \begin{cases} \frac{\mathbf{d}(X_0)}{1-\rho\cdot\mathbf{p}_{ND}^U(X_0)} & \text{for } X_D \leq X_0 \leq X_U \\ \frac{\mathbf{d}(X_0)}{1-\rho\cdot\mathbf{p}_{DS}^U(X_0)} & \text{for } X_B < X_0 < X_D \end{cases} \tag{A.10}$$

A.3 Adjustment Costs

We assume that the firm incurs adjustment costs each time it changes its capital structure. These adjustment costs are equal to a percentage λ of the debt being issued. At the time of refinancing, the total value of the adjustment costs are equal to the flotation costs for the debt currently being issued plus the expected adjustment costs that the firm will pay for subsequent debt issues. We can write the value of the adjustment costs at the time of issuance as follows

$$\mathbf{AC}(X_0, C) = \begin{cases} \lambda\mathbf{D}(X_0, C) + \mathbf{p}_{ND}^U(X_0) \cdot \mathbf{AC}(X_U, C_U) & \text{for } X_D \leq X_0 < X_U \\ \underbrace{\lambda\mathbf{D}(X_0, C)}_{\substack{\text{Adjustment cost} \\ \text{for current} \\ \text{debt issued}}} + \underbrace{\mathbf{p}_{DS}^U(X_0) \cdot \mathbf{AC}(X_U, C_U)}_{\substack{\text{NPV of Adjustment} \\ \text{costs for future} \\ \text{debt issues}}} & \text{for } X_B < X_0 < X_D \end{cases} \tag{A.11}$$

where X_U is the restructuring boundary and C_U is the new coupon paid by the firm after having adjusted its capital structure. As for any claim in our model, Equation (A.11) differentiates between the time when the firm is not distressed ($X_D \leq X_0 < X_U$) and when it is distressed and is incurring pre-default costs ($X_B < X_0 < X_D$).

By the scaling property, $\mathbf{AC}(X_U, C_U) = \rho\mathbf{AC}(X_0, C)$. We can simplify Equation (A.11) as follows

$$\mathbf{AC}(X_0, C) = \begin{cases} \frac{\lambda \mathbf{D}(X_0,C)}{1-\rho\cdot \mathbf{p}^U_{ND}(X_0)} & \text{for } X_D \leq X_0 < X_U \\ \frac{\lambda \mathbf{D}(X_0,C)}{1-\rho\cdot \mathbf{p}^U_{DS}(X_0)} & \text{for } X_B < X_0 < X_D \end{cases} \tag{A.12}$$

After having issued debt, the total value of the adjustment costs is equal to the expected adjustment costs that the firm will incur over its entire life which we can write as follows

$$\mathbf{AC}(X_t, C) = \begin{cases} \mathbf{p}^U_{ND}(X_t)\rho\mathbf{AC}(X_0, C) & \text{for } X_D \leq X_t < X_U \\ \mathbf{p}^U_{DS}(X_t)\ \underbrace{\rho\mathbf{AC}(X_0, C)}_{\substack{\mathbf{AC}(X_U,C_U) \\ \text{NPV of future} \\ \text{adjustment costs}}} & \text{for } X_B < X_t < X_D \end{cases} \tag{A.13}$$

A.4 Firm and Equity Value

At any time t, the levered asset value of the firm, $\mathbf{v}(X_t, C)$, is the sum of the present value of cash flows to shareholders plus cash flows to all debtholders minus the net present value of the adjustment costs. It is given by

$$\mathbf{v}(X_t, C) = \underbrace{\mathbf{NI}(X_t, C)}_{\substack{\text{NPV of claim} \\ \text{on net income}}} + \underbrace{\mathbf{TD}(X_t, C)}_{\substack{\text{NPV of claim} \\ \text{on total debt}}} - \underbrace{\mathbf{AC}(X_t, C)}_{\substack{\text{Adjustment} \\ \text{costs}}} \tag{A.14}$$

Equity is a residual claim and its value is the difference between the total value of the firm $\mathbf{v}(X_t, C)$ and the value of current debt $\mathbf{D}(X_t, C)$:

$$\mathbf{E}(X_t, C) = \mathbf{v}(X_t, C) - \mathbf{D}(X_t, C) \tag{A.15}$$

Appendix B Arrow-Debreu securities

For ease of notation, let $V(X) \equiv V$ where $V(X)$ is the unlevered asset value defined in Equation (9). Denote the state when the firm is not distressed, $X_B < X_D \leq X_t \leq X_U$, as ND (No Distress state) and denote the state when it is distressed, $X_B < X_t < X_D < X_U$, as DS (Distressed State). Let $\mathbf{p}^U_{ND}(X)$ and $\mathbf{p}^U_{DS}(X)$ be the present value of \$1 to be received at the time of restructuring, contingent on restructuring occurring before default when the state is ND and DS, respectively. Using the standard no-arbitrage argument, these claims must satisfy the following system of partial differential equations (PDEs)

$$\begin{cases} \frac{(\sigma^L)^2}{2} V^2 \frac{\partial^2 \mathbf{p}^U_{ND}(\cdot)}{\partial V^2} + \mu V \frac{\partial \mathbf{p}^U_{ND}(\cdot)}{\partial V} - r\mathbf{p}^U_{ND}(\cdot) = 0 & \text{for } X_{i,B} < X_{i,D} \leq X_{it} \\ \frac{(\sigma^H)^2}{2} V^2 \frac{\partial^2 \mathbf{p}^U_{DS}(\cdot)}{\partial V^2} + (\mu - \gamma) V \frac{\partial \mathbf{p}^U_{DS}(\cdot)}{\partial V} - r\mathbf{p}^U_{DS}(\cdot) = 0 & \text{for } X_{i,B} < X_{it} < X_{iD} \end{cases}$$

The general solution to this system of PDEs is

$$\begin{cases} \mathbf{p}^U_{ND}(X) = H_{1,ND} V^{\beta_{1,ND}} + H_{2,ND} V^{\beta_{2,ND}} \\ \mathbf{p}^U_{DS}(X) = H_{1,DS} V^{\beta_{1,DS}} + H_{2,DS} V^{\beta_{2,DS}} \end{cases}$$

The constants $\beta_{1,ND}$, $\beta_{2,ND}$, $\beta_{1,DS}$, and $\beta_{2,DS}$ are

$$\beta_{1,ND} = \frac{1}{(\sigma^L)^2}\left[-(\mu - 0.5(\sigma^L)^2) + \sqrt{(\mu - 0.5(\sigma^L)^2)^2 + 2r(\sigma^L)^2)}\right]$$
$$\beta_{2,ND} = -\frac{1}{(\sigma^L)^2}\left[(\mu - 0.5(\sigma^L)^2) + \sqrt{(\mu - 0.5(\sigma^L)^2)^2 + 2r(\sigma^L)^2)}\right]$$
$$\beta_{1,DS} = \frac{1}{(\sigma^H)^2}\left[-(\mu - \gamma - 0.5(\sigma^H)^2) + \sqrt{(\mu - \gamma - 0.5(\sigma^H)^2)^2 + 2r(\sigma^H)^2)}\right]$$
$$\beta_{2,DS} = -\frac{1}{(\sigma^H)^2}\left[(\mu - \gamma - 0.5(\sigma^H)^2) + \sqrt{(\mu - \gamma - 0.5(\sigma^H)^2)^2 + 2r(\sigma^H)^2)}\right]$$

The constants $H_{1,ND}$, $H_{2,ND}$, $H_{1,DS}$, and $H_{2,DS}$ are solved by imposing the following boundary conditions

$$\mathbf{p}^U_{ND}(X_U) = 1 \qquad \mathbf{p}^U_{DS}(X_B) = 0$$
$$\mathbf{p}^U_{ND}(X_D) = \mathbf{p}^U_{DS}(X_D) \qquad \left.\frac{\partial \mathbf{p}^U_{ND}(X)}{\partial X}\right|_{X=X_D} = \left.\frac{\partial \mathbf{p}^U_{DS}(X)}{\partial X}\right|_{X=X_D}$$

Re-writing the above conditions in matrix form yields the following solution

$$\begin{bmatrix} H_{1,ND} & H_{2,ND} & H_{1,DS} & H_{2,DS} \end{bmatrix}' = \mathbf{M}^{-1} \cdot \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}' \tag{B.1}$$

where

$$\mathbf{M} = \begin{bmatrix} V_U^{\beta_{1,ND}} & V_U^{\beta_{2,ND}} & 0 & 0 \\ 0 & 0 & V_B^{\beta_{1,DS}} & V_B^{\beta_{2,DS}} \\ V_D^{\beta_{1,ND}} & V_D^{\beta_{2,ND}} & -V_D^{\beta_{1,DS}} & -V_D^{\beta_{2,DS}} \\ \beta_{1,ND}V_D^{\beta_{1,ND}} & \beta_{2,ND}V_D^{\beta_{2,ND}} & -\beta_{1,DS}V_D^{\beta_{1,DS}} & -\beta_{2,DS}V_D^{\beta_{2,DS}} \end{bmatrix} \tag{B.2}$$

Let $\mathbf{p}^B_{ND}(X)$ and $\mathbf{p}^B_{DS}(X)$ be the present value of \$1 to be received at the time of default, contingent on default occurring before refinancing when the state is ND and DS, respectively. Using the standard no-arbitrage argument, these claims must satisfy the following PDEs

$$\begin{cases} \frac{(\sigma^L)^2}{2}V^2\frac{\partial^2 \mathbf{p}^B_{ND}(\cdot)}{\partial V^2} + \mu V\frac{\partial \mathbf{p}^B_{ND}(\cdot)}{\partial V} - r\mathbf{p}^B_{ND}(\cdot) = 0 & \text{for } X_{it} \geq X_{i,D} > X_{i,B} \\ \frac{(\sigma^H)^2}{2}V^2\frac{\partial^2 \mathbf{p}^B_{DS}(\cdot)}{\partial V^2} + (\mu - \gamma)V\frac{\partial \mathbf{p}^B_{DS}(\cdot)}{\partial V} - r\mathbf{p}^B_{DS}(\cdot) = 0 & \text{for } X_{i,D} > X_{it} > X_{i,B} \end{cases}$$

The general solution to this pair of PDEs is

$$\begin{cases} \mathbf{p}^B_{ND}(X) = J_{1,ND}V^{\beta_{1,ND}} + J_{2,ND}V^{\beta_{2,ND}} \\ \mathbf{p}^B_{DS}(X) = J_{1,DS}V^{\beta_{1,DS}} + J_{2,DS}V^{\beta_{2,DS}} \end{cases}$$

The constants $J_{1,ND}$, $J_{2,ND}$, $J_{1,DS}$, and $J_{2,DS}$ are solved by imposing the following boundary conditions

$$\mathbf{p}^B_{ND}(X_U) = 0 \qquad \mathbf{p}^B_{DS}(X_B) = 1$$
$$\mathbf{p}^B_{ND}(X_D) = \mathbf{p}^B_{DS}(X_D) \qquad \left.\frac{\partial \mathbf{p}^B_{ND}(X)}{\partial X}\right|_{X=X_D} = \left.\frac{\partial \mathbf{p}^B_{DS}(X)}{\partial X}\right|_{X=X_D}$$

Re-writing the above conditions in matrix form yields the following solution

$$\begin{bmatrix} J_{1,ND} & J_{2,ND} & J_{1,DS} & J_{2,DS} \end{bmatrix}' = \mathbf{M}^{-1} \cdot \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}' \tag{B.3}$$

Appendix C Simulated Method of Moments

For each firm i, we calculate a vector of empirical moments, $h(Y_i)$, using the empirical data $Y_i = [y_{i,1}, y_{i,2}, \cdots, y_{i,T_i}]$ where T_i is the empirical sample length for firm i. We use six empirical moments: the operating ROA, the quasi-market leverage, the excess return of firm's equity with respect to the risk-free rate, the probability of default at 5 years, the variance and returns of equity. We estimate the parameters of the model, $\theta = [\alpha, \gamma, \mu, \sigma_F, \beta]$, using the Simulated Method of Moments (SMM) (Gourieroux and Monfort, 1996). The SMM searches for the vector of parameters to "fit" the simulated moments to their empirical counterparts. More specifically, we search for the vector of parameters that minimizes the weighted distance between the simulated and empirical moments, $\Lambda(\theta)$:

$$\theta^* = \arg\min_{\theta} \Lambda(\theta) \tag{C.1}$$

where

$$\Lambda(\theta) = g(\theta)'\hat{W}g(\theta)$$
$$g(\theta) = h(\mathrm{Y}_i) - \frac{1}{S}\sum_{s=1}^{S} h\left(Y_s(\theta)\right)$$

where S is the number of simulations, θ is the vector of parameters, $Y_k(\theta)$ is the vector of the simulated data for the k-th simulation given parameters θ, and $\hat{W}$ is a positive definite weighting matrix. This estimator is known to be asymptotically normal for fixed S; for $T_i \to \infty$ the estimator's asymptotic distribution is

$$\sqrt{T_i}(\theta^* - \theta) \xrightarrow{d} \mathcal{N}\left(0, \mathrm{Var}(\theta^*)\right) \tag{C.2}$$

where

$$\mathrm{Var}(\theta^*) = \left(1 + \frac{1}{S}\right)\left[D' \cdot W \cdot D\right]^{-1}$$
$$D = \left.\frac{\partial g(\theta)}{\partial \theta'}\right|_{\theta=\theta^*}$$

W is the optimal weighting matrix. The optimal weighting matrix is chosen as to place greater weights on more precisely estimated moments (i.e. moments with lower variance):

$$\hat{W} = \left[\hat{\mathrm{Var}}(h(Y_i))\right]^{-1} \tag{C.3}$$

The estimated variance-covariance matrix, $\hat{\mathrm{Var}}(h(Y_i)$, is calculated using the influence function approach described in Erickson and Whited (2002) which has better finite sample properties as shown in Bazdresch, Kahn, and Whited (2017). This methodology ensures that $\hat{\mathrm{Var}}(h(Y_i) \xrightarrow{p} \mathrm{Var}(h(Y_i)$.

We conduct the estimation in three steps. First, we build a fine grid containing 20 equally spaced points for each parameter which implies evaluating the model on 3,200,000 points.[18] Second, we use the minimum found in the previous step as the mid-point and we build another grid of 20 equally spaced points for each parameter. Third, we use the results from the previous step as the starting values of a local minimization algorithm (Nelder-Mead) to achieve a more precise minimum.

Appendix D NPV of loss given default and pre-default costs

In this section we provide the pricing formulas for the present value of loss given default and pre-default costs. We calculate the value of the securities for the following 2 regions

$$\mathcal{R}_1 : X_B \leq X_t < X_D \tag{D.1}$$

$$\mathcal{R}_2 : X_D \leq X_t < X_U \tag{D.2}$$

where X_B is the default boundary, X_D is the distress threshold which we set equal to the current coupon, and X_U is the restructuring threshold.

NPV of loss given default

Here we provide the general solution as well as the boundary conditions to obtain the net present value of the loss given default. Let us define the loss given default security as $LGD(X_t, s)$ where s denotes whether the firm is distressed D (i.e. it is in region $\mathcal{R}_1$) or healthy H (i.e. it is in region $\mathcal{R}_2$). In the region $\mathcal{R}_1 : X_B \leq X_t < X_D$, the firm is distressed and it incurs pre-default costs. The LGD is realized when EBIT reaches X_B therefore the value function for a claim that pays the loss given default $LGD(X_t, D)$ has to satisfy the following ODE

$$r\, LGD(X_t, D) = (\mu - \gamma) X_t \frac{\partial LGD(X_t, D)}{\partial X_t} + \frac{1}{2}\sigma_X^2 X_t^2 \frac{\partial LGD(X_t, D)}{\partial X_t^2} \tag{D.3}$$

We guess that the functional form for $LGD(X_t, D)$ is

$$LGD(X_t, D) = a_{D1}^L X_t^{\psi_{D1}} + a_{D2}^L X_t^{\psi_{D2}} \tag{D.4}$$

where ψ_{D1} and ψ_{D2} are the positive and negative root of

$$\frac{1}{2}\sigma_X^2 y(y-1) + (\mu - \gamma) y - r = 0 \tag{D.5}$$

In the region $\mathcal{R}_2 : X_D \leq X_t < X_U$, the firm is not distressed and the value function for the $LGD(X_t, H)$ claim is:

$$rLGD(X_t, H) = \mu X_t \frac{\partial LGD(X_t, H)}{\partial X_t} + \frac{1}{2}\sigma_X^2 X_t^2 \frac{\partial LGD(X_t, H)}{\partial X_t^2} \tag{D.6}$$

We guess that the functional form for $LGD(X_t, H)$ is

$$LGD(X_t, H) = e_{H1}^L X_t^{\psi_{H1}} + e_{H2}^L X_t^{\psi_{H2}} \tag{D.7}$$

[18]Computations were performed on the Niagara supercomputer at the SciNet HPC Consortium (Ponce et al., 2019; Loken et al., 2010). SciNet is funded by: the Canada Foundation for Innovation; the Government of Ontario; Ontario Research Fund - Research Excellence; and the University of Toronto.

where ψ_{H1} and ψ_{H2} are the roots of

$$\frac{1}{2}\sigma_X^2 y(y-1) + \mu y - r = 0 \tag{D.8}$$

In order to find the 4 parameters a_{D1}^L, a_{D2}^L, e_{H1}^L, and e_{H2}^L, we use the following boundary conditions.

$$\lim_{X_t \downarrow X_B} LGD(X_t, D) = \alpha \times \frac{X_t}{r-\mu+\gamma} \tag{D.9}$$

$$\lim_{X_t \uparrow X_D} LGD(X_t, D) = \lim_{X_t \downarrow X_D} LGD(X_t, H) \tag{D.10}$$

$$\lim_{X_t \uparrow X_D} \frac{\partial LGD(X_t, D)}{\partial X_t} = \lim_{X_t \downarrow X_D} \frac{\partial LGD(X_t, H)}{\partial X_t} \tag{D.11}$$

$$\lim_{X_t \uparrow X_U} LGD(X_t, H) = \frac{X_U}{X_0} LDG(X_0, H) \tag{D.12}$$

Equation (D.9) implies that when the firm defaults (i.e. X_t reaches X_B) it incurs a deadweight loss proportional to the value of its assets. Equation (D.10) and Equation (D.11) are, respectively, a value-matching and smooth-pasting condition at X_D. Equation (D.12) implies that when firm reaches the restructuring threshold X_U, the value of all securities (including $LGD(\cdot)$) is scaled by the factor X_U/X_0 where X_0 is the value of EBIT at the time of the last refinancing.[19]

NPV of pre-default costs of financial distress

Here we provide the general solution as well as the boundary conditions to obtain the net present value of pre-default costs. Let us define the pre-default cost security as $PDC(X_t, s)$ where s denotes whether the firm is distressed D (i.e. it is in region $\mathcal{R}_1$) or healthy H (i.e. it is in region $\mathcal{R}_2$). In the region $\mathcal{R}_1 : X_B \leq X_t < X_D$, the firm is distressed and it incurs pre-default costs as a loss of EBIT at a rate γ. Upon default, the firm incurs a loss given default but it stops accumulating pre-default costs of financial distress. The value function for such a claim has to satisfy the following ODE

$$r\, PDC(X_t, D) = \gamma X_t + (\mu - \gamma) X_t \frac{\partial PDC(X_t, D)}{\partial X_t} + \frac{1}{2}\sigma_X^2 X_t^2 \frac{\partial PDC(X_t, D)}{\partial X_t^2} \tag{D.13}$$

We guess that the functional form for $PDC(X_t, D)$ is

$$PDC(X_t, D) = \frac{\gamma X_t}{r-\mu+\gamma} + a_{D1}^P X_t^{\psi_{D1}} + a_{D2}^P X_t^{\psi_{D2}} \tag{D.14}$$

where ψ_{D1} and ψ_{D2} are provided in Equation (D.5)

In the region $\mathcal{R}_2 : X_D \leq X_t < X_U$, the firm is not distressed and the value function for the $PDC(X_t, H)$ claim is:

$$rPDC(X_t, H) = \mu X_t \frac{\partial PDC(X_t, H)}{\partial X_t} + \frac{1}{2}\sigma_X^2 X_t^2 \frac{\partial PDC(X_t, H)}{\partial X_t^2} \tag{D.15}$$

We guess that the functional form for $PDC(X_t, H)$ is

$$PDC(X_t, H) = e_{H1}^P X_t^{\psi_{H1}} + e_{H2}^P X_t^{\psi_{H2}} \tag{D.16}$$

[19] See Section 2 for a discussion of the scaling property in this class of models.

where ψ_{H1} and ψ_{H2} are provided in Equation (D.8).

In order to find the 4 parameters a^P_{D1}, a^P_{D2}, e^P_{H1}, and e^P_{H2}, we use the following boundary conditions.

$$\lim_{X_t \downarrow X_B} PDC(X_t, D) = 0 \tag{D.17}$$

$$\lim_{X_t \uparrow X_D} PDC(X_t, D) = \lim_{X_t \downarrow X_D} PDC(X_t, H) \tag{D.18}$$

$$\lim_{X_t \uparrow X_D} \frac{\partial PDC(X_t, D)}{\partial X_t} = \lim_{X_t \downarrow X_D} \frac{\partial PDC(X_t, H)}{\partial X_t} \tag{D.19}$$

$$\lim_{X_t \uparrow X_U} PDC(X_t, H) = \frac{X_U}{X_0} LDG(X_0, H) \tag{D.20}$$

Equation (D.17) implies that upon default (i.e. X_t reaches X_B) the firms stops incurring pre-default costs of financial distress. Equation (D.18) and Equation (D.19) are, respectively, a value-matching and smooth-pasting condition at X_D. Equation (D.20) implies that when firm reaches the restructuring threshold X_U, the value of all securities (including $PDC(\cdot)$) is scaled by the factor X_U/X_0 where X_0 is the value of EBIT at the time of the last refinancing.