Adversarial Coordination and Public Information Design*

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Abstract

We study flexible public information design in global games. In addition to receiving public information from the designer, agents are endowed with exogenous private information and must decide whether or not to "attack" a status-quo. The designer does not trust the agents to play favorably to her and evaluates any policy under the "worst-case scenario." First, we show that the optimal policy removes any strategic uncertainty by inducing all agents to take the same action, but without permitting them to perfectly learn the fundamentals or the beliefs that rationalize other agents' actions. Second, we identify conditions under which the optimal policy takes the form of a simple pass/fail test. Finally, we show that, when the designer cares only about the probability of regime change, the optimal policy need not be monotone in fundamentals but then identify conditions on payoffs and exogenous beliefs under which the optimal policy is monotone.

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1 Introduction

Coordination plays a major role in many socio-economic environments. The damages to society of mis-coordination can be severe and often call for government intervention. Think of the possibility of default by major financial institutions in case investors run or refrain from rolling over their short-term positions. Such defaults can trigger a collapse in financial markets, with severe consequences for the real economy. Confronted with such prospects, governments and supervising authorities have incentives to intervene. These interventions often take the form of public information disclosures, such as stress testing or, more broadly, releases of information aimed at influencing market beliefs.

In this paper, we study *public information design* in markets in which a large number of receivers (e.g., investors in financial markets) must choose whether to play an action favorable to the designer (e.g., pledging funds to the banking sector), or an "adversarial" action (e.g., refraining from pledging). A policy maker can flexibly design a policy disclosing information to market participants about relevant economic fundamentals. The analysis delivers results that are important for various situations in which coordination plays a major role, including bank runs, currency crises, technology and standards adoption. In the context of stress testing, the policy maker may represent a supervising authority attempting to prevent a run against the banking sector. In the case of currency crises, the policy maker may represent a central bank attempting to dissuade speculators from short-selling the domestic currency by releasing information about the bank's reserves and/or domestic economic fundamentals. In the case of technology adoption, the policy maker may represent the owners of an intellectual property trying to persuade heterogenous market users of the merits of a new product (Lerner and Tirole (2006)).

The backbone of the model is a *global game of regime change* in which multiple agents must choose between "attacking" a status quo or "refraining from doing so," and where the success of the attack depends on its aggregate size and on exogenous fundamentals. In addition to receiving public information from the designer, agents are endowed with exogenous private information. The designer does not trust the agents to play favorably to her and evaluates any policy of its choice under the "worst-case" scenario. That is, when multiple rationalizable strategy profiles are consistent with the information disclosed, the designer takes a "robust approach" by looking at the outcome that prevails when investors play according to the rationalizable profile least favorable to her.[1]

We assume the policy maker can flexibly design a policy that disseminates publicly information about relevant economic fundamentals. We use the model to address the following questions: (a) Are there benefits to preventing market participants from predicting each others' actions and beliefs? (b) When are simple policies such as pass/fail tests optimal? (c) Are there merits to non-monotone rules that induce the market to play favorably for intermediate fundamentals but not necessarily for stronger ones?

Our first result establishes that, despite the fear of adversarial coordination, the optimal policy satisfies the "*perfect coordination property.*" In each state, it induces all market participants to take the same action, but without creating homogenous beliefs among market participants. In other words, the optimal policy completely removes any *strategic uncertainty* while preserving *structural uncertainty.* Given the public information disclosed, each receiver can perfectly predict the action of any other receiver, but not the beliefs that rationalize such actions. For example, an investor who is induced to pledge must not be able to determine whether other investors pledge because they know that the fundamentals are so strong that default will never occur, irrespective of the aggregate pledge, or because they are confident that other investors pledge. The optimal policy leverages the heterogeneity of investors' primitive beliefs by making pledging dominant for some investors based on their first-order beliefs, but only iteratively dominant for others based on their higher-order beliefs.[2] Under adversarial coordination, preserving uncertainty over beliefs is key to the minimization of the risk of an undesirable outcome such as default, or more, generally, regime change. When the designer trusts the agents to follow her recommendations, the optimality of the perfect coordination property is straightforward and follows from arguments similar to those establishing the Revelation Principle (e.g., Myerson (1986)). This is not the case under adversarial coordination, for information that facilitates perfect coordination may also favor rationalizable profiles in which some of the agents play adversarially to the designer.

[1]Such a robust approach is motivated by the applications the analysis is meant for. For example, when concerned about runs to the banking sector, policy makers typically do not trust the market to play favorably.

[2]The optimal policy does not ensure that pledging is the unique rationalizable action based on first-order beliefs for all investors. It relies on a contagion argument *through higher-order beliefs* to induce all investors to pledge under the unique rationalizable profile.

Our second result shows that when the economic fundamentals and the agents' beliefs co-move in the sense that states in which fundamentals are strong are also states in which most agents expect other agents to expect the fundamentals to be strong, and so on, then the optimal policy takes the form of a simple "pass/fail" test, with no further information disclosed to the market. It is known that, when the distribution from which the agents' private signals are drawn is log-supermodular, or, equivalently, satisfies the *Monotone Likelihood Ratio Property*– in short MLRP, all agents follow monotone (i.e., cut-off) strategies, no matter the public information. This is because, under MLRP, the agents' "optimism ranking" is preserved under Bayesian updating. If agent j is more optimistic than agent i before the public announcement is made (formally, j's beliefs dominate i's beliefs according to the MLRP order), then this continues to be the case after *any* public announcement. When this is the case, disclosing information to the market in addition to whether or not the policy maker expects regime change to happen when agents play adversarially does not help. We also show that MLRP is key to the optimality of simple pass/fail policies. When the information the policy maker discloses can be used to change the ranking of the investors' optimism, the policy maker can leverage the optimism reversal to spare more fundamentals from regime change by disclosing information in addition to whether or not she expects regime change.[3]

In the context of stress testing, these results provide a foundation for the optimality of simple pass/fail policies. Importantly, optimal stress tests should be *transparent*, in the sense of facilitating coordination among investors, but should not generate consensus among market participants about the soundness of the financial institutions under scrutiny.

Our third result is about the optimality of monotone pass/fail policies, that is, rules that grant a pass grade if and only if fundamentals are above a given threshold. We show that the optimality of such rules is related to the extent to which the policy maker's preferences for avoiding regime change vary with the fundamentals. We identify precise conditions involving the policy maker's preferences and the agents' payoffs and exogenous beliefs under which monotone rules are optimal. Such conditions are fairly sharp in the sense that, when violated,

[3]When, instead, the designer trusts her ability to coordinate the receivers on the course of action most favorable to her, optimal policies always take the form of action recommendations, and hence pass/fail policies are optimal, irrespective of the investors' primitive beliefs. This is not the case under adversarial/robust design.

one can identify instances in which non-monotone rules strictly outperform monotone ones.[4] The reason is that non-monotone rules make it more difficult for the agents to *commonly learn* the fundamentals and hence permit the policy maker to give a pass grade to a larger set of fundamentals. When the policy maker preferences for avoiding regime change do not vary much with the fundamentals (in particular, when they are constant), non-monotone rules may thus be optimal.

Organization. The rest of the paper is organized as follows. Below, we wrap up the introduction with a brief review of the most pertinent literature. Section 2 presents the model. Section 3 contains all the results about properties of optimal policies (perfect-coordination, pass/fail, monotonicity). Section 4 discusses enrichments of the baseline model that accommodate for more general payoffs and the possibility that the policy maker faces uncertainty about the fate of the regime. Section 5 concludes. The Appendix contains all proofs with the exception of the proof of Example 3 which is in the Online Appendix.

(Most) pertinent literature. The paper is related to a few strands of the literature.

The first one is the literature on *adversarial coordination and unique implementation*. See, among others, Segal (2003), Winter (2004), Sakovics and Steiner (2012), Frankel (2017), Halac et al. (2020), Halac et al. (2021). These papers study the design of transfers implementing the desired outcome (e.g., the financing of a public good) as a unique equilibrium. Our paper shares with these works the idea that iterative dominance can be exploited to economize on costs when the designer does not trust the agents to play favorably to her. Contrary to these papers, however, we consider a setting in which (a) the designer has no transfers, (b) the agents are endowed with exogenous private information, and (c) iterative dominance is induced by manipulating the agents' first and *higher-order beliefs*.[5]

The second one is the literature on *global games with endogenous information*. Angeletos et al. (2006) and Angeletos and Pavan (2013) study signaling in global games, where a policy maker, with no commitment power, engages in costly actions (e.g, raising interest rates) to influence the investors' behavior. Angeletos and Werning (2006) investigate the role of prices

[4]We also show that the conditions guaranteeing the optimality of monotone rules are more stringent when the policy maker faces multiple privately-informed receivers than when she faces either a single (possibly privately-informed) receiver, or multiple receivers who possess no exogenous private information.

[5]See also Halac et al. (2022) for a recent paper in which unique implementation is achieved through a combination of transfers and information provision.

as a vehicle for information aggregation. Angeletos et al. (2007) consider a dynamic model in which investors learn from the accumulation of private information and from the (possibly noisy) observation of past outcomes. Cong et al. (2020) consider a dynamic setting similar to the one in Angeletos et al. (2007) but allowing for policy interventions. Edmond (2013) considers propaganda in global games, in a setting in which the policy maker manipulates the investors' private signals. Szkup and Trevino (2015), Yang (2015), Morris and Yang (2022), and Denti (2023) study the acquisition of private information in global games.

The contribution of our paper vis-a-vis the above body of work is in identifying properties of the optimal provision of public information when the sender can fully commit to her policy but does not trust the receivers to play favorably to her. Goldstein and Huang (2016) characterize the lowest fundamental threshold below which regime change occurs when the market play adversarially and the policy maker restricts herself to binary monotone rules, whereas Galvão and Shalders (2022) characterize the optimal monotone partitional structure by imposing that, when two states are pooled into the same cell, all in-between states also pooled into the same cell. Relative to these works, our paper establishes three key results: (a) it proves that inducing all agents to take the same action is always optimal, despite the fear of adversarial coordination; (b) it shows why, in general, binary policies are sub-optimal but then identifies sharp conditions under which such policies are optimal; (c) it shows why, in general, non-monotone rules permit to avoid regime change over a larger set of fundamentals but then identifies sharp conditions under which optimal policies are monotone.

In a coordination setting with two privately-informed receivers and two states, Alonso and Zachariadis (2021) show that, when the precision of the receivers' exogenous information is high, private and public information are complements in that an increase in the precision of the investors' private information leads to the provision of more accurate public information.

Li et al. (2021), and Morris et al. (2020) consider the design of *private* information in supermodular games in which the receivers possess no exogenous private information and play adversarially. In contrast, we assume that the receivers are endowed with exogenous private information and study the optimal design of public information. In most applications of interest, agents are endowed with private information before hearing from the policy maker. Therefore, allowing for private information when studying properties of optimal public disclosures is important given the type of applications the theory is meant for. Importantly, the structure of the optimal policy cannot be derived from simple extrapolations from the case where agents are homogeneously informed. For example, the optimality of coordinating all market participants to take the same action while preserving heterogeneity in beliefs about fundamentals has special meaning only when agents are privately informed. Likewise, when agents are privately informed, the optimal policy need not be binary or monotone. These results are important both theoretically and for the implications they have in applications.

At a broad level, the paper is related also to the literature on *information design*, in particular the one with multiple receivers. See, among others, Alonso and Camara (2016a), Arieli and Babichenko (2019), Bardhi and Guo (2017), Basak and Zhou (2020a), Che and Hörner (2018), Doval and Ely (2020), Galperti and Perego (2020), Gick and Pausch (2012), Gitmez and Molavi (2022), Heese and Lauermann (2021), Inostroza (2023), Laclau and Renou (2017), Mathevet et al. (2020), Shimoji (2021), and Taneva (2019). The key contribution vis-a-vis this literature is in showing how the interaction between (a) adversarial coordination and (b) exogenous private information among the receivers shapes the optimal provision of public information.[6]

2 Model

Global games have been used to study the interaction between information and coordination in many socio-economic environments, including bank-runs, debt crises, currency attacks, investment in technologies with network externalities, technological spillovers, and political change.

To ease the exposition, hereafter we describe the model and all the results in the context of a specific game in the spirit of Rochet and Vives (2004) in which the agents are investors (e.g., fund managers, or unsecured bank depositors) deciding whether or not to pledge funds to one, or multiple financial institutions, and where regime change occurs when these institutions default on their obligations.[7] The analysis, however, readily extends to many other global games.

[6]See Bergemann and Morris (2019) and Kamenica (2019) for an overview on information design.

[7]Rochet and Vives (2004) consider a three-period economy a' la Diamond and Dybvig (1983) but with heterogenous investors, in which banks may fail early or late. As shown in that paper, the full model admits a reduced-form version similar to the one considered here.

Players and Actions. A policy maker designs an information disclosure policy, e.g., stress tests, call reports, publication of accounting standards, and disclosure of various macro and financial variables that are jointly responsible for the profitability of the investors' decisions. The market is populated by a measure-one continuum of investors (the receivers) distributed uniformly over $[0,1]$. Each investor may either take a "friendly" action, $a_i = 1$, or an "adversarial" action, $a_i = 0$. The friendly action is interpreted as the decision to pledge (more generally, to "refrain from attacking" a given status quo). The adversarial action is interpreted as the decision not to pledge (more generally, to "attack"). We denote by $A \in [0,1]$ the size of the aggregate pledge.

Fundamentals and Exogenous Information. Consistently with the rest of the literature, we parameterize the relevant fundamentals by $\theta \in \mathbb{R}$. The fundamentals are exogenous to the policy maker's choice of a disclosure policy. It is commonly believed (by the policy maker and the investors alike) that θ is drawn from a distribution F, absolutely continuous over an interval $\Theta \supsetneqq [0,1]$, with a smooth density f strictly positive over Θ. In addition, each investor $i \in [0,1]$ is endowed with private information summarized by a uni-dimensional statistic $x_i \in \mathbb{R}$ drawn independently across investors given θ from an absolutely continuous cumulative distribution function $P(x|\theta)$ with smooth density $p(x|\theta)$ strictly positive over an (open) interval $\varrho_\theta \equiv (\underline{\varrho}_\theta, \bar{\varrho}_\theta)$ containing θ, with $\underline{\varrho}_\theta, \bar{\varrho}_\theta$ monotone in θ. The bounds $\underline{\varrho}_\theta, \bar{\varrho}_\theta$ can be either finite or infinite. For example, when $x_i = \theta + \sigma\varepsilon_i$, with ε_i drawn from a uniform distribution over $[-1,+1]$, then, for any θ, $\underline{\varrho}_\theta = \theta - \sigma$ and $\bar{\varrho}_\theta = \theta + \sigma$. When, instead, $x_i = \theta + \sigma\varepsilon_i$ with ε_i drawn from a standard Normal distribution, then, for any θ, $\underline{\varrho}_\theta = -\infty$ and $\bar{\varrho}_\theta = +\infty$. Furthermore, in this latter case, $P(x|\theta) = \Phi((x-\theta)/\sigma)$, where Φ is the cumulative distribution function of the standard Normal distribution. We denote by $\mathbf{x} \equiv (x_i)_{i\in[0,1]}$ a profile of private signals and by $\mathbf{X}(\theta)$ the collection of all $\mathbf{x} \in \mathbb{R}^{[0,1]}$ that are consistent with the fundamentals being equal to θ. As usual, we assume that any pair of signal profiles $\mathbf{x}, \mathbf{x}' \in \mathbf{X}(\theta)$ has the same cross-sectional distribution of signals, with the latter equal to $P(x|\theta)$.

Regime change. The fundamentals θ parameterize the critical size of the aggregate pledge that is necessary to avoid default. If $A > 1 - \theta$, short-term obligations are met and default is avoided. If, instead, $A \leq 1 - \theta$, default occurs. We denote by $r = 1$ the event in which default is avoided and by $r = 0$ the event in which default occurs.[8]

[8]The model assumes that, given A and θ, the regime outcome is binary. The case in which default is

Dominance Regions. For any $\theta \leq 0$, default occurs irrespective of the size of the aggregate pledge, whereas for any $\theta > 1$ default is averted with certainty. For $\theta \in (0, 1]$, instead, whether or not default occurs is determined by the behavior of the market.

Payoffs. Each investor's payoff differential between the friendly and the adversarial action is equal to $g(\theta) > 0$ in case default is avoided and $b(\theta) < 0$ otherwise. In turn, the policy maker's payoff is equal to $W(\theta)$ in case default is avoided and $L(\theta)$ in case of default, with $W(\theta) > L(\theta)$ for all θ. When W and L are invariant in θ, the policy maker's objective reduces to minimizing the probability of default. The functions b, g, W, and L are all bounded. For any $(\theta, A) \in \Theta \times [0, 1]$, then let

$$\begin{aligned} u(\theta, A) &\equiv g(\theta)\mathbf{1}(A > 1-\theta) + b(\theta)\mathbf{1}(A \leq 1-\theta), \\ U^P(\theta, A) &\equiv W(\theta)\mathbf{1}(A > 1-\theta) + L(\theta)\mathbf{1}(A \leq 1-\theta) \end{aligned}$$

denote the payoffs of a representative investor and of the policy maker, respectively, when the fundamentals are θ and the aggregate size of the pledge is A. In Section 4, we extend the analysis to a setting in which g, b, W, L also depend on the size of the pledge A, and on variables that are orthogonal to θ and the investors' exogenous signals. We also accommodate for the possibility that default may be influenced by such additional variables.[9]

Policy. Let $\mathcal{S}$ be a compact Polish space defining the set of possible signal realizations. A *policy* $\Gamma = (\mathcal{S}, \pi)$ consists of the set $\mathcal{S}$ along with a measurable mapping $\pi : \Theta \to \Delta(\mathcal{S})$ specifying, for each θ, a probability distribution over the information disclosed to the market.

Timing. The sequence of events is the following:

1. The policy maker publicly announces the policy $\Gamma = (\mathcal{S}, \pi)$ and commits to it.[10]

2. The fundamentals θ are drawn from the distribution F and the investors' exogenous signals $\mathbf{x} \in \mathbf{X}(\theta)$ are drawn from the distribution $P(x|\theta)$.

3. The public signal s is drawn from the distribution $\pi(\theta)$ and is publicly observed.

"partial" is qualitatively similar, from a strategic standpoint, to the case where, given A and θ, the regime outcome is stochastic and determined by variables that are not observable by the policy maker at the time of her public announcement (see the discussion in Section 4).

[9] In the baseline model, investors are heterogenous only in terms of their beliefs. In the file "Additional Material" on our websites, we show, however, that Theorem 1 extends to richer economies in which the investors also have heterogeneous payoffs.

[10] See Leitner and Williams (2023) for a discussion of the commitment assumption in stress testing.

4. Investors simultaneously choose whether or not to pledge.

5. The regime outcome is determined (i.e., whether or not default occurred) and payoffs are realized.

Adversarial Coordination and Robust Information Design. The policy maker does not trust the market to follow her recommendations and play favorably to her (i.e., pledge whenever $\theta > 0$).[11] Instead, she adopts a robust/conservative approach. She evaluates any policy Γ under the "worst-case" scenario, i.e., she assumes that the market plays according to the rationalizable strategy profile that is most adversarial to her, among all those consistent with the policy Γ.

Definition 1. Given any policy Γ, the **most aggressive rationalizable profile** (MARP) consistent with Γ is the strategy profile $a^{\Gamma} \equiv (a_i^{\Gamma})_{i\in[0,1]}$ that minimizes the policy maker's ex-ante expected payoff over all profiles surviving *iterated deletion of interim strictly dominated strategies* (henceforth IDISDS).

In the IDISDS procedure leading to MARP, investors use Bayes rule to update their beliefs about the fundamentals θ and the other investors' exogenous information $\mathbf{x} \in \mathbf{X}(\theta)$ using the common prior F, the distribution of private signals $P(x|\theta)$, and the policy Γ. Under MARP, given (x, s), each investor $i \in [0, 1]$, after receiving exogenous information x from Nature and endogenous information s from the policy maker, refrains from pledging whenever there exists at least one conjecture over (θ, A) consistent with the above Bayesian updating and supported by all other investors playing strategies surviving IDISDS, under which refraining from pledging is a best response for the individual.

Remarks. Hereafter, we confine attention to policies Γ for which MARP exists.[12] Because the game among the investors is supermodular (no matter the prior F, the distribution P from which the exogenous signals are drawn, and the policy Γ), the strategy profile a^{Γ} coincides with the "smallest" Bayes-Nash equilibrium (BNE) of the continuation game among the investors, and minimizes the policy maker's payoff state-by-state, and not just in expectation.

[11] If she did, a simple monotone policy revealing whether or not $\theta > 0$ would be optimal.

[12] Because the state is continuous, in principle, one can think of policies Γ for which the investors' common posteriors are not well-defined or, when combined with the investors' exogenous information, are such that the investors' hierarchies of beliefs are not well-defined, in which case MARP may not exist.

The reason why we consider MARP is that, in general, without imposing specific assumptions on F, P, and Γ, the only way the "smallest" BNE can be identified is by the iterated deletion of interim dominated strategies. In standard global games, the "smallest" BNE is typically identified by assuming the investors' signals are drawn from a distribution P satisfying the monotone likelihood property (MLRP), which is also used to guarantee equilibrium uniqueness. Here, we allow for arbitrary policies Γ, and do not require that, given Γ, the continuation equilibrium be unique.

Furthermore, given a policy $\Gamma = (\mathcal{S}, \pi)$, when describing the investors' behavior, we do not distinguish between pairs (x, s) that are mutually consistent given Γ (meaning that the joint density of (x, s) is positive, i.e., $\int_{\theta: s \in \text{supp}(\pi(\theta))} p(x|\theta) \mathrm{d}F(\theta) > 0$) and those that are not. Because the policy maker commits to the policy Γ, the abuse is legitimate and permits us to ease the exposition. Any claim about the optimality of the investors' behavior, however, should be interpreted to apply to pairs (x, s) that are mutually consistent given Γ.

3 Properties of optimal policies

We now introduce and discuss three key properties of optimal policies.

3.1 Perfect-coordination property

Definition 2. A policy $\Gamma = (\mathcal{S}, \pi)$ satisfies the **perfect-coordination property** (PCP) if, for any $\theta \in \Theta$, any exogenous information $\mathbf{x} \in \mathbf{X}(\theta)$, any public announcement $s \in \text{supp}(\pi(\theta))$, and any pair of individuals $i, j \in [0, 1]$, $a_i^\Gamma(x_i, s) = a_j^\Gamma(x_j, s)$, where $a^\Gamma = (a_i^\Gamma)_{i \in [0,1]}$ is the most aggressive rationalizable profile (MARP) consistent with the policy Γ.

A disclosure policy thus has the perfect-coordination property if it coordinates all market participants on the same action, after any information it discloses. For any $\theta \in \Theta$, any $s \in \text{supp}(\pi(\theta))$, let $r^\Gamma(\theta, s) \in \{0, 1\}$ denote the regime outcome that prevails when investors play according to a^Γ, that is, $r^\Gamma(\theta, s) = 1$ (alternatively, $r^\Gamma(\theta, s) = 0$) means that default does not occur (alternatively, occurs) when, given (θ, s), market participants play according to MARP consistent with Γ. That the investors' signals are drawn independently from $P(x|\theta)$, conditional on θ implies that the cross-sectional distribution of signals is pinned down by $P(x|\theta)$, and hence the regime outcome (that is, whether default occurs or not) is the same across any pair of signal profiles $\mathbf{x}, \mathbf{x}' \in \mathbf{X}(\theta)$ and thus depends only on Γ, θ, and s. We also establish that Theorem 1 below extends to a larger class of economies in which signals are not conditionally independent (see Section AM3 of the file "Additional Material" on our webpages the discussion in Section 4 in the current document). The key property required for the result to hold is the possibility for the designer to have access to information that is a *sufficient statistic* of the investors' information when predicting the sign of the investors' payoff differential (between attacking and not attacking) under MARP. This property holds when, for example, the correlated noise in the investors' exogenous beliefs originates in public signals the policy maker also has access to.[13] Hereafter, we say that the policy Γ is *regular* if MARP under Γ is well-defined and the regime outcome under a^{Γ} is measurable in (θ, s).

Theorem 1. *Given any (regular) policy* Γ*, there exists another (regular) policy* Γ^* *satisfying the perfect coordination property and such that, for any* θ*, the probability of default under* Γ^* *is the same as under* Γ*.*

The policy Γ^* is obtained from the original policy Γ by disclosing, for each θ, in addition to the information $s \in \text{supp}(\pi(\theta))$ disclosed by the original policy Γ, a second piece of information that reveals to the market whether at (θ, s), under MARP consistent with the original policy Γ, a^{Γ}, the investors' expected payoff differential (between pledging and not pledging) is positive or negative. Because in this simple economy, the sign of this differential is given by the regime outcome, this additional piece of information coincides in the baseline model with the regime outcome $r^{\Gamma}(\theta, s) \in \{0, 1\}$.

That, under the new policy Γ^*, it is rationalizable for all investors to pledge when the policy discloses the information $(s, 1)$, and to refrain from pledging when the policy discloses the information $(s, 0)$, is fairly straight-forward. In fact, the announcement of $(s, 1)$ (alternatively, of $(s, 0)$) makes it common certainty among the investors that $\theta > 0$ (alternatively, that $\theta \leq 1$).

The reason why the result is not obvious is that the designer does not content herself with one rationalizable profile delivering the desired outcome; she is concerned with the possibility of adversarial coordination and, as a result, when she recommends to the investors to pledge,

[13] We conjecture that, as long as the above sufficient statistic property holds, Theorems 2 and 3 also extend to settings in which the investors' signals are not conditionally independent given θ. Whether the results extend to some environments in which the sufficient statistic property is violated is an interesting question for future work.

she must guarantee that pledging is the *unique* rationalizable action for each investor, irrespective of his exogenous signal x. The proof in the Appendix shows that, when the additional information is $r^{\Gamma}(\theta, s)$, this is indeed the case.

To fix ideas, consider first the case where, under the original policy Γ, the regime outcome $r^{\Gamma}(\theta, s)$ is monotone in θ. The announcement that $r^{\Gamma}(\theta, s) = 1$ makes it common certainty among the investors that $\theta > \hat{\theta}(s)$, for some threshold $\hat{\theta}(s)$. In this case, all investors revise their first-order beliefs about θ upward when receiving the additional information that $r^{\Gamma}(\theta, s) = 1$. That each investor is more optimistic about the strength of the fundamentals, however, does not guarantee that MARP under the new policy is less aggressive than under the original one. In fact, the new piece of information changes not only the investor's first-order beliefs about θ but also his higher-order beliefs and the latter matter for the determination of the most-aggressive rationalizable profile. More generally, $r^{\Gamma}(\theta, s)$ need not be monotone in θ. This is because MARP, under the original policy Γ, need not entail strategies that are monotone in x. As a result, in general, the announcement that $r^{\Gamma}(\theta, s) = 1$ need not trigger an upward revision of the investors' beliefs.

Furthermore, in richer settings, whether regime change occurs or not may also depend on variables other than θ for which both the policy maker and the market have imperfect information about. Lastly, in more general settings, the investors' payoffs may depend on A beyond the effect that this variable has on the regime outcome.[14]

The result in Theorem 1 follows from the fact that, at any stage n of the IDISDS procedure, any investor who, under the original policy Γ pledges under the most aggressive strategy profile surviving $n-1$ rounds of deletion, does so also under the new policy Γ^*. In the Appendix, we show that this last property in turn follows from the game being supermodular along with the fact that Bayesian updating preserves the likelihood ratio of any two states that are consistent with no default under the original policy Γ. Formally, for any $s \in \text{supp}(\pi(\Theta))$, any pair of states θ' and θ'' such that (a) $s \in \text{supp}\ \pi(\theta') \cap \text{supp}\ \pi(\theta'')$, and (b) $r^{\Gamma}(\theta', s) = r^{\Gamma}(\theta'', s) = 1$, the likelihood ratio of such two states under Γ^* is the same as under the original policy Γ. This property, together with the announcement that the payoff differential under MARP consistent with the original policy Γ was positive, and the fact that the most aggressive strategy profile

[14]In Section 4, we explain that in these richer economies, perfect coordination is attained by announcing to the market the sign of the investors' expected payoff differential at (θ, s).

surviving n rounds of deletion of dominated strategies is less aggressive than the profile surviving $n-1$ rounds guarantees that, for any private signal x for which pledging was optimal under MARP consistent with the original policy Γ, pledging is the unique rationalizable action under the new policy Γ^*.[15]

The policy Γ^* thus completely eliminates any strategic uncertainty. Indeed, when $(s, 1)$ (alternatively, $(s, 0)$) is announced, each investor knows that, under MARP, all other investors will pledge (alternatively, will refrain from pledging), irrespective of their exogenous private information. Importantly, while the policy Γ^* removes any strategic uncertainty, it preserves structural uncertainty, that is, heterogeneity in the investors' first and higher-order beliefs about θ. As explained in the Introduction, it is essential that investors who pledge are uncertain as to whether other investors pledge because they find it dominant to do so, or because when they count on other investors pledging, they find it iteratively dominant to do so, which requires heterogeneity in posterior beliefs.

When it comes to disclosures in financial markets, Theorem 1 implies that optimal policies should combine the announcement of a pass/fail result (captured by $r \in \{0, 1\}$) with the disclosure of additional information (captured by s) whose role is to guarantee that, when a pass grade is given, the extra information s the investors receive from the policy maker makes pledging the unique rationalizable action. This structure appears broadly consistent with common practice. The theorem, however, says more. It indicates that optimal disclosure policies should be transparent but not in the sense of creating conformism in beliefs about fundamentals. Rather, they should leave no room to ambiguity as to whether or not default will be averted when a pass grade is announced. Preserving heterogeneity in beliefs about fundamentals is key to minimizing the probability of default.

3.2 Pass/Fail

Our next result provides a foundation for policies that take a simple pass/fail form; it identifies a key property of the investors' beliefs under which such policies are optimal.

[15] Formally, the properties above imply that each investor's posterior beliefs after hearing $r^{\Gamma}(\theta,\mathrm{s})=1$ are a "truncation" that eliminates from the support states θ at which, under the most aggressive profile surviving n rounds of IDISDS under Γ, the investor's payoff from pledging would have been negative.

Theorem 2. *Suppose that $p(x|\theta)$ is log-supermodular. Then, given any policy Γ satisfying the perfect coordination property, there exists a binary policy $\Gamma^* = (\{0,1\}, \pi^*)$ that also satisfies the perfect coordination property and such that, for any θ, the default probability under Γ^* is the same as under Γ.*[16]

As anticipated in the Introduction, the log-supermodularity of $p(x|\theta)$ (equivalently, the assumption that $p(x|\theta)$ satisfies the *monotone likelihood ratio property* – in short, MLRP)) implies that the policy maker cannot reverse the ranking in the investors' optimism through her public announcements. Whenever investor j is more optimistic than investor i (in the MLRP order) based on her exogenous private information x_j, she continues to be more optimistic after hearing the policy maker's announcement, irrespectively of the shape of the policy Γ. In turn, this implies that MARP is always in monotone strategies, and hence that the policy maker does not benefit from disclosing any information beyond the sign of the investors' expected payoff differential under MARP (which, in the baseline model, is determined by the fate of the regime $r^{\Gamma}(\theta, s)$).

To see this more formally, take any policy $\Gamma = (\mathcal{S}, \pi)$ satisfying the perfect coordination property. Given the result in Theorem 1, without loss of generality, assume that $\Gamma = (\mathcal{S}, \pi)$ is such that $\mathcal{S} = \{0,1\} \times S$, for some measurable set S, and is such that, under MARP, when the policy discloses any signal $(s, 1)$, pledging is the unique rationalizable action for each investor, irrespective of their exogenous private information. Given the policy Γ, let $U^{\Gamma}(x, (s,1)|k)$ denote the expected payoff differential of an investor with exogenous private information x who receives public information $(s, 1)$ and who expects all other investors to pledge if and only if their exogenous signal exceeds a cut-off k. No matter the shape of the policy Γ, when $p(x|\theta)$ is log-supermodular, then MARP associated with the policy Γ is in monotone (i.e., cut-off) strategies.[17] Hence, each investor's expected payoff differential when all other investors play according to MARP can be written as $U^{\Gamma}(x, (s,1)|k)$ for some k that depends on s. That the original policy Γ satisfies the perfect-coordination policy in turn implies that, for

[16]The property that $p(x|\theta)$ is log-supermodular means that, for any $x', x'' \in \mathbb{R}$, with $x' < x''$, and any $\theta', \theta'' \in \Theta$, with $\theta'' > \theta'$, then $p(x''|\theta'')p(x'|\theta') \geq p(x''|\theta')p(x'|\theta'')$.

[17]Assume $p(x|\theta)$ is log-supermodular. Then, no matter Γ, when j's beliefs dominate i's beliefs (in the MLRP order) before observing s, the same is true after observing s. Thus, at any round of the IDISDS procedure, investors follow monotone strategies.

any s and k such that $(k,(s,1))$ are mutually consistent,[18] $U^{\Gamma}(k,(s,1)|k) > 0$. That is, the expected payoff differential of any investor whose private signal x coincides with the cutoff k must be strictly positive. If this were not the case, the continuation game would also admit a rationalizable profile (in fact, a continuation equilibrium) in which some of the investors refrain from pledging, contradicting the fact that pledging irrespectively of x is the unique rationalizable profile following the announcement of $(s,1)$.

Now consider a policy Γ^* that, for any θ, draws the signal $(s,1)$ (alternatively, $(s,0)$) from the distribution $\pi(\theta)$ of the original policy $\Gamma = (\mathcal{S},\pi)$ but conceals the information s and only discloses $r=1$ (alternatively, $r=0$). By the law of iterated expectations, for all k with $(k,(s,1))$ mutually consistent, because $U^{\Gamma}(k,(s,1)|k) > 0$ then $U^{\Gamma^*}(k,1|k) > 0$. This implies that the new policy Γ^* also satisfies the perfect-coordination property. The policy maker can thus drop the additional signals s from the original policy Γ and still guarantee that after $r=1$ is announced, pledging is the unique rationalizable action for all investors.

The inability to change the ranking in the investors' beliefs through public announcements is key to the optimality of simple pass/fail policies, as the next example shows.

Example 1. Suppose that θ is drawn from a uniform distribution over $[-1,2]$. Given θ, each investor $i \in [0,1]$ receives an exogenous signal $x_i \in \{x^L, x^H\}$, drawn independently across investors from a Bernoulli distribution with probability

$$\Pr\left\{x^L|\theta\right\} = \begin{cases} 2/3 & \text{if } \theta \in (0,1/3) \cup [2/3,5/6) \cup [1,7/6) \cup [4/3,5/3) \\ 1/3 & \text{if } \theta \in [1/3,2/3) \cup [5/6,1) \cup [7/6,4/3) \cup [5/3,2). \end{cases}$$

The value of $\Pr\left\{x^L|\theta\right\}$ for $\theta \in [-1,0]$ plays no role in this example, so it can be taken arbitrarily. Suppose that investors' payoffs are such that $g(\theta) = 1-c$ and $b(\theta) = -c$, for all θ, with $c \in (1/2, 8/15)$. There exits a deterministic policy that satisfies PCP and guarantees that default does not occur for $\theta > 0$, whereas no pass/fail policy can guarantee that default does not occur for all $\theta > 0$.[19]

[18] This means that the set $\theta \in \Theta$ such that (a) $k \in \varrho_\theta$ and (b) $(s,1) \in \text{supp}(\pi(\theta))$ has positive measure.

[19] The example features signals drawn from a distribution with finite support. This property, however, is not essential. Conclusions similar to those in the example obtain when the investors' signals are drawn from a continuous distribution. We thank Tommaso Denti for suggesting a similar example with finite signals and Leifu Zhang for suggesting an example with continuous signals.

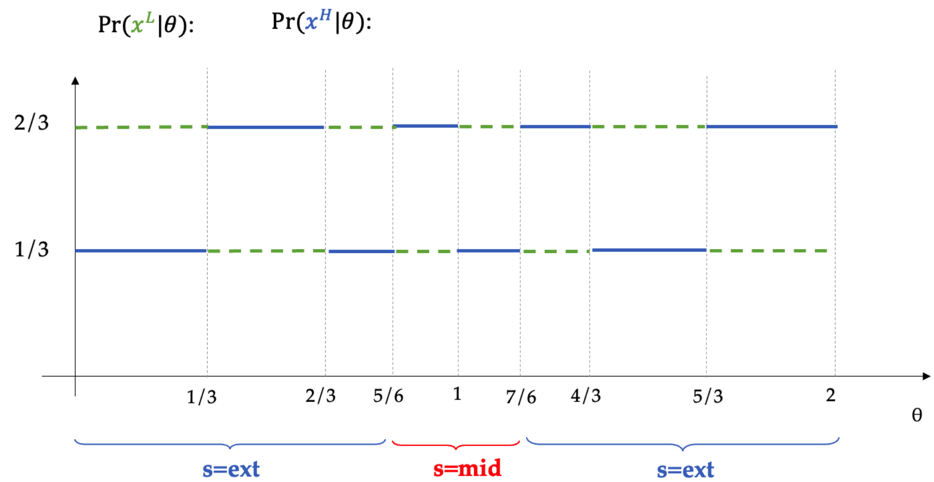

Figure 1: Sub-optimality of simple pass/tail tests

Proof of Example 1. Figure 1 illustrates the signal structure considered in Example 1. The dash line depicts the probability of signal x^L whereas the solid line the complementary probability of signal x^H, as a function of θ.

Note that the investors' posterior beliefs under the signal structure of Example 1 can be ranked according to FOSD, but not according to MLRP. Each investor observing x^H has posterior beliefs about θ that dominate those of each investor observing x^L in the FOSD order. Nonetheless, the ratio $p(x^H|\theta)/p(x^L|\theta)$ is not increasing in θ over the entire domain, meaning that $p(x|\theta)$ is not log-supermodular and hence posteriors cannot be ranked according to MLRP. Also note that, under the payoff specification in the example, pledging is optimal for an investor assigning probability to default no greater than $1-c$, whereas not pledging is optimal if such a probability is at least $1-c$.

To see that there exists no pass/fail policy guaranteeing that default does not occur for all $\theta > 0$, note that, by virtue of Theorem 1, if such a policy existed, there would also exist a binary policy satisfying PCP and such that $\pi(1|\theta) = 0$ for all $\theta \leq 0$ and $\pi(1|\theta) = 1$ for all $\theta > 0$, with $\pi(1|\theta)$ denoting the probability that the policy discloses signal 1 when the fundamentals are θ. Under such a policy, after hearing that $s = 1$, no matter the private signal x, each investor assigns probability $1/2$ to $\theta \in [0, 1]$ and probability $1/2$ to $\theta \in [1, 2]$. Because $c > 1/2$, each investor expecting all other investors to refrain from pledging (and hence default to occur for all $\theta \in [0,1]$) then finds it optimal to do the same. Hence, under MARP consistent with the above policy, after the signal $s = 1$ is announced, all investors refrain from pledging, meaning that the above policy fails to spare types $\theta \in [0,1]$ from default, when the investors play adversarially.

To see that, instead, the policy maker can avoid default for all $\theta > 0$ using a richer policy, consider the policy $\Gamma = (\{0, (1, mid), (1, ext)\}, \pi)$ that, in addition to publicly announcing a pass grade, also announces whether the fundamentals are *extreme* (i.e., $\theta \in (0, 5/6) \cup (7/6, 2]$), or *intermediate* (i.e., $\theta \in [5/6, 7/6]$). Formally, for any $\theta \in [-1, 0]$, $\pi(0|\theta) = 1$, meaning that the policy maker assigns a failing grade. For any $\theta \in [5/6, 7/6]$, instead, $\pi(1, mid|\theta) = 1$, meaning that the the policy maker announces a pass grade and that fundamentals are intermediate. Finally, for any $\theta \in (0, 5/6) \cup (7/6, 2]$, $\pi(1, ext|\theta) = 1$ meaning that the policy maker announces a pass grade and that fundamentals are extreme. See Figure 1 for a graphical representation.

Under such a policy, pledging is the unique rationalizable action for any investor observing a pass grade, no matter whether the investor also learns that the fundamentals are intermediate or extreme; but hearing this extra information is precisely what guarantees the uniqueness of the rationalizable action.

To see this, consider first the case in which the fundamentals are extreme, i.e., $\theta \in (0, 5/6) \cup (7/6, 2]$. All investors with exogenous information x^H find it dominant to pledge when hearing $s = (1, ext)$. In fact, even if all other investors refrained from pledging, the probability that each investor with signal x^H assigns to $\theta > 1$ (and hence to the bank surviving) is $\Pr\left\{\theta > 1 | x^H, ext\right\} = 8/15 > c$, making it dominant to pledge. As a consequence of this property, each investor with exogenous private information x^L finds it *iteratively* dominant to pledge. This is because, for any $\theta \in [1/3, 5/6]$, even if all investors with exogenous information equal to x^L refrained from pledging, the aggregate size of the pledge from those investors with information x^H would suffice for the bank to survive. This means that the probability that each investor with information x^L assigns to the bank surviving is at least equal to $\Pr\left\{\theta > 1/3 | x^L, (1, ext)\right\} = 11/15$, implying that it is optimal for the investor to pledge.

Next, consider the case in which fundamentals are intermediate, i.e., $\theta \in [5/6, 7/6]$. In this case, the ranking of the investors' optimism is reversed, with those investors observing the x^L signal assigning higher probability to higher states. In particular, because each investor with information x^L assigns probability $2/3 > c$ to $\theta \geq 1$, any such investor finds it dominant to pledge. Because, for any $\theta \in (5/6, 1)$, 1/3 of the investors receives information x^L, the minimal size of the pledge that each investor with signal equal to x^H can expect at any $\theta \in (5/6, 1)$ is equal to $\Pr\left\{x^L|\theta\right\} = 1/3 > 1 - \theta$, implying that even if all the less optimistic investors with signal equal to x^H refrained from pledging, the bank would survive. But this means, that pledging is iteratively dominant for those investors receiving the x^H signal.

Hence, the proposed policy spares any $\theta > 0$ from default. Because all investors pledge when they observe a pass grade, no matter whether they learn that the fundamentals are extreme or intermediate, one may find it surprising that the policy maker needs to provide the extra information. This is a consequence of the policy maker not trusting the market to play favorably to her. The extra information is precisely what guarantees the uniqueness of the rationalizable action. □

As anticipated above, the benefits from disclosing information in addition to the pass (or fail) grade stem from the possibility to reverse the ranking of the investors' optimism, which is possible only when the distribution $p(x|\theta)$ is not log-supermodular. In the example above, the most optimistic investors are those observing the x^L-signals when the fundamentals are intermediate, whereas they are those observing the x^H-signals when the fundamentals are extreme. The reversal in the investors' optimism in turn permits the policy maker to guarantee that pledging is the unique rationalizable action over a larger set of fundamentals (the entire set $\theta > 0$ in the example).

The above example also illustrates the failure of the Revelation Principle when the policy maker is concerned with unique implementation (equivalently, when the market is expected to play according to MARP). It is well known that, in this case, confining attention to policies that take the form of action recommendations is *with* loss of generality. The contribution of Theorem 2 is in showing that, notwithstanding such a qualification, the optimal policy does take the form of actions recommendation in the special case in which beliefs co-move with fundamentals according to MLRP.

3.3 Monotone rules

We now turn to the optimality of policies that fail with certainty institutions with weak fundamentals and pass with certainty those with strong fundamentals. As anticipated in the Introduction, the optimality of such rules crucially depends on whether the policy maker's preferences for avoiding regime change (i.e., default) for stronger fundamentals are large enough to compensate for the possibility that non-monotone rules may permit her to reduce the ex-ante probability of regime change.

In this subsection, we identify a condition relating the policy maker's preferences to the investors' exogenous beliefs and payoffs under which monotone rules are optimal. We show that the condition is fairly sharp in that, when violated, one can identify economies in which non-monotone rules do strictly better than monotone ones.

The analysis below extends to a richer class of economies in which the investors' payoffs $u(\theta, A)$ depend on the size of the aggregate pledge above and beyod the determination of the regime outcome. We assume that $u(\theta, \cdot)$ is monotone in A and that the following condition holds.

Condition FB. *For any x, $u(\theta, 1 - P(x|\theta)) \geq 0$ (alternatively, $u(\theta, 1 - P(x|\theta)) \leq 0$) implies that $u(\theta'', 1 - P(x|\theta'')) > 0$ for all $\theta'' > \theta$ (alternatively, $u(\theta', 1 - P(x|\theta')) < 0$ for all $\theta' < \theta$).*

Condition FB requires that, for any x, $u(\theta, 1 - P(x|\theta))$ changes sign only once, from negative to positive. This property clearly holds when $u(\theta, A)$, in addition to being non-decreasing in A as assumed above, is also non-decreasing in θ. It also holds when the default outcome is a deterministic function of (θ, A), as in the baseline model, because, for any (θ, A), $g(\theta, A) > 0 > b(\theta, A)$, and $r(\theta, A)$ is non-decreasing in A.

We also assume that

$$\left\{x \in \mathbb{R} : \int_{\Theta} u\left(\theta, 1 - P\left(x|\theta\right)\right) \mathbf{1}(\theta > 0) p\left(x|\theta\right) \mathrm{d}F\left(\theta\right) \leq 0\right\} \neq \emptyset. \tag{1}$$

When Condition (1) is violated, the expected payoff differential between pledging and not pledging is positive for any investor who is informed that fundamentals are non-negative and who expects each investor to pledge when receiving a signal above hers and not to pledge when receiving a signal below hers. In this case, the information-design problem is uninteresting because the policy maker can save all $\theta > 0$ through a policy that announces whether or not $\theta > 0$. Then, let

$$\bar{x}_G \equiv \sup \left\{ x \in \mathbb{R} : \int_\Theta u\left(\theta, 1 - P\left(x|\theta\right)\right) \mathbf{1}(\theta > 0) p\left(x|\theta\right) \mathrm{d}F\left(\theta\right) \leq 0 \right\}. \tag{2}$$

As we show in the Appendix, $\bar{x}_G$ is an upper bound for the set of cut-offs characterizing the strategies consistent with MARP across all disclosure policies Γ satisfying the perfect coordination property.

For any x, let $\mathit{\Theta}(x) \equiv \{\theta \in \Theta : x \in \varrho_\theta\}$ denote the set of fundamentals that, given the distribution $P(\cdot|\theta)$ from which the investors' signals are drawn, are consistent with private information x.

Condition-M. The following properties hold:

(1) $\inf \mathit{\Theta}(\bar{x}_G) \leq 0$;

(2') $|u(\theta, 1 - P(x|\theta))|$ *is log-supermodular over*[20]

$$\{(\theta, x) \in [0,1] \times \mathbb{R} : u(\theta, 1 - P(x|\theta)) \leq 0\}, \text{ and}$$

(2") *for any* $\theta_0, \theta_1 \in [0,1]$, *with* $\theta_0 < \theta_1$, *and* $x \leq \bar{x}_G$ *such that (a)* $u(\theta_1, 1 - P(x|\theta_1)) \leq 0$ *and (b)* $x \in \varrho_{\theta_0}$,

$$\frac{U^P(\theta_1, 1) - U^P(\theta_1, 0)}{U^P(\theta_0, 1) - U^P(\theta_0, 0)} > \frac{p\left(x|\theta_1\right) u\left(\theta_1, 1 - P\left(x|\theta_1\right)\right)}{p\left(x|\theta_0\right) u\left(\theta_0, 1 - P\left(x|\theta_0\right)\right)}. \tag{3}$$

Property (1) in Condition M says that the lower bound of the support of the beliefs of the marginal investor with signal $\bar{x}_G$, where $\bar{x}_G$ is the threshold defined in (2), is non-positive and therefore that, according to this investor, there is a positive probability that default is unavoidable, no matter the size of the pledge. Clearly, this property trivially holds when, for any θ, the investors' signals are drawn from a distribution whose support is large enough (and hence, a fortiori, when the noise in the investors' signals is drawn from a distribution with

[20] The log-supermodularity of $|u(\theta, 1 - P(x|\theta))|$ means that, for any $x', x'' \in \mathbb{R}$, with $x' < x''$, and any $\theta', \theta'' \in \Theta$, with $\theta'' > \theta'$, such that $u(\theta'', 1 - P(x'|\theta'')) \leq 0$,

$$u(\theta'', 1 - P(x''|\theta''))u(\theta', 1 - P(x'|\theta')) \geq u(\theta'', 1 - P(x'|\theta''))u(\theta', 1 - P(x''|\theta')).$$

unbounded support, e.g., a Normal distribution).

Property (2') says that the percentage reduction in the investors' loss from pledging when default occurs due to higher fundamentals is larger when more investors pledge. Precisely, for any $\theta' < \theta''$ and $x' < x''$ such that $u(\theta'', 1 - P(x'|\theta'')) < 0$,

$$\frac{u(\theta', 1 - P(x''|\theta')) - u(\theta'', 1 - P(x''|\theta''))}{u(\theta', 1 - P(x''|\theta'))} \leq \frac{u(\theta', 1 - P(x'|\theta')) - u(\theta'', 1 - P(x'|\theta''))}{u(\theta', 1 - P(x'|\theta'))}. \tag{4}$$

Note that $u(\theta'', 1 - P(x'|\theta'')) < 0$ implies that $u(\theta', 1 - P(x'|\theta')), u(\theta', 1 - P(x''|\theta')), u(\theta'', 1 - P(x''|\theta'')) < 0$. The left-hand side of (4) is thus the percentage reduction in the loss from pledging under regime change (i.e., default) when fundamentals improve from θ' to θ'' and investors pledge when, and only when, they receive signal $x \geq x''$. The right-hand side of (4), instead, is the analogous reduction when investors are more lenient and pledge when, and only when, they receive signals $x \geq x'$, with $x' < x''$. Importantly, this property is required to hold only for fundamentals θ and signal thresholds x for which the investors' expected payoff from pledging, $u(\theta, 1 - P(x|\theta))$, is non-positive. Also note that this property trivially holds when payoffs $u(\theta, A)$ are invariant in A conditional on the regime outcome as in the baseline model. The reason for stating the condition in these more general terms is that Condition M above plays a key role for the optimality of monotone policies also under richer payoff specifications in which $u(\theta, A)$ depends on A over and above the effect that the latter variable has on the default outcome (see the discussion in section 4).

Finally, Property (2") of Condition M says that the value the policy maker assigns to avoiding regime change must increase at a significant rate when fundamentals become stronger. Specifically, the property requires that the benefit that the policy maker derives from changing the investors' behavior (inducing all investors to pledge starting from a situation in which no investor pledges) must increase with the fundamentals at a sufficiently high rate, with the critical rate determined by a combination of the investors' payoffs and beliefs.

Theorem 3. *Suppose that $p(x|\theta)$ is log-supermodular and Condition M holds. Given any policy Γ, there exists a deterministic monotone policy $\Gamma^{\hat{\theta}} = (\{0,1\}, \pi^{\hat{\theta}})$ satisfying the perfect-coordination property and yielding the policy maker a payoff weakly higher than Γ. The policy $\Gamma^{\hat{\theta}}$ is such that there exists a threshold $\hat{\theta} \in [0,1]$ such that, for any $\theta \leq \hat{\theta}$, $\pi^{\hat{\theta}}(\theta)$ assigns probability one to $s = 0$, whereas for any $\theta > \hat{\theta}$, $\pi^{\hat{\theta}}(\theta)$ assigns probability one to $s = 1$.*

When Condition M holds, the choice of the optimal policy reduces to the choice of the smallest threshold $\hat{\theta}$ such that, when investors commonly learn that $\theta > \hat{\theta}$, under the unique rationalizable profile, all investors pledge irrespective of their exogenous private information. For this to be the case, it must be that, for any $x \in \mathbb{R}$, $\int_{\hat{\theta}}^{\infty} u(\theta, 1 - P(x|\theta))p(x|\theta)\mathrm{d}F(\theta) > 0$.

The above problem, however, does not have a formal solution, due to the lack of upper-hemicontinuity of the policy maker's payoff in $\hat{\theta}$. Notwithstanding these complications, hereafter we follow the pertinent literature and refer to the "optimal monotone policy" as the one defined as follows.

For any $\theta \in (0, 1)$, let $x^*(\theta)$ be the critical signal threshold such that, when investors follow a cut-off strategy with threshold $x^*(\theta)$, default occurs if and only if the fundamentals are below θ.[21] Let

$$\theta^* \equiv \inf \left\{ \hat{\theta} \geq 0 : \int_{\hat{\theta}}^{\infty} u\left(\tilde{\theta}, 1 - P\left(x^*(\theta)|\tilde{\theta}\right)\right) p\left(x^*(\theta)|\tilde{\theta}\right) \mathrm{d}F(\tilde{\theta}) \geq 0 \text{ for all } \theta \in \left[\hat{\theta}, 1\right) \right\} \quad (5)$$

be the lowest truncation point $\hat{\theta}$ such that, when the policy reveals that fundamentals are above $\hat{\theta}$, then for any possible default threshold $\theta \in \left[\hat{\theta}, 1\right)$, if default were to occur for fundamentals below θ and not for fundamentals above θ, then the marginal investor with signal $x^*(\theta)$ would find it optimal to pledge. Hereafter, we assume that θ^* is well-defined, which is always the case when[22]

$$\theta^{\#\#} \equiv \sup \left\{ \theta \in (0, 1) : \int_{\Theta} u\left(\tilde{\theta}, 1 - P\left(x^*(\theta)|\tilde{\theta}\right)\right) p\left(x^*(\theta)|\tilde{\theta}\right) \mathrm{d}F(\tilde{\theta}) \leq 0 \right\} < 1.$$

The optimal monotone policy is the one with cut-off $\hat{\theta} = \theta^*$.[23]

[21] For any $\theta \in (0, 1)$, the threshold $x^*(\theta)$ is implicitly defined by $P(x^*(\theta)|\theta) = \theta$. When the noise in the investors' signals is bounded, the definition of $x^*(\theta)$ can be extended to $\theta = 0$ and $\theta = 1$. When the noise is unbounded, abusing notation, one can extend the definition to $\theta = 0$ and $\theta = 1$ by letting $x^*(0) = -\infty$ and $x^*(1) = +\infty$.

[22] For any $\hat{\theta} \in (\theta^{\#\#}, 1)$, and any $\theta \in \left[\hat{\theta}, 1\right)$,

$$0 < \int_{-\infty}^{\infty} u\left(\tilde{\theta}, 1 - P\left(x^*(\theta)|\tilde{\theta}\right)\right) p\left(x^*(\theta)|\tilde{\theta}\right) \mathrm{d}F(\tilde{\theta}) < \int_{\hat{\theta}}^{\infty} u\left(\tilde{\theta}, 1 - P\left(x^*(\theta)|\tilde{\theta}\right)\right) p\left(x^*(\theta)|\tilde{\theta}\right) \mathrm{d}F(\tilde{\theta}).$$

Hence, when $\theta^{\#\#} < 1$, θ^* is well defined.

[23] The reason why this is an abuse is that, under the monotone policy with cut-off θ^*, in the continuation game that starts after the policy maker announces $s = 1$, there exists a rationalizable profile in which some of the investors refrain from pledging. However, there exists a monotone policy with cut-off $\hat{\theta}$ arbitrarily close to the threshold θ^* such that, after the policy maker announces $s = 1$ (equivalently, that $\theta \geq \hat{\theta}$), the unique

The previous literature characterized the threshold θ^* by restricting attention to monotone rules. The contribution of Theorem 3 is in identifying the conditions under which such rules are optimal. Importantly, these conditions are not met in the works that restrict attention to monotone rules. As the examples below suggest, monotone rules can be improved upon in those settings.

As we show in the Appendix, Property (1) in Condition M guarantees that, starting from the optimal monotone policy (the one with cut-off θ^*), one cannot perturb the policy by assigning a pass grade also to a small interval of fundamentals $[\theta', \theta'']$, with $0 \leq \theta' < \theta'' < \theta^*$, while guaranteeing that pledging remains the unique rationalizable action when the policy maker announces a pass grade (i.e., when signal $s = 1$ is disclosed). This property trivially holds when the noise in the investors' signals is large (and hence, a fortiori, when noise is unbounded), but plays a key role when the noise is drawn from a bounded interval of small size (see Example 2 below for an illustration).

Property (2') of Condition M in turn guarantees that, given any binary non-monotone rule, perturbations of the original policy that swap the probability of inducing all investors to pledge from low to high fundamentals in a way that keep the the payoff of the marginal investor constant, preserve the uniqueness of the investors' rationalizable action after hearing that $s = 1$. Again, this property is trivially satisfied in the baseline model where the investors' payoffs depend on the size of the aggregate pledge A only through the determination of the regime outcome. The property allows us to extend the result to a richer class of economies where investors' payoffs depend on A conditional on the regime outcome, as well as economies in which the regime outcome also depends on additional variables that imperfectly correlate with the fundamentals θ (see Section 4).

Property (2") of Condition M in turn guarantees that the higher value that the policy maker derives, under the perturbed policy, from avoiding default when fundamentals are stronger compensates for the possibility that, from an ex-ante perspective, the probability of regime change (i.e., default) may be larger under monotone policies than under non-monotone ones (see Example 3 for an illustration of why non-monotone rules may permit the policy maker

rationalizable profile features all investors pledging. Because the policy maker's payoff under the latter policy is arbitrarily close to the one she obtains when all investors pledge for $\theta > \theta^*$ and refrain from pledging when $\theta \leq \theta^*$, the abuse appears justified.

to avoid default over a larger measure of fundamentals).

As anticipated above, Condition M is fairly sharp in the sense that, when violated, one can identify economies in which the optimal policy is non-monotone. We provide two such examples below. Example 2 illustrates the role of Property (1) in Condition M, whereas Example 3 illustrates the role of Property (2”) in Condition M. These examples also illustrate why non-monotone rules, in general, may reduce the set of fundamentals over which regime change happens.

Let $\theta^{MS} \in (0,1)$ be implicitly defined by the unique solution to

$$\int_0^1 u(\theta^{MS}, A)\mathrm{d}A = 0. \tag{6}$$

The threshold θ^{MS} corresponds to the value of the fundamentals at which an investor who knows θ and holds *Laplacian beliefs* with respect to the measure of investors pledging is indifferent between pledging and not pledging.[24] Importantly, θ^{MS} is independent of the initial common prior F and of the distribution of the investors' signals.

Example 2. Suppose that there exist scalars $g, b \in \mathbb{R}$, with $g > 0 > b$, such that, for any θ, $g(\theta) = g$, and $b(\theta) = b$. Assume that θ is drawn from a uniform distribution with support $[-K, 1+K]$, for some $K \in \mathbb{R}_{++}$. Finally, assume that the investors' exogenous signals are given by $x_i = \theta + \sigma\epsilon_i$, with $\sigma \in \mathbb{R}_{++}$ and with each ϵ_i drawn independently across investors from a uniform distribution over $[-1, 1]$, with $\sigma < K/2$. Let θ^*_σ be the threshold defined in (5), applied to the primitives described in this example.[25] There exists $\sigma^\# \in (0, K/2)$ such that (a) $\inf \Theta(x^*_{\sigma^\#}(\theta^{MS})) > 0$, and (b) for all $\sigma \in (0, \sigma^\#)$, starting from the optimal monotone policy with cut-off θ^*_σ, there exists a deterministic *non-monotone* policy satisfying the perfect-coordination property and permitting the policy maker to avoid default over a set of fundamentals of strictly larger probability measure than the optimal monotone policy.

The proof is in the Appendix. Here we sketch the key arguments. To fix ideas, let $g = 1-c$ and $b = -c$, with $c \in (0,1)$, as in Example 1, and recall that, under such a payoff specification,

[24] This means that the investor believes that the proportion of investors pledging is uniformly distributed over $[0, 1]$. See Morris and Shin (2006).

[25] Hereafter, the subscript σ in θ^*_σ and x^*_σ is meant to highlight that these thresholds are those for the economy in which the noise in the investors' exogenous private signals is scaled by σ.

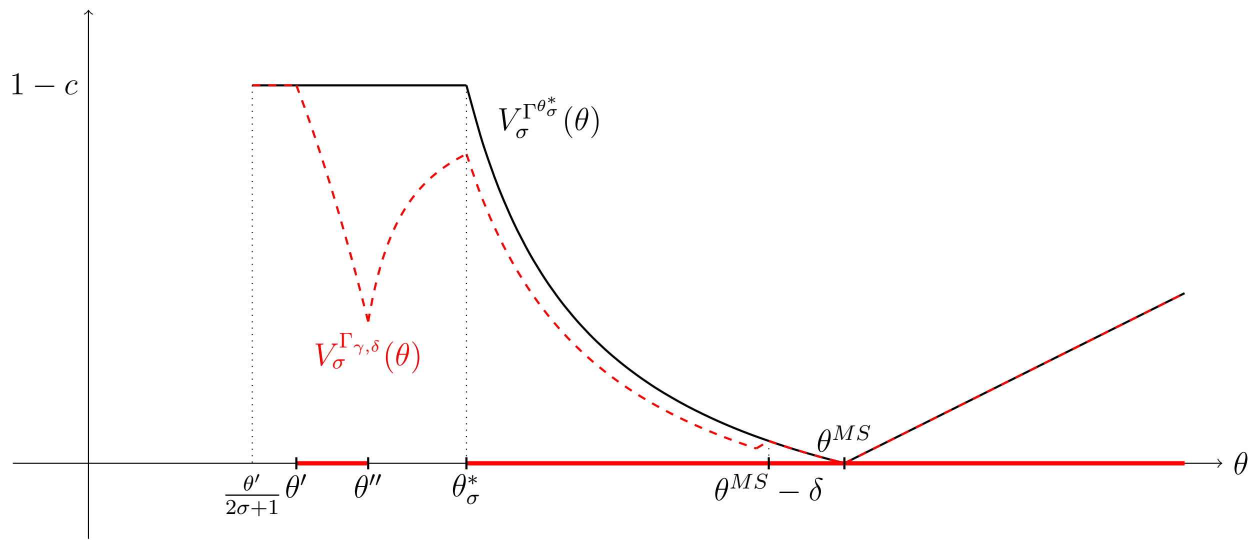

Figure 2: Sub-optimality of deterministic binary monotone policies.

pledging is optimal when the probability of regime change (i.e., default) is no greater than $1-c$, whereas not pledging is optimal when such a probability exceeds $1-c$.

For any $\theta \in [0,1]$, let $x^*_\sigma(\theta)$ be the critical signal threshold such that, when all investors pledge for $x > x^*_\sigma(\theta)$ and refrain from pledging for $x < x^*_\sigma(\theta)$, regime change occurs if and only if the fundamentals are below θ. For any binary policy $\Gamma = (\{0,1\},\pi)$, and any threshold $\theta \in [0,1]$ such that $(x^*_\sigma(\theta),1)$ are mutually consistent, then let

$$V_\sigma^\Gamma(\theta) \equiv U_\sigma^\Gamma\left(x^*_\sigma(\theta), 1 | x^*_\sigma(\theta)\right),$$

denote the payoff of the marginal investor with signal $x^*_\sigma(\theta)$, after the policy Γ announces that $s=1$, where U_σ^Γ is the function defined after Theorem 2.[26]

Now, for any $\hat{\theta} \in \Theta$, let $\Gamma^{\hat{\theta}} = (\{0,1\},\pi^{\hat{\theta}})$ be the deterministic, binary, monotone rule with cut-off $\hat{\theta}$. Note that the absence of any public disclosure is equivalent to a monotone policy with cut-off $\hat{\theta} = \min\Theta = -K$ and that, under such a policy, default occurs if and only if $\theta \leq \theta^{MS} = c$.

A necessary and sufficient condition for all investors to pledge under MARP after the monotone policy $\Gamma^{\hat{\theta}}$ announces that $s=1$ is that, for any possible default threshold $\theta > \hat{\theta}$, $V_\sigma^{\Gamma^{\hat{\theta}}}(\theta) > 0$. Next note that the lowest fundamental in the support of $x^*_\sigma(\theta)$'s beliefs is

[26] Recall that the payoff of the marginal investor is computed under the expectation that each investor with signal below $x^*_\sigma(\theta)$ refrains from pledging whereas each investor with signal above $x^*_\sigma(\theta)$ pledges.

$x_\sigma^*(\theta) - \sigma$. Hence, when $x_\sigma^*(\theta) - \sigma > \hat{\theta}$, the marginal investor with signal $x_\sigma^*(\theta)$ *already knows* from his private information that fundamentals are above $\hat{\theta}$ and thus learns nothing from the announcement that $s = 1$. Because, in the absence of any public disclosure, the payoff of the marginal investor is strictly negative for all $\theta < \theta^{MS}$, this implies that the cut-off θ_σ^* for the optimal deterministic monotone rule is $\theta_\sigma^* = x_\sigma^*(\theta^{MS}) - \sigma$.

Now to see that the optimal monotone policy is improvable, assume that σ is small so that $x_\sigma^*\left(\theta^{MS}\right) - \sigma > 0$. Next, pick $\gamma, \delta > 0$ small and let $\theta'' \equiv x_\sigma^*(\theta^{MS} - \delta) - \sigma$ and $\theta' \equiv \theta'' - \gamma$, with $\theta' > 0$. Consider a binary policy $\Gamma_{\gamma,\delta} = (\{0,1\}, \pi_{\gamma,d})$ that, in addition to announcing a pass grade $s = 1$ when fundamentals are above θ_σ^* (as the optimal monotone rule does) also announces $s = 1$ when $\theta \in [\theta', \theta'']$. Let $V_\sigma^{\Gamma_{\gamma,\delta}}(\theta)$ be the payoff of the marginal investor with signal $x_\sigma^*(\theta)$ under the new rule $\Gamma_{\gamma,\delta}$, after the policy maker announces that $s = 1$. This payoff is represented in Figure 2 along with the payoff $V_\sigma^{\Gamma^{\theta_\sigma^*}}(\theta)$ under the optimal monotone rule. Provided that γ and δ are small, $V_\sigma^{\Gamma_{\gamma,\delta}}(\theta) \geq 0$ for all θ for which $(x_\sigma^*(\theta), 1)$ are mutually consistent under $\Gamma_{\gamma,\delta}$, with $V_\sigma^{\Gamma_{\gamma,\delta}}(\theta) = 0$ if and only if $\theta = \theta^{MS}$. Starting from $\Gamma_{\gamma,\delta}$, one can then further perturb the policy $\Gamma_{\gamma,\delta}$ by giving a fail grade to banks with fundamentals in $[\theta_\sigma^*, \theta_\sigma^* + \varepsilon]$, with $\varepsilon > 0$ small. The new policy $\tilde{\Gamma}$ so constructed is such that $V_\sigma^{\tilde{\Gamma}}(\theta) > 0$ for all θ for which $(x_\sigma^*(\theta), 1)$ are mutually consistent under $\tilde{\Gamma}$, meaning that, when the policy maker announces that $s = 1$, pledging is the *unique* rationalizable action for all investors. The policy $\tilde{\Gamma}$ thus satisfies the perfect-coordination property and guarantees that default occurs over a set of fundamentals of strictly smaller measure than the optimal monotone one.

The reason why the non-monotone policy $\tilde{\Gamma}$ constructed in the proof of Example 2 guarantees that default occurs over a smaller set of fundamentals than the optimal deterministic monotone policy (the one with threshold θ_σ^*) is that investors receiving signals around θ^{MS} are highly sensitive to the grade the policy gives to banks with fundamentals around θ^{MS} but not so much so to the grade given to fundamentals far from θ^{MS}. In the above example with bounded noise, an investor receiving a signal $x_\sigma^*(\theta^{MS})$ is not sensitive at all to the grade the policy gives to fundamentals below $x_\sigma^*(\theta^{MS}) - \sigma$ because his private signal informs him that the fundamentals are above $x_\sigma^*(\theta^{MS}) - \sigma$. Hence, while it is impossible to amend the optimal deterministic monotone policy (the one with cut-off $\theta_\sigma^* = x_\sigma^*(\theta^{MS}) - \sigma$) by giving a pass grade also to fundamentals slightly below θ_σ^* without inducing some of the investors to refrain from pledging, it is possible to amend the optimal deterministic monotone policy by extending the pass grade to an interval $[\theta', \theta'']$ of fundamentals sufficiently "far away" from θ^*_σ, while continuing to induce all investors to pledge under MARP. The reason why such improvements are not feasible under Condition M in Theorem 3 is that Property 1 in Condition M implies that $x^*_\sigma(\theta^{MS}) - \sigma < 0$, thus making the above construction unfeasible.[27] Interestingly, when $\theta \in [\theta', \theta'']$, because of the bounded support of investors' beliefs, a positive-measure set of investors know with certainty that $\theta \in [\theta', \theta'']$, and yet the unique rationalizable action for them and the rest of investors remains to be pledging; this is because, by design, the policy $\tilde{\Gamma}$ prevents that, when $\theta \in [\theta', \theta'']$, such an event is commonly learned.

The next example considers an economy in which the noise in the investors' exogenous signals is drawn from an unbounded distribution (in which case, Property 1 in Condition M trivially holds), but Property (2") is violated.

Given any binary, deterministic policy $\Gamma = (\{0,1\}, \pi)$ (for any θ, $\pi(\theta)$ assigns probability 1 either to $s = 1$ or to $s = 0$), let $D^\Gamma = \left\{(\underline{\theta}_i, \bar{\theta}_i] : i = 1, ..., N\right\}$ denote the partition of $\left(0, \theta^{MS}\right]$ induced by π, with $N \in \mathbb{N}$, $\underline{\theta}_1 = 0$, and $\bar{\theta}_N = \theta^{MS}$.[28] Let $d \in D^\Gamma$ denote a generic cell of the partition D^Γ and, for any $\theta \in (0, \theta^{MS}]$, denote by $d^\Gamma(\theta) \in D^\Gamma$ the cell that contains θ. Finally, let $M(\Gamma) \equiv \max_{i=1,...,N} |\bar{\theta}_i - \underline{\theta}_i|$ denote the *mesh* of D^Γ, that is, the Lebesgue measure of the cell of D^Γ of maximal Lebesgue measure.

Example 3 below shows that, when the noise in the investors' information is small, any deterministic binary policy of large mesh can be improved upon by a non-monotone deterministic binary policy with a smaller mesh. This property in turn implies that, when the noise is small, optimal policies are highly non-monotone.

Example 3. Suppose that θ is drawn from an improper uniform prior over $\mathbb{R}$ and that the investors' signals are given by $x_i = \theta + \sigma\varepsilon_i$ with ε_i drawn from a standard Normal distribution.[29] Further assume that there exist scalars $g, b, W, L \in \mathbb{R}$, with $g > 0 > b$ and $W > L$, such that,

[27] Under Property (1), the marginal investor with signal $x^*_\sigma(\theta^{MS})$ does not rule out any fundamental in $(0, \theta^{MS})$. Hence, any perturbation of the optimal monotone policy passing fundamentals to the left of θ^{MS} induces the investor to refrain from pledging.

[28] Either (a) $\pi(\theta) = 0$ for all $\theta \in \bigcup_{i=2k, k \leq N} \left(\underline{\theta}_i, \bar{\theta}_i\right]$ and $\pi(\theta) = 1$ for all $\theta \in \bigcup_{i=2k-1, k \leq N} \left(\underline{\theta}_i, \bar{\theta}_i\right]$, or (b) $\pi(\theta) = 1$ for all $\theta \in \bigcup_{i=2k, k \leq N} \left(\underline{\theta}_i, \bar{\theta}_i\right]$ and $\pi(\theta) = 0$ for all $\theta \in \bigcup_{i=2k-1, k \leq N} \left(\underline{\theta}_i, \bar{\theta}_i\right]$.

[29] The improperness of the prior simplifies the exposition but is not important. The investors' hierarchies of beliefs are still well-defined.

for any θ, $g(\theta) = g$, $b(\theta) = b$, $W(\theta) = W$ and $L(\theta) = L$. There exists a scalar $\bar{\sigma} > 0$ and a function $\mathcal{E} : (0, \bar{\sigma}] \to \mathbb{R}_+$, with $\lim_{\sigma \to 0^+} \mathcal{E}(\sigma) = 0$, such that, for any $\sigma \in (0, \bar{\sigma}]$, in the game in which the noise in the investors' information is σ, the following is true: given any deterministic pass/fail policy $\Gamma = (\{0,1\}, \pi)$ satisfying the perfect-coordination property and such that $M(\Gamma) > \mathcal{E}(\sigma)$, there exists another deterministic pass/fail policy Γ^* with $M(\Gamma^*) < \mathcal{E}(\sigma)$ that also satisfies the perfect-coordination property and such that the ex-ante probability of default under Γ^* is strictly smaller than under Γ.

See the Online Appendix for a detailed derivation of the result. Here we discuss the main ideas. Non-monotone policies permit the policy maker to avoid default over a larger set of fundamentals by making it difficult for the investors to commonly learn the fundamentals when the latter are between 0 and θ^{MS} and the policy maker announces a pass grade. Intuitively, if the policy maker assigned a pass grade to an interval $(\theta', \theta''] \subset (0, \theta^{MS}]$ of large Lebesgue measure, when σ is small and $\theta \in (\theta', \theta'']$, most investors would receive private signals $x_i \in (\theta', \theta'']$. No matter the grade assigned to fundamentals outside the interval $(\theta', \theta'']$, in the continuation game that starts after the policy maker announces a pass grade, most investors with signals $x_i \in (\theta', \theta'']$ would then assign high probability to the joint event that $\theta \in (\theta', \theta'']$, that other investors assign high probability to $\theta \in (\theta', \theta'']$, and so on. When this is the case, it is rationalizable for such investors to refrain from pledging. Hence, when σ is small, the only way the policy maker can guarantee that, when $\theta \in (0, \theta^{MS}]$, the investors pledge after hearing a pass grade is by dividing the subset of $(0, \theta^{MS}]$ into a collection of disjoint intervals, each of small Lebesgue measure. This guarantees that the support of each investor's posterior beliefs after a pass grade is announced is not connected. Connectedness of the supports facilitates rationalizable profiles where some investors refrain from pledging.

Next, suppose that the intervals $\left(\underline{\theta}_i, \bar{\theta}_i\right] \subset \left(0, \theta^{MS}\right]$, $i = 1, ..., N$, receiving a pass grade are far apart, implying that the policy maker fails an interval $(\theta', \theta''] \subset (0, \theta^{MS}]$ of large Lebesgue measure (note that this is indeed the case under the optimal monotone deterministic rule with cutoff θ^*_σ, where θ^*_σ is as defined in (5).[30] The detailed derivations in the Online Appendix then show that, starting from Γ, the policy maker could assign a pass grade to fundamentals in the middle of $[\theta', \theta'']$ and a fail grade to some fundamentals to the right of θ'', in such a way

[30]The subscript simply highlights the dependence of the cutoff θ^*_σ on σ.

that (a) pledging continues to be the unique rationalizable action for all investors after hearing a pass grade, and (b) the set of fundamentals receiving a pass grade under the new policy is strictly larger than under the original one. Furthermore, the construction sketched above can be iterated till one arrives at a new policy with a mesh smaller than $\mathcal{E}(\sigma)$ under which regime change (i.e., default)occurs over a set of fundamentals of strictly smaller measure than under the original policy. When the benefit $W(\theta) - L(\theta)$ of avoiding default is constant in θ , the new policy thus yields the policy maker a strictly higher payoff than the original one.

Finally, one can show that, when σ is small, a pass grade can be given to all $\theta > \theta^{MS} + \varepsilon$, with $\varepsilon > 0$ small, while guaranteeing that all investors pledge after the policy maker announces the pass grade $s = 1$.[31]

The above properties thus also imply that if the policy maker is restricted to deterministic policies (arguably, the most relevant case in practice), when the precision of the investors' exogenous information is large, the optimal policy is highly non-monotone over $(0, \theta^{MS})$ and announces a pass grade when fundamentals are above θ^{MS}. □

3.3.1 Discussion: role of multiplicity of receivers and exogenous private information

It is worth contrasting the above results about the sub-optimality of monotone rules (when Condition M is violated) to those for economies featuring either a single privately-informed receiver, or multiple receivers with no exogenous private information.

Single receiver. The optimal policy is a simple monotone pass/fail policy with cutoff equal to $\theta^* = 0$. This is because, in this model, the policy maker's and the receiver's payoffs are aligned (they both want to avoid default when possible). With a single receiver, there is no risk of adversarial coordination and hence the optimal policy coincides with the one that the designer would select if she trusted the receiver to play favorably to her.

Things are different when preferences are misaligned. To see this, suppose the policy maker's payoff is equal to W in case of no default, and L in case of default, with $W > L$ as in Examples 2 and 3 above. However, now suppose that the receiver's payoff differential

[31] Formally, for any $\varepsilon > 0$, there exists $\sigma(\varepsilon)$ such that, for any $\sigma < \sigma(\varepsilon)$, given any pass/fail policy Γ satisfying PCP, there exists another pass/fail policy Γ' also satisfying PCP that agrees with Γ on any $\theta < \theta^{MS}$ and gives a pass grade to any $\theta \geq \theta^{MS} + \varepsilon$.

between pledging and not pledging is equal to $-g$ in case of default and $-b$ in case of no default, with $g > 0 > b$. Such a payoff differential may reflect the idea that the receiver is a speculator whose payoff is zero when he refrains from speculating (equivalently, when he pledges), is positive when he speculates and default occurs, and is negative when he speculates and default does not occur. Using the results in Guo and Shmaya (2019), one can then show that the optimal policy in this case has the *interval structure*: each type x of the receiver is induced to play the action favorable to the policy maker (abstain from speculating) over an interval of fundamentals $[\theta_1(x), \theta_2(x)]$, with $\theta_1(x) < 1 < \theta_2(x)$, for all x, and with $\theta_1(x)$ decreasing in x and $\theta_2(x)$ increasing in x. Such a policy requires disclosing more than two signals and hence cannot be implemented through a simple pass/fail test. In contrast, with a continuum of heterogeneously informed receivers with the same payoffs as in the variant above, the optimal policy is a pass-fail test that is typically non-monotone in θ.[32] Furthermore, when the optimal policy is not monotone, it does not have the interval structure, as each receiver with signal x is induced to pledge over a non-connected set of fundamentals. The reason for these differences is that, with a single receiver, to avoid an attack, the policy maker must persuade the receiver that the fundamentals are likely to be above 1, in which case the attack is unsuccessful. With multiple receivers, instead, the policy maker must persuade each receiver that enough other receivers are not attacking, which, as shown above, is best accomplished by a non-monotone policy that makes it difficult for the receivers to commonly learn the fundamentals, when the latter are between 0 and θ^{MS}.[33]

Multiple receivers with no exogenous private information. When all receivers have the same posterior beliefs, no matter whether payoffs are aligned or mis-aligned, under MARP, each receiver plays the friendly action only if it is dominant to do so. The optimal policy is a simple monotone pass/fail policy with cutoff θ^* implicitly defined by

$$\int_{\theta^*}^{1} b \mathrm{d}F(\theta) + \int_{1}^{\infty} g \mathrm{d}F(\theta) = 0.$$

[32] This is because, under MARP, all investors play the friendly action if and only if it is iteratively dominant for them to do so, irrespective of the alignment in payoffs.

[33] Mensch (2021) characterizes general conditions under which the optimal policy is monotone with a single, *uninformed*, receiver. Goldstein and Leitner (2018) studies an economy in which these conditions are not satisfied and the optimal policy is non-monotone. The analysis in these works is very different in that it does not identify the role that coordination and the receivers' private information play for the optimal policy.

The reason why the optimal policy is monotone when the receivers possess no exogenous private information is that the policy maker needs to convince each of them that θ is above 1 with sufficiently high probability to make the friendly action dominant.

4 Extensions

4.1 Generalizations

In this section we discuss richer economies.

The fundamentals are given by (θ, z), with θ drawn from Θ according to F, and with z drawn from $[\underline{z}, \bar{z}]$ according to $Q_\theta(z)$, with the cdf $Q_\theta(z)$ weakly decreasing in θ, for any z.[34]

The variable θ continues to parametrize the maximal information the policy maker can collect about the fundamentals. The additional variable z parametrizes risk that the investors and the policy maker face at the time of the disclosure (e.g., macroeconomic variables that are only imperfectly correlated with the fundamentals).[35]

There exists a function $R : \Theta \times [0, 1] \times [\underline{z}, \bar{z}] \to \mathbb{R}$ such that, given (θ, A, z), default occurs (i.e., $r = 0$) if, and only if, $R(\theta, A, z) \leq 0$. The function R is continuous, strictly increasing in (θ, z, A), and such that $R(\underline{\theta}, 1, \bar{z}) = R(\bar{\theta}, 0, \underline{z}) = 0$, for some $\underline{\theta}, \bar{\theta} \in \mathbb{R}$, with $\underline{\theta} < \bar{\theta}$. The thresholds $\underline{\theta}$ and $\bar{\theta}$ define the "critical region" $(\underline{\theta}, \bar{\theta}]$ where the regime outcome depends on the response of the market. For any (θ, A), the probability of avoiding default is thus given by $r(\theta, A) \equiv \Pr\{R(\theta, A, z) > 0|\theta, A\}$.

The policy maker's payoff is equal to

$$\hat{U}^P(\theta, A, z) = \hat{W}(\theta, A, z)\mathbf{1}\{R(\theta, A, z) > 0\} + \hat{L}(\theta, A, z)\mathbf{1}\{R(\theta, A, z) \leq 0\}, \tag{7}$$

whereas the investors' payoff differential between playing the "friendly" action (e.g., pledging funds to the banking sector) and the "adversarial" action is equal to

$$\hat{u}(\theta, A, z) = \hat{g}(\theta, A, z)\mathbf{1}\{R(\theta, A, z) > 0\} + \hat{b}(\theta, A, z)\mathbf{1}\{R(\theta, A, z) \leq 0\}. \tag{8}$$

[34] All the results extend to the case where $Q_\theta(z)$ has unbounded support.

[35] As in the baseline model, conditional on θ, the private signals $(x_i)_{i\in[0,1]}$ are i.i.d. draws from an (absolutely continuous) cumulative distribution function $P(x|\theta)$, with associated density $p(x|\theta)$ strictly positive over the interval $\varrho_\theta \in \mathbb{R}$.

with $\hat{g}(\theta, A, z) > 0 > \hat{b}(\theta, A, z)$, for any (θ, A, z). For any (θ, A), then let

$$g(\theta, A) \equiv \frac{\mathbb{E}\{\mathbf{1}(R(\theta, A, z) > 0)\hat{g}(\theta, A, z)|\theta, A\}}{r(\theta, A)} \text{ and } b(\theta, A) \equiv \frac{\mathbb{E}\{\mathbf{1}(R(\theta, A, z) \leq 0)\hat{b}(\theta, A, z)|\theta, A\}}{1 - r(\theta, A)}$$

denote the agents' expected payoff differential, respectively, in case of no default and in case of default, and, likewise, let

$$W(\theta, A) \equiv \frac{\mathbb{E}\{\mathbf{1}(R(\theta, A, z) > 0)\hat{W}(\theta, A, z)|\theta, A\}}{r(\theta, A)} \text{ and } L(\theta, A) \equiv \frac{\mathbb{E}\{\mathbf{1}(R(\theta, A, z) \leq 0)\hat{L}(\theta, A, z)|\theta, A\}}{1 - r(\theta, A)}$$

denote the policy maker's expected payoff, again in case of no default and default, respectively.

The investors' and the policy maker's expected payoffs can then be conveniently expressed as a function of θ and A only, by letting

$$u(\theta, A) \equiv r(\theta, A)g(\theta, A) + (1 - r(\theta, A))b(\theta, A) \text{ and } U^P(\theta, A) \equiv r(\theta, A)W(\theta, A) + (1 - r(\theta, A))L(\theta, A).$$

Hereafter, we assume that both $u(\theta, A)$ and $U^P(\theta, A)$ are non-decreasing in A and that $U^P(\theta, 1) > U^P(\theta, 0)$ for all $\theta \in (\underline{\theta}, \bar{\theta}]$.[36]

4.2 Results.

We now identify conditions under which Theorems 1-3 extend to these enriched economies.

Given any common posterior $G \in \Delta(\Theta)$, for any x such that $\int p(x|\theta)G(\mathrm{d}\theta) > 0$, let

$$\bar{U}^G(x) \equiv \left(\int u(\theta, 1 - P(x|\theta))p(x|\theta)G(\mathrm{d}\theta)\right) / \int p(x|\theta)G(\mathrm{d}\theta)$$

denote the expected payoff differential of an investor with signal x who expects all other investors to pledge if their private signal exceeds x and to not pledge otherwise. Let ξ^G be the largest solution to $\bar{U}^G(x) = 0$ if such an equation admits a solution, $\xi^G = +\infty$ if $\bar{U}^G(x) < 0$ for all x such that $\int p(x|\theta)G(\mathrm{d}\theta) > 0$, and $\xi^G = -\infty$ if $\bar{U}^G(x) > 0$ for all x such that $\int p(x|\theta)G(\mathrm{d}\theta)$. Finally, let $\theta^G \equiv \inf\left\{\theta : u(\theta, 1 - P(\xi^G|\theta)) \geq 0\right\}$. The interpretation of ξ^G and θ^G is the following. Suppose that the policy maker induces a common posterior G

[36] That $u(\theta, A)$ is monotone in A implies that the continuation game remains supermodular. That $U^P(\theta, A)$ is non-decreasing in A implies that, for any Γ, MARP continues to coincide with the "smallest" rationalizable profile, that is, the one involving the smallest measure of agents pledging. Finally, that, for any θ in the critical region, the policy maker strictly prefers that all investors pledge to no agent pledging guarantees that, when the optimal policy has a pass/fail structure, it is obtained by maximizing the probability that a pass grade is given when fundamentals are in the critical range.

over Θ, $p(x|\theta)$ is log-supermodular, and Condition FB holds. Then, in the continuation game that starts after the policy Γ induces the common posterior G, MARP is in cut-off strategies and is defined by the cut-off ξ^G.[37] When investors play according to MARP given the induced posterior G, their expected payoff differential is non-positive for all $\theta \leq \theta^G$ and non-negative for all $\theta > \theta^G$.

Condition PC. *For any distribution $\Lambda \in \Delta(\Delta(\Theta))$ over posterior beliefs consistent with the common prior F (i.e., such that $\int G\Lambda(\mathrm{d}G) = F$), the following condition holds:*

$$\int \left(\int_{-\infty}^{\theta^G} U^P(\theta, 0) G(\mathrm{d}\theta) + \int_{\theta^G}^{+\infty} U^P(\theta, 1) G(\mathrm{d}\theta) \right) \Lambda(\mathrm{d}G) \geq \int \left(\int U^P(\theta, 1 - P(\xi^G|\theta)) G(\mathrm{d}\theta) \right) \Lambda(\mathrm{d}G).$$

Condition PC trivially holds when the policy maker faces no aggregate uncertainty (i.e., when each distribution Q_θ is degenerate), and W and L are invariant in A, as in the baseline model. More generally, Condition PC accommodates for the possibility that both W and L depend on A, possibly non-monotonically, provided that, on average, the loss to the policy maker from having no agent pledge in states $\theta \leq \theta^G$ in which the agents' expected payoff differential (under MARP) is negative is more than compensated by the benefit from having all agents pledge in states $\theta > \theta^G$ in which the differential is positive. The average is over both the posteriors induced by the policy maker and the fundamentals. Roughly, the condition says that the loss to the policy maker from having no investor pledging in those states in which, under MARP consistent with the policy Γ, the investors' expected payoff differential is negative is more than compensated by the benefit from having all investors pledging in those states in which their expected payoff differential is positive. The condition thus requires that the policy maker's and the investors' payoffs be not too misaligned.

Theorem 4. (a) Given any regular policy Γ, there exists a regular policy Γ^* satisfying *PCP* and such that, when agents play according to MARP, at any θ, their expected payoff differential under Γ^* is at least as high as under Γ.[38] Furthermore, when, under MARP, θ perfectly predicts the default outcome, the probability of default under Γ^* is the same as under Γ; (b) Suppose that $p(x|\theta)$ is log-supermodular and Condition FB holds. The policy Γ^* from part (a) is a pass/fail policy; (c) If in addition to the conditions in part (b), Condition PC also

[37] The proof follows from arguments similar to those in the proof of Theorem 2 in the main text.

[38] A policy Γ is regular if MARP under Γ is well defined and the sign of the agents' expected payoff differential under MARP is measurable in θ.

holds, then the policy maker's payoff under Γ^* is at least as high as under Γ; (d) Suppose that, in addition to the conditions in part (c), Conditions M also holds. Then, Γ^* is a deterministic monotone policy.

Proof. See the Appendix.

Theorem 4 qualifies in what sense the results in the baseline model extend to richer economies. The value of the generalizations is that they permit one to study the comparative statics of optimal policies in various micro-founded economies in which for example financial institutions issue equity (alternatively, debt) to finance their short-term obligations and where the price of the securities is endogenous and influenced by the outcome of the disclosure policy (see the file "Additional Material" on the authors' webpages for a few examples along these lines).

5 Conclusions

We consider the design of public information in coordination settings in which the designer does not trust the receivers to play favorably to her. We show that, despite the fear of adversarial coordination, the optimal policy induces all receivers to take the same action. Importantly, while each investor can perfectly predict the action of any other investor, she is not able to predict the beliefs that rationalize such actions. We identify conditions under which the optimal policy has a pass/fail structure, as well as conditions under which the optimal policy is monotone, passing institutions with strong fundamentals and failing the others.

The results are worth extending in a few directions. The analysis assumes that the policy maker is Bayesian and knows the distribution from which the investors' exogenous private information is drawn. While this is a natural starting point, in future work it would be interesting to investigate how the structure of the optimal policy is affected by the policy maker's uncertainty about the investors' information sources.[39]

The analysis in the present paper is static. Many applications of interest are dynamic, with investors coordinating on multiple attacks and/or learning over time (for the role of dynamics in global games, see, among others, Angeletos et al. (2007)). In future work, it would be

[39]See Dworczak and Pavan (2022) for a notion of robustness in information design that accounts for this type of ambiguity.

interesting to consider dynamic extensions and investigate how the timing of information disclosures is affected by the investors' behavior in previous periods.[40]

Finally, the analysis is conducted by assuming that the maximal information that the designer can collect about the fundamentals (in the paper, θ) is exogenous. In future work, it would be interesting to accommodate for the possibility that part of the information is provided by the banks themselves. This creates an interesting screening+persuasion problem in the spirit of the literature on privacy in sequential contacting (see, e.g., Calzolari and Pavan (2006a) Calzolari and Pavan (2006b), Dworczak (2020)).

Appendix

Proof of Theorem 1. Given any regular policy $\Gamma = (\mathcal{S}, \pi)$ and any $n \in \mathbb{N}$, let $T^{\Gamma}_{(n)}$ be the set of strategies surviving n rounds of IDISDS, with $T^{\Gamma}_{(0)}$ denoting the entire set of strategy profiles $a = (a_i(\cdot))_{i\in[0,1]}$, where for any $i \in [0,1]$, $a_i(x,s)$ denotes the probability investor i pledges, given (x,s). Let $a^{\Gamma}_{(n)} \equiv \left(a^{\Gamma}_{(n),i}(\cdot)\right)_{i\in[0,1]} \in T^{\Gamma}_{(n)}$ denote the most aggressive profile surviving n rounds of IDISDS (that is, the profile in $T^{\Gamma}_{(n)}$ that is most adversarial to the policy maker, in the sense that it minimizes the policy maker's ex-ante payoff). The profiles $\left(a^{\Gamma}_{(n)}\right)_{n\in\mathbb{N}}$ can be constructed inductively as follows. The profile $a^{\Gamma}_{(0)} \equiv \left(a^{\Gamma}_{(0),i}(\cdot)\right)_{i\in[0,1]}$ prescribes that all investors refrain from pledging, irrespective of (x,s). Next, let $U^{\Gamma}_i(x_i, s; a)$ denote the payoff differential between pledging and not pledging for investor i when, under Γ, all other investors follow the strategy in a. Then, $a^{\Gamma}_{(n),i}(x_i,s) = 0$ if $U^{\Gamma}_i\left(x_i, s; a^{\Gamma}_{(n-1)}\right) \leq 0$ and $a^{\Gamma}_{(n),i}(x_i,s) = 1$ if $U^{\Gamma}_i\left(x_i, s; a^{\Gamma}_{(n-1)}\right) > 0$. MARP consistent with Γ is then the profile $a^{\Gamma} = (a^{\Gamma}_i(\cdot))_{i\in[0,1]}$ given by $a^{\Gamma}_i(\cdot) = \lim_{n\to\infty} a^{\Gamma}_{(n),i}(\cdot)$, all $i \in [0,1]$.

Next, consider the policy $\Gamma^+ = (\mathcal{S}^+, \pi^+)$, $\mathcal{S}^+ \equiv \mathcal{S} \times \{0,1\}$, that, for each θ, draws the score s from the same distribution $\pi(\theta) \in \Delta(\mathcal{S})$ as the original policy Γ, and then, for each s it draws, it also announces the sign of the investors' payoff differential at (θ, s), when investors play according to MARP a^{Γ} consistent with the original policy Γ. In the baseline model of Section 2, the sign of such payoff differential is uniquely determined by the regime outcome $r^{\Gamma}(\theta, s)$. For any θ, and any $s \in supp(\pi(\theta))$, the new policy Γ^+ thus announces $\left(s, r^{\Gamma}(\theta, s)\right)$.

Define $T^{\Gamma+}_{(n)}$ and $a^{\Gamma+}_{(n)}$ analogously to $T^{\Gamma}_{(n)}$ and $a^{\Gamma}_{(n)}$, but with respect to the policy Γ^+.

[40]For models of dynamic persuasion, see, among others, Ely (2017) and Basak and Zhou (2020b).

The proof is in three steps. Steps 1 and 2 show that any investor i who, given (x_i, s), finds it dominant (alternatively, iteratively dominant) to pledge under Γ also finds it dominant (alternatively, iteratively dominant) to pledge under Γ^+ when receiving information $(x_i, (s, 1))$. Step 3 uses the above property to establish that, because the game is supermodular and $a^{\Gamma+}$ is "less aggressive" than a^Γ (meaning that any investor who, given (x, s), pledges under a^Γ also pledges under $a^{\Gamma+}$ when receiving the information $(x, (s, 1))$, then, under $a^{\Gamma+}$, all investors pledge (alternatively, refrain from pledging) when receiving information $(s, 1)$ (alternatively, $(s, 0)$).

Step 1. First, we prove that, $(x_i, s) : U_i^\Gamma(x_i, s; a) > 0\ \forall a\} \subseteq \{(x_i, s) : U_i^{\Gamma+}(x_i, (s, 1); a) > 0\ \forall a\}$, for all $i \in [0, 1]$. That is, any investor i who, under Γ, finds it dominant to pledge, given the information (x_i, s), also finds it dominant to pledge under Γ^+ when receiving the information $(x_i, (s, 1))$.

First, note that the supermodularity of the game implies that $\{(x_i, s) : U_i^\Gamma(x_i, s; a) > 0\ \forall a\} = \{(x_i, s) : U_i^\Gamma(x_i, s; a_{(0)}^\Gamma) > 0\}$ and $\{(x_i, s) : U_i^{\Gamma+}(x_i, (s, 1); a) > 0\ \forall a\} = \{(x_i, s) : U_i^{\Gamma+}(x_i, (s, 1); a_{(0)}^{\Gamma+}) > 0\}$.

Now let $\Lambda_i^\Gamma(\theta, \mathbf{x}|x_i, s)$ denote the beliefs of investor $i \in [0, 1]$ over θ and the cross-sectional distribution of signals, $\mathbf{x} \in \mathbb{R}^{[0,1]}$, when receiving information $(x_i, s) \in \mathbb{R} \times \mathcal{S}$ under Γ, and $\Lambda_i^{\Gamma+}(\theta, \mathbf{x}|x_i, (s, 1))$ the corresponding beliefs under Γ^+. Bayesian updating implies that

$$\Lambda_i^{\Gamma+}(\mathrm{d}(\theta, \mathbf{x})|x_i, (s, 1)) = \frac{\mathbf{1}\left(r^\Gamma(\theta, s) = 1\right)}{\Lambda_i^\Gamma(1|x_i, s)} \Lambda_i^\Gamma(\mathrm{d}(\theta, \mathbf{x})|x_i, s), \tag{9}$$

where $\mathbf{1}\left(r^\Gamma(\theta, s) = 1\right)$ is the indicator function, taking value 1 if θ is such that $r^\Gamma(\theta, s) = 1$, and 0 otherwise, and where $\Lambda_i^\Gamma(1|x_i, s) \equiv \int_{\{(\theta,\mathbf{x}):r^\Gamma(\theta,s)=1\}} \Lambda_i^\Gamma(\mathrm{d}(\theta, \mathbf{x})|x_i, s)$.

Next, observe that, under both $a_{(0)}^\Gamma$ and $a_{(0)}^{\Gamma+}$, default occurs if, and only if, $\theta \le 1$. Take any $i \in [0, 1]$ and $(x_i, s) \in \mathbb{R} \times \mathcal{S}$ such that

$$U_i^\Gamma\left(x_i, s; a_{(0)}^\Gamma\right) = \int_{(\theta,\mathbf{x})} \left(b(\theta)\mathbf{1}\left(\theta \le 1\right) + g(\theta)\mathbf{1}(\theta > 1)\right) \Lambda_i^\Gamma(\mathrm{d}(\theta, \mathbf{x})|x_i, s) > 0. \tag{10}$$

The aforementioned property of Bayesian updating implies that

$$\begin{aligned} U_i^{\Gamma+}(x_i, (s, 1); a_{(0)}^{\Gamma+})\Lambda_i^\Gamma(1|x_i, s) &= \textstyle\int_{(\theta,\mathbf{x})} \left(b(\theta)\mathbf{1}\left(\theta \le 1\right) + g(\theta)\mathbf{1}(\theta > 1)\right) \mathbf{1}(r^\Gamma(\theta, s) = 1)\Lambda_i^\Gamma(\mathrm{d}(\theta, \mathbf{x})|x_i, s) \\ &\ge \textstyle\int_{(\theta,\mathbf{x})} \left(b(\theta)\mathbf{1}\left(\theta \le 1\right) + g(\theta)\mathbf{1}(\theta > 1)\right) \Lambda_i^\Gamma(\mathrm{d}(\theta, \mathbf{x})|x_i, s) = U_i^\Gamma((x_i, s); a_{(0)}^\Gamma) > 0, \end{aligned}$$

where the first equality follows from (9), the first inequality from the fact that, for all θ such that $r^{\Gamma}(\theta,s)=0$, $b(\theta)\mathbf{1}(\theta\leq 1)+g(\theta)\mathbf{1}(\theta>1)=b(\theta)<0$, the second equality follows from the definition of $U_i^{\Gamma}\left(x_i,s;a_{(0)}^{\Gamma}\right)$, and the second inequality from (10). Thus, any investor for whom pledging was dominant after receiving information (x_i,s) under Γ, continues to find it dominant to pledge after receiving information $(x_i,(s,1))$ under Γ^{+}.

Step 2. Next, take any $n>1$. Assume that, for any $1\leq k\leq n-1$, any $i\in[0,1]$,

$$\{(x_i,s):U_i^{\Gamma}(x_i,s;a)>0\ \ \forall a\in T_{(k-1)}^{\Gamma}\}\subseteq\{(x_i,s):U_i^{\Gamma^{+}}(x_i,(s,1);a)>0,\ \ \forall a\in T_{(k-1)}^{\Gamma^{+}}\}.\quad(11)$$

Arguments similar to those establishing the result in Step 1 above imply that

$$\{(x_i,s):U_i^{\Gamma}(x_i,s;a)>0\ \ \forall a\in T_{(n-1)}^{\Gamma}\}\subseteq\{(x_i,s):U_i^{\Gamma^{+}}(x_i,(s,1);a)>0,\ \ \forall a\in T_{(n-1)}^{\Gamma^{+}}\}.\quad(12)$$

Intuitively, the result follows from the following two properties: (a) because the game is supermodular, $\{(x_i,s):U_i^{\Gamma}(x_i,s;a)>0\ \ \forall a\in T_{(n-1)}^{\Gamma}\}=\{(x_i,s):U_i^{\Gamma}\left(x_i,s;a_{(n-1)}^{\Gamma}\right)>0\}$ where recall that $a_{(n-1)}^{\Gamma}$ is the most aggressive profile surviving $n-1$ rounds of IDISDS (clearly, the same property holds for Γ^{+}); (b) $a_{(n-1)}^{\Gamma^{+}}$ is "less aggressive" than $a_{(n-1)}^{\Gamma}$, in the sense that any investor who, given (x,s), pledges under $a_{(n-1)}^{\Gamma}$ also pledges under Γ^{+} when receiving information $(x,(s,1))$; and (c) the observation that $r^{\Gamma}(\theta,s)=1$ removes from the support of the investors' posterior beliefs states in which default would have occurred under a^{Γ} and hence under $a_{(n-1)}^{\Gamma}$ as well (observe that $a_{(n-1)}^{\Gamma}$ is more aggressive that a^{Γ}, meaning that any investor who, given (x,s), pledges under $a_{(n-1)}^{\Gamma}$, also pledges under a^{Γ} when receiving the same information $(x,1)$).

Step 3. Equipped with the results in steps 1 and 2 above, we now prove that, for all $\theta\in\Theta$ and all $s\in supp(\pi(\theta))$ such that $r^{\Gamma}(\theta,s)=1$, for any $\mathbf{x}\in\mathbf{X}(\theta)$, and any $i\in[0,1]$, $a_i^{\Gamma^{+}}(x_i,(s,1))\equiv\lim_{n\to\infty}a_{(n),i}^{\Gamma^{+}}(x_i,(s,1))=1$. This follows directly from the fact that, as shown above, $a_i^{\Gamma}(x_i,s)=1\Rightarrow a_i^{\Gamma^{+}}(x_i,(s,1))=1$. The announcement that θ is such that $r^{\Gamma}(\theta,s)=1$ thus reveals to each investor that, when investors play according to MARP $a^{\Gamma^{+}}$ consistent with the new policy Γ^{+}, default does not occur. Because the payoff from pledging is strictly positive when default does not occur, any investor i receiving information $(s,1)$ under Γ^{+} thus necessarily pledges, no matter x_i. Under the new policy Γ^{+}, all investors thus pledge when they learn that θ is such that $r^{\Gamma}(\theta,s)=1$. That they all refrain from pledging when they learn that θ is such that $r^{\Gamma}(\theta, 0) = 0$ follows from the fact that such an announcement makes it common certainty that $\theta \leq 1$.

We conclude that the new policy Γ^+ satisfies the perfect-coordination property and is such that, for any θ, the probability of default under Γ^+ is the same as under Γ. The result in the theorem then follows by taking $\Gamma^* = \Gamma^+$. Q.E.D.

Proof of Theorem 2. The proof is in 2 steps. Step 1 shows that, when $p(x|\theta)$ is log-supermodular, i.e., it satisfies MLRP, then, irrespective of Γ, MARP is in cut-off strategies. Step 2 then shows that, starting from any Γ satisfying the perfect-coordination property, one can drop any signal other than the predicted regime outcome without changing the investors' behavior.

Step 1. Fix an arbitrary policy $\Gamma = (\mathcal{S}, \pi)$ and, for any pair $(x, s) \in \mathbb{R} \times \mathcal{S}$, let $\Lambda^{\Gamma}(\theta|x, s)$ represent the endogenous posterior beliefs over Θ of each investor receiving exogenous information x and endogenous information s. Let $u(\theta, A) \equiv g(\theta)\mathbf{1}(A > 1-\theta) + b(\theta)\mathbf{1}(A \leq 1-\theta)$ be the payoff differential between pledging and not pledging when the fundamentals are θ and the aggregate size of the pledge is A.

Next, let $U^{\Gamma}(x, s|k) \equiv \int u(\theta, 1 - P(k|\theta))\mathrm{d}\Lambda^{\Gamma}(\theta|x, s)$ denote the expected payoff differential of an investor with information (x, s), when all other investors follow a cut-off strategy with cut-off k (i.e., they pledge if their private signal exceeds k and refrain from pledging if it is below k). The following result establishes that, when the distribution $p(x|\theta)$ from which the signals are drawn satisfies MLRP, no matter Γ, MARP is in cut-off strategies:

Lemma 1. Suppose that $p(x|\theta)$ is log-supermodular. Given any policy $\Gamma = (\mathcal{S}, \pi)$, for any $s \in \mathcal{S}$, there exists $\xi^{\Gamma;s} \in \mathbb{R}$ such that MARP consistent with Γ is given by the strategy profile $a^{\Gamma} \equiv (a_i^{\Gamma})_{i\in[0,1]}$ such that, for any $s \in \mathcal{S}$, $x \in \mathbb{R}$, $i \in [0, 1]$, $a_i^{\Gamma}(x, s) = \mathbf{1}\{x > \xi^{\Gamma;s}\}$ with $\xi^{\Gamma;s} \equiv \sup\{x : U^{\Gamma}(x, s|x) \leq 0\}$ if $\{x : U^{\Gamma}(x, s|x) \leq 0\} \neq \emptyset$, and $\xi^{\Gamma;s} \equiv -\infty$ otherwise. Moreover, the strategy profile a^{Γ} is a BNE of the continuation game that starts with the announcement of the policy Γ.

Proof of Lemma 1. Fix the policy $\Gamma = (S, \pi)$. For any $s \in \mathcal{S}$, let $\xi_{(1)}^{\Gamma;s} \equiv \sup\{x : \lim_{k\to\infty} U^{\Gamma}(x, s|k) \leq 0\}$. Given the public signal s, it is dominant for any investor with private signal x exceeding $\xi_1^{\Gamma;s}$ to pledge. Next, recall that, for any $n \in \mathbb{N}$, $T_{(n)}^{\Gamma}$ denotes the set of strategy profiles that survive the first n rounds of IDISDS and $a_{(n)}^{\Gamma} \equiv \left(a_{(n),i}^{\Gamma}\right)_{i\in[0,1]}$ denotes the most aggressive profile in $T^{\Gamma}_{(n)}$. Observe that the profile $a^{\Gamma}_{(1)}$ is given by $a^{\Gamma}_{(1),i}(x,s) = \mathbf{1}\{x > \xi^{\Gamma;s}_{(1)}\}$ for all $(x,s) \in \mathbb{R} \times \mathcal{S}$, and all $i \in [0,1]$, and minimizes the policy maker's payoff not just in expectation but for any (θ, s). This follows from the fact that, when nobody else pledges, the expected payoff differential $\int u(\theta,0)\mathrm{d}\Lambda^{\Gamma}(\theta|x,s)$ between pledging and not pledging crosses 0 only once and from below at $x = \xi^{\Gamma;s}_{(1)}$. The single-crossing property of $\int u(\theta,0)\mathrm{d}\Lambda^{\Gamma}(\theta|x,s)$ in turn is a consequence of the fact that $u(\theta,0)$ crosses 0 only once from below at $\theta = 1$ along with Property SCB below.

Property SCB. *Suppose that $h : \mathbb{R} \to \mathbb{R}$ crosses 0 only once from below at $\theta = \theta_0$ (that is, $h(\theta) \leq 0$ for all $\theta \leq \theta_0$ and $h(\theta) \geq 0$ for all $\theta > \theta_0$). Let $q : \mathbb{R}^2 \to \mathbb{R}_+$ be a log-supermodular function and suppose that, for any θ, there is an open interval $\varrho_\theta = (\underline{\varrho}_\theta, \bar{\varrho}_\theta) \subset \mathbb{R}$ containing θ such that $q(x,\theta) > 0$ for all $x \in \varrho_\theta$ and $q(x,\theta) = 0$ for (almost) all $x \in \mathbb{R} \setminus \varrho_\theta$, with the bounds $\underline{\varrho}_\theta, \bar{\varrho}_\theta$ non-decreasing in θ. Choose any (Lebesgue) measurable subset $\Omega \subseteq \mathbb{R}$ containing θ_0 and, for any $x \in \mathbb{R}$, let $\Psi(x;\Omega) \equiv \int_\Omega h(\theta)q(x,\theta)d\theta$. Suppose there exists $x^\star \in \varrho_{\theta_0}$ such that* $\Psi(x^\star;\Omega) = 0$. Then, necessarily, $\Psi(x;\Omega) \geq 0$ for all $x \in \varrho_{\theta_0}$ with $x > x^\star$, and $\Psi(x;\Omega) \leq 0$ for all $x \in \varrho_{\theta_0}$ with $x < x^\star$, *with both inequalities strict if (a) $\{\theta \in \Omega : h(\theta) \neq 0\}$ has strict positive Lebesgue measure, (b) q is strictly log-supermodular over $\mathbb{R}^2$.* [41]

Proof of Property SCB. For any $x \in \mathbb{R}$, let $\Omega_x \equiv \{\theta \in \Omega : x \in \varrho_\theta\}$. The monotonicity of ϱ_θ in θ implies that Ω_x is monotone in x in the strong-order sense. Pick any $x' \in \varrho_{\theta_0}$ with $x' > x^\star$. That $x^\star$ and x' belong to ϱ_{θ_0} implies that $\theta_0 \in \Omega_{x^\star} \cap \Omega_{x'}$. Next, observe that

$$\begin{aligned}
\Psi(x';\Omega) &= \int_{\Omega_{x'}} h(\theta)q(x',\theta)\mathrm{d}\theta = \int_{\Omega_{x'}\cap\Omega_{x^\star}} h(\theta)q(x',\theta)\mathrm{d}\theta + \int_{\Omega_{x'}\setminus\Omega_{x^\star}} h(\theta)q(x',\theta)\mathrm{d}\theta \\
&= \int_{\Omega_{x^\star}\cap\Omega_{x'}\cap(-\infty,\theta_0)} h(\theta)q(x^\star,\theta)\frac{q(x',\theta)}{q(x^\star,\theta)}\mathrm{d}\theta + \int_{\Omega_{x^\star}\cap\Omega_{x'}\cap(\theta_0,\infty)} h(\theta)q(x^\star,\theta)\frac{q(x',\theta)}{q(x^\star,\theta)}\mathrm{d}\theta \\
&\quad + \int_{\Omega_{x'}\setminus\Omega_{x^\star}} h(\theta)q(x',\theta)\mathrm{d}\theta \\
&\geq \frac{q(x',\theta_0)}{q(x^\star,\theta_0)}\left(\int_{\Omega_{x^\star}\cap\Omega_{x'}\cap(-\infty,\theta_0)} h(\theta)q(x^\star,\theta)\mathrm{d}\theta + \int_{\Omega_{x^\star}\cap\Omega_{x'}\cap(\theta_0,\infty)} h(\theta)q(x^\star,\theta)\mathrm{d}\theta\right) + \\
&\quad + \int_{\Omega_{x'}\setminus\Omega_{x^\star}} h(\theta)q(x',\theta)\mathrm{d}\theta \\
&\geq \frac{q(x',\theta_0)}{q(x^\star,\theta_0)}\underbrace{\Psi(x^\star;\Omega)}_{=0} + \int_{\Omega_{x'}\setminus\Omega_{x^\star}} h(\theta)q(x',\theta)\mathrm{d}\theta \geq 0.
\end{aligned}$$

[41] That q is strictly log-supermodular over $\mathbb{R}^2$ also implies that $q(x,\theta) > 0$ for all $(x,\theta) \in \mathbb{R}^2$.

The first equality follows from the fact that $q(x', \theta) = 0$ for almost all $\theta \in \Omega \setminus \Omega_{x'}$. The second equality follows from the fact that $\Omega_{x'}$ can be partitioned into $\Omega_{x'} \cap \Omega_{x^\star}$ and $\Omega_{x'} \setminus \Omega_{x^\star}$. The third equality follows from noting that $q(x^\star, \theta) > 0$ for all $\theta \in \Omega_{x^\star}$. The first inequality obtains from the monotonicity of $q(x', \theta)/q(x^\star, \theta)$ over $\Omega_{x^\star} \cap \Omega_{x'}$ as a consequence of q being log-supermodular, along with the fact that $\theta_0 \in \Omega_{x^\star} \cap \Omega_{x'}$ and the assumption that h crosses 0 once from below at $\theta = \theta_0$. The second inequality follows from the fact that, for any $\theta \in (\Omega_{x^\star} \setminus \Omega_{x'}) \cap (-\infty, \theta_0)$, $h(\theta) \leq 0$, along with the fact that $\Omega_{x^\star} \cap (\theta_0, +\infty) = \Omega_{x^\star} \cap \Omega_{x'} \cap (\theta_0, \infty)$, with the last property following from noting that the sets Ω_x are ranked in the strong-order sense. The last inequality follows from the observation that, for any $\theta \in \Omega_{x'} \setminus \Omega_{x^\star}$, $h(\theta) \geq 0$, which in turn is a consequence of (i) the monotonicity of the sets Ω_x in x, (ii) the assumption that h crosses 0 only once from below at $\theta = \theta_0$, and (iii) the assumption that $\theta_0 \in \Omega_{x^\star} \cap \Omega_{x'}$.

Similar arguments imply that, for $x < x^\star$, $\Psi(x; \Omega) \leq 0$. The same arguments also imply that, when (a) $\{\theta \in \Omega : h(\theta) \neq 0\}$ has strict positive Lebesgue measure and (b) q is strictly log-supermodular over $\mathbb{R}^2$, then $\Psi(x; \Omega) < 0$ for all $x < x^\star$ and $\Psi(x; \Omega) > 0$ for all $x > x^\star$. This completes the proof of Property SCB. □

The facts that (a) the continuation game is supermodular, (b) the density $p(x|\theta)$ is log-supermodular, and (c) when investors follow monotone strategies, the regime outcome is monotone in θ imply that, for any $s \in \mathcal{S}$, there exists a unique sequence $\left(\xi_{(n)}^{\Gamma;s}\right)_{n \in \mathbb{N}}$ such that, for any $n \geq 1$, $a_{(n)}^{\Gamma}$ is such that $a_{(n),i}^{\Gamma}(x, s) = \mathbf{1}\{x > \xi_{(n)}^{\Gamma;s}\}$ for all i and all $(x, s) \in \mathbb{R} \times \mathcal{S}$, with each $\xi_{(1)}^{\Gamma;s}$ as defined above, and with all other cut-offs $\xi_{(n)}^{\Gamma;s}$, $n > 1$, $s \in \mathcal{S}$, defined inductively by $\xi_{(n)}^{\Gamma;s} \equiv \sup\{x : U^{\Gamma}(x, s|\xi_{(n-1)}^{\Gamma;s}) \leq 0\}$.

Let $T^{\Gamma} \equiv \cap_{n=1}^{\infty} T_n^{\Gamma}$ denote the set of strategy profiles that are *rationalizable* for investors under Γ. The most aggressive strategy profile in T^{Γ} is then given by $a_i^{\Gamma}(x, s) \equiv \mathbf{1}\{x > \xi^{\Gamma;s}\}$ for all i and all $(x, s) \in \mathbb{R} \times \mathcal{S}$, where, for any $s \in \mathcal{S}$, $\xi^{\Gamma;s} \equiv \lim_{n \to \infty} \xi_{(n)}^{\Gamma;s}$. The sequence $(\xi_{(n)}^{\Gamma;s})_n$ is monotone and its limit is given by $\xi^{\Gamma;s} = \sup\{x : U^{\Gamma}(x, s|x) \leq 0\}$ if $\{x : U^{\Gamma}(x, s|x) \leq 0\} \neq \emptyset$, and $\xi^{\Gamma;s} \equiv -\infty$ otherwise. This establishes the first part of the lemma. That the profile a^{Γ} is a BNE for the continuation game that starts with the announcement of the policy Γ follows from the fact that, given any $s \in \mathcal{S}$, when all investors follow a cut-off strategy with cutoff $\xi^{\Gamma;s}$, the best response for each investor $i \in [0, 1]$ is to pledge for $x_i > \xi^{\Gamma;s}$ and to refrain from pledging for $x_i < \xi^{\Gamma;s}$. This completes the proof of the lemma. ■

Step 2. Now take any policy $\Gamma = (\mathcal{S}, \pi)$ satisfying the perfect-coordination property. Given the result in Theorem 1, without loss of generality, assume that $\Gamma = (\mathcal{S}, \pi)$ is such that $\mathcal{S} = \{0,1\} \times \hat{S}$, for some measurable set $\hat{S}$, and is such that (a) when the policy discloses any signal $s = (\hat{s}, 1)$, all investors pledge and default does not happen, whereas (b) when the policy discloses any signal $s = (\hat{s}, 0)$, all investors refrain from pledging and default happens.

Equipped with the result in Lemma 1, we show that, starting from $\Gamma = (\mathcal{S}, \pi)$, one can construct a binary policy $\Gamma^* = (\{0,1\}, \pi^*)$ also satisfying the perfect-coordination property and such that the probability of default under Γ^* is the same as under Γ. The policy $\Gamma^* = (\{0,1\}, \pi^*)$ is such that, for any θ, $\pi^*(1|\theta) = \int_{\hat{\mathcal{S}}} \pi\left(\mathrm{d}\left(\hat{s},1\right)|\theta\right)$. That is, for each θ, the binary policy Γ^* recommends to pledge with the same total probability as the original policy Γ discloses signals leading all investors to pledge.[42]

We now show that, under Γ^*, when the policy announces that $s = 1$, the unique rationalizable action for each investor is to pledge. To see this, for any $(x, 1)$ that are mutually consistent given Γ^*, let $U^{\Gamma^*}(x, 1|k)$ denote the expected payoff differential for any investor with private signal x, when the policy Γ^* announces $s = 1$, and all other investors follow a cut-off strategy with cut-off k.[43] From the law of iterated expectations, we have that

$$U^{\Gamma^*}(x,1|k) = \int_{\hat{\mathcal{S}}} U^{\Gamma}(x,(\hat{s},1)|k)\varsigma^{\Gamma}(\mathrm{d}\hat{s}|x,1) \tag{13}$$

where $\varsigma^{\Gamma}(\cdot|x,1)$ is the probability measure over $\hat{S}$ obtained by conditioning on the event $(x, 1)$, under Γ. For any signal $s = (\hat{s}, 1)$ in the range of π, MARP consistent with Γ is such that $a_i^{\Gamma}(x, (\hat{s}, 1)) = 1$ all $x \in \mathbb{R}$, meaning that pledging is the unique rationalizable action after Γ announces $s = (\hat{s}, 1)$. Lemma 1 in turn implies that, for all $s = (\hat{s}, 1)$ in the range of π, $\hat{s} \in \hat{S}$, all $k \in \mathbb{R}$, $U^{\Gamma}(k, (\hat{s}, 1)|k) > 0$. From (13), we then have that, for all all $k \in \mathbb{R}$, $U^{\Gamma^*}(k, 1|k) > 0$. In turn, this implies that, given the new policy Γ^*, when $s = 1$ is disclosed, under the unique rationalizable profile, all investors pledge, that is, $a_i^{\Gamma^*}(x, 1) = 1$ all x, all $i \in [0, 1]$. It is also easy to see that, when the policy Γ^* discloses the signal $s = 0$, it becomes common certainty among the investors that $\theta \leq 1$. Hence, under MARP consistent with Γ^*, after $s = 0$ is disclosed, all investors refrain from pledging, irrespective of their private signals. The new

[42] $\int_{\hat{\mathcal{S}}} \pi\left(\mathrm{d}\left(\hat{s},1\right)|\theta\right)$ represents the total probability that the measure $\pi(\theta)$ assigns to signal $(\hat{s}, 1)$.
[43] Recall that $(x, 1)$ are mutually consistent under Γ^* if $p^{\Gamma^*}(x,1) \equiv \int p(x|\theta)\pi^*(1|\theta)\mathrm{d}F(\theta) > 0$.

pass/fail policy Γ^* so constructed thus (a) satisfies the perfect-coordination property, and (b) is such that, for any θ, the probability of default under Γ^* is the same as under Γ. Q.E.D.

Proof of Theorem 3. Without loss of generality, assume that the policy $\Gamma = (\mathcal{S}, \pi)$ (a) is a (possibly stochastic) "pass/fail"policy (i.e., $\mathcal{S} = \{0, 1\}$, with $\pi(1|\theta) = 1 - \pi(0|\theta)$ denoting the probability that signal $s = 1$ is disclosed when the fundamentals are θ), (b) is such that $\pi(1|\theta) = 0$ for all $\theta \leq 0$ and $\pi(1|\theta) = 1$ for all $\theta > 1$, and (c) satisfies the perfect-coordination property (PCP). Theorems 1 and 2 imply that, if Γ does not satisfy these properties, there exists another policy Γ' that satisfies these properties and yields the policy maker a payoff weakly higher than Γ. The proof then follows from applying the arguments below to Γ' instead of Γ.

Suppose that Γ is such that there exists no $\hat{\theta}$ such that $\pi(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi(1|\theta) = 1$ for F-almost all $\theta > \hat{\theta}$.[44] We establish the result by showing that there exists a deterministic monotone policy $\Gamma^{\hat{\theta}} = (\{0, 1\}, \pi^{\hat{\theta}})$ satisfying PCP that yields the policy maker a payoff strictly higher than Γ.

For the policy Γ to satisfy PCP, it must be that, when the policy discloses the signal $s = 1$, $U^{\Gamma}(x, 1|x) > 0$ for all x such that $(x, 1)$ are mutually consistent, where $U^{\Gamma}(x, 1|x)$ is the expected payoff of an investor with signal x who hears that $s = 1$ and who expects all other investors to follow a cut-off policy with cut-off x.

Let $\mathbb{G}$ denote the set of policies $\Gamma' = (\mathcal{S}, \pi')$ that, in addition to properties (a) and (b) above, are such that $U^{\Gamma'}(x, 1|x) \geq 0$ for all x such that $(x, 1)$ are mutually consistent. [45] For any $\Gamma \in \mathbb{G}$, let $\mathcal{U}^P[\Gamma]$ denote the policy maker's ex-ante expected payoff when, under Γ, investors pledge after hearing that $s = 1$ and refrain from pledging after hearing that $s = 0$. Denote by $\arg\max_{\tilde{\Gamma} \in \mathbb{G}}\{\mathcal{U}^P[\tilde{\Gamma}]\}$ the set of policies that maximize the policy maker's payoff over $\mathbb{G}$.[46]

Step 1 below shows that any $\Gamma \in \arg\max_{\tilde{\Gamma} \in \mathbb{G}}\{\mathcal{U}^P[\tilde{\Gamma}]\}$ is such that $\pi(1|\theta) = 0$ for F-almost all $\theta \leq \theta^*$ and $\pi(1|\theta) = 1$ for F-almost all $\theta > \theta^*$, with θ^* as defined in (5). Step 2 then shows that the policy maker's payoff under the optimal deterministic monotone policy

[44] If this not the case, then the deterministic monotone policy $\Gamma^{\hat{\theta}} = (\{0, 1\}, \pi^{\hat{\theta}})$ with cut-off $\hat{\theta}$ also satisfies PCP and yields the policy maker the same payoff as Γ, in which case the result trivially holds.

[45] As explained in the main text, some policies Γ' in $\mathbb{G}$ need not satisfy PCP, namely those for which there exists x, with $(x, 1)$ mutually consistent, such that $U^{\Gamma'}(x, 1|x) = 0$.

[46] That $\arg\max_{\tilde{\Gamma} \in \mathbb{G}}\{\mathcal{U}^P[\tilde{\Gamma}]\} \neq \emptyset$ follows from the compactness of $\mathbb{G}$ and the upper hemi-continuity of $\mathcal{U}^P$.

$\Gamma^{\theta^*} = (\{0,1\}, \pi^{\theta^*})$ with cut-off θ^* can be approximated arbitrarily well by a deterministic monotone policy $\Gamma^{\hat{\theta}} = (\{0,1\}, \pi^{\hat{\theta}}) \in \mathbb{G}$ that satisfies PCP, thus establishing the theorem.

Step 1. Given any policy Γ, let

$$X^{\Gamma} \equiv \{x : (x,1)\ \Gamma\text{-mutually consistent and } U^{\Gamma}(x,1|x) = 0\}.$$

Take any policy $\Gamma' \in \mathbb{G}$ for which there exists no $\hat{\theta}$ such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \hat{\theta}$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \hat{\theta}$. Clearly, if $X^{\Gamma'} = \emptyset$, there exists another policy $\Gamma'' \in \mathbb{G}$ that yields the policy maker a payoff strictly higher than Γ'.[47] Thus, assume that $X^{\Gamma'} \neq \emptyset$, and let $\bar{x} \equiv \sup X^{\Gamma'}$. Claim A below shows that the set $\{\theta \in \Theta(\bar{x}) : \pi'(1|\theta) < 1\}$ has strict positive F-measure. Claim B shows that, given any $\Gamma' \in \mathbb{G}$ for which the posterior beliefs of the marginal investor with signal $\bar{x}$ differ from those obtained by Bayes rule conditioning on the event that fundamentals are above some threshold $\hat{\theta}$, there exists another policy $\Gamma'' \in \mathbb{G}$ that yields the policy maker a payoff strictly higher than Γ'. Finally, Claim C shows that, under the properties in Condition M, the only policies $\Gamma' \in \mathbb{G}$ that generate posterior beliefs for the marginal investors with signal $\bar{x}$ equal to those obtained from Bayes rule by conditioning on the event that fundamentals are above some threshold $\hat{\theta}$ are such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \theta^*$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \theta^*$. Jointly, the three claims thus establish the result that any policy $\Gamma \in \arg\max_{\tilde{\Gamma} \in \mathbb{G}} \{\mathcal{U}^P[\tilde{\Gamma}]\}$, is such that $\pi(1|\theta) = 0$ for F-almost all $\theta \leq \theta^*$ and $\pi(1|\theta) = 1$ for F-almost all $\theta > \theta^*$.

Given any x, let $\theta_0(x)$ be the fundamental threshold below which the investors' expected payoff differential is negative and above which it is positive, when all investors follow a cut-off strategy with cut-off x.[48] For any policy $\Gamma = (\{0,1\}, \pi) \in \mathbb{G}$, let $p^{\Gamma}(x,1) \equiv \int_{-\infty}^{+\infty} \pi(1|\theta) p(x|\theta) \mathrm{d}F(\theta)$ denote the joint probability density of the exogenous signal x and the endogenous signal $s = 1$.

Claim A. *For any $\Gamma' = (\{0,1\}, \pi') \in \mathbb{G}$ such that $X^{\Gamma'} \neq \emptyset$, $\{\theta \in \Theta(\bar{x}) : \pi'(1|\theta) < 1\}$ has strict positive F-measure.*

[47] In fact, because there exists no such a $\hat{\theta}$, there must exists a set $(\theta', \theta'') \subseteq [0,1]$ of F-positive measure over which $\pi'(1|\theta) < 1$. The policy Γ'' can then be obtained from Γ' by increasing $\pi'(1|\theta)$ over such a set. Provided the increase is small, $\Gamma'' \in \mathbb{G}$. Because $U^P(\theta,1) > U^P(\theta,0)$ over $[0,1]$, the policy maker's payoff under Γ'' is strictly higher than under Γ'.

[48] When the default outcome is a function of A and θ only, as in the baseline model of Section 2 in the main text, $\theta_0(x)$ coincides with the threshold below which default occurs and above which it does not occur, when agents follow a cut-off strategy with cut-off x.

Proof of Claim A. Suppose, by contradiction, that $\pi'(1|\theta) = 1$ for F-almost all $\theta \in \Theta(\bar{x})$. Property 1 in Condition M then implies that $\bar{x} > \bar{x}_G$, where $\bar{x}_G$ is defined as in (2).In fact, if this was not the case, the monotonicity of $\Theta(\cdot)$ would imply that $\inf \Theta(\bar{x}) \leq \inf \Theta(\bar{x}_G) < 0$. That $\pi'(1|\theta) = 1$ for F-almost all $\theta \in \Theta(\bar{x})$ would then imply that $\pi'(1|\theta) = 1$ for a set of fundamentals $\theta < 0$ of strict positive F-measure, which is inconsistent with the assumption that $\Gamma' \in \mathbb{G}$. Thus, necessarily, $\bar{x} > \bar{x}_G$.

Now suppose that $\inf \Theta(\bar{x}) \geq 0$. That $\pi'(1|\theta) = 1$ for F-almost all $\theta \in \Theta(\bar{x})$ means that, from the perspective of an investor with signal $\bar{x}$, the information conveyed by the announcement that $s = 1$ under Γ' is the same as under the monotone deterministic policy $\Gamma^0 = (\{0,1\}, \pi^0)$ with cut-off $\hat{\theta} = 0$. As a result, $U^{\Gamma'}(\bar{x}, 1|\bar{x})p^{\Gamma'}(x, 1) = U^{\Gamma^0}(\bar{x}, 1|\bar{x})p^{\Gamma^0}(x, 1)$. Because $\bar{x} > \bar{x}_G$, and because, by definition of $\bar{x}_G$, $U^{\Gamma^0}(x, 1|x) > 0$ for all $x > \bar{x}_G$, it must be that $U^{\Gamma'}(\bar{x}, 1|\bar{x}) > 0$, which contradicts the assumption that $\bar{x} \in X^{\Gamma'}$. Hence, it must be that $\inf \Theta(\bar{x}) < 0$. As explained above, however, this is inconsistent with the assumption that $\Gamma' \in \mathbb{G}$. $\square$

Next, for any $\Gamma' = (\{0,1\}, \pi') \in \mathbb{G}$, let

$$\theta_H \equiv \sup \{\theta \in \Theta : \exists \delta > 0 \, \text{s.t.} \, \pi'(1|\theta') < 1 \, \text{for} \, F\text{-almost all} \, \theta' \in [\theta - \delta, \theta)\}.$$

The result in Claim A above implies that θ_H is such that $\theta_H > \inf \Theta(\bar{x})$.

Claim B. *Take any* $\Gamma' = (\{0,1\}, \pi') \in \mathbb{G}$ *such that* $X^{\Gamma'} \neq \emptyset$. *Suppose that*

$$\{\theta \in (\inf \Theta(\bar{x}), \theta_H) : \pi'(1|\theta) > 0\} \text{ has strict positive } F\text{-measure.} \tag{14}$$

Then, there exists another policy $\Gamma'' \in \mathbb{G}$ *that yields the policy maker a strictly higher payoff.*

Proof of Claim B. The proof below distinguishes two cases.

Case 1: $\inf \Theta(\bar{x}) < \theta_0(\bar{x}) < \theta_H$. Consider the policy $\Gamma^{\epsilon,\delta} = (\{0,1\}, \pi^{\epsilon,\delta})$ defined by $\pi^{\epsilon,\delta}(1|\theta) = \pi'(1|\theta)$ for all $\theta \leq \theta_0(\bar{x} + \delta)$, with $\delta > 0$ small so that $\theta_0(\bar{x} + \delta) < \theta_H$, and $\pi^{\epsilon,\delta}(1|\theta) = \min\{\pi'(1|\theta) + \epsilon, 1\})$ for all $\theta > \theta_0(\bar{x} + \delta)$, with $\epsilon > 0$ also small. To see that, when ϵ and δ are small, $\Gamma^{\epsilon,\delta} \in \mathbb{G}$, note that, by definition of $\theta_0(\cdot)$, for any x, and any $\theta > \theta_0(x)$, $u(\theta, 1 - P(x|\theta)) > 0$. This property, together with the monotonicity of $\theta_0(\cdot)$, jointly imply that, for any $x \leq \bar{x} + \delta$,

$$\int_{-\infty}^{\infty} u(\theta, 1 - P(x|\theta)) \left[\pi'(1|\theta)\mathbf{1}\left\{\theta \leq \theta_0\left(\bar{x} + \delta\right)\right\} + \min\{\pi'(1|\theta) + \epsilon, 1\}\mathbf{1}\left\{\theta > \theta_0\left(\bar{x} + \delta\right)\right\}\right] p(x|\theta)\mathrm{d}F(\theta)$$
$$\geq \int_{-\infty}^{\infty} u(\theta, 1 - P(x|\theta))\pi'(1|\theta)p(x|\theta)\mathrm{d}F(\theta). \tag{15}$$

The inequality follows from the fact that, when $x \leq \bar{x} + \delta$, $u\left(\theta, 1 - P\left(x|\theta\right)\right) > 0$ for any $\theta > \theta_0\left(\bar{x} + \delta\right)$. Because $\Gamma' \in \mathbb{G}$, the right-hand side of (15) is non-negative.[49] Hence, for any $x \leq \bar{x} + \delta$ such that $(x, 1)$ are mutually consistent under $\Gamma^{\epsilon,\delta}$, because the left-hand side of (15) is equal to $U^{\Gamma^{\epsilon,\delta}}(x, 1|x)p^{\Gamma^{\epsilon,\delta}}(x, 1)$ and because, for such x, $p^{\Gamma^{\epsilon,\delta}}(x, 1) > 0$, we have that $U^{\Gamma^{\epsilon,\delta}}(x, 1|x) \geq 0$. That $U^{\Gamma^{\epsilon,\delta}}(x, 1|x) \geq 0$ also for all $x > \bar{x} + \delta$ such that $(x, 1)$ are mutually consistent under $\Gamma^{\epsilon,\delta}$ follows from the fact that, by definition of $\bar{x}$, for any $x \geq \bar{x} + \delta$, the function $J(x) \equiv \int_{-\infty}^{+\infty} u(\theta, 1 - P(x|\theta))\pi'(1|\theta)p(x|\theta)\mathrm{d}F(\theta)$ is bounded away from 0, along with the fact that, for any $\delta > 0$, the function family $\left(J^{\epsilon,\delta}(\cdot)\right)_\epsilon$ whose elements $J^{\epsilon,\delta}(\cdot)$ are given by $J^{\epsilon,\delta}(x) \equiv \int_{-\infty}^{+\infty} u(\theta, 1 - P(x|\theta))\pi^{\epsilon,\delta}(1|\theta)p(x|\theta)\mathrm{d}F(\theta)$, is continuous in ϵ in the sup-norm in a neighborhood of 0.[50] Because the new policy $\Gamma^{\epsilon,\delta} \in \mathbb{G}$ is such that $\pi^{\epsilon,\delta}(1|\theta) \geq \pi'(1|\theta)$ for all θ, with the inequality strict over a set of fundamentals $\theta < 1$ of F-positive measure, the policy maker's payoff under $\Gamma^{\epsilon,\delta}$ is strictly higher than under Γ', as claimed.

Case 2: $\inf \Theta\left(\bar{x}\right) < \theta_H \leq \theta_0\left(\bar{x}\right)$. Consider the monotone deterministic policy $\Gamma^0 = \left\{\{0, 1\}, \pi^0\right\}$ with cut-off $\hat{\theta} = 0$. Then, for any $x \geq \bar{x}$,

$$\int_{0}^{+\infty} u(\theta, 1 - P(x|\theta))\pi'(1|\theta)p(x|\theta)\mathrm{d}F(\theta) \geq \int_{0}^{+\infty} u(\theta, 1 - P(x|\theta))p(x|\theta)\mathrm{d}F(\theta),$$

where the inequality follows from (i) the fact that, for any $x \geq \bar{x}$ and any $\theta \leq \theta_0\left(\bar{x}\right)$, $u\left(\theta, 1 - P\left(x|\theta\right)\right) < 0$, along with (ii) the fact that $\pi'(1|\theta) = 1$ for F-almost all $\theta \geq \theta_0(x) \geq \theta_0\left(\bar{x}\right) \geq \theta_H$. Furthermore, when $x = \bar{x}$, the above inequality is strict and, because $p^{\Gamma^0}\left(\bar{x}, 1\right) > p^{\Gamma'}\left(\bar{x}, 1\right) > 0$, it implies that $U^{\Gamma^0}(\bar{x}, 1|\bar{x}) < U^{\Gamma'}(\bar{x}, 1|\bar{x}) = 0$. By continuity of $U^{\Gamma^0}(x, 1|x)$ in x, we thus have that $\bar{x} < \bar{x}_G$. This property in turn permits us to apply Property (2”) of Condition M to $\bar{x}$ in the arguments below.

Next, let $\theta_L \equiv \inf\{\theta \in \Theta : \exists\delta > 0\,\text{s.t.}\,\pi'(1|\theta') > 0,\ F\text{-almost all}\,\theta' \in [\theta, \theta + \delta)\}$. That $\theta_L <$

[49] Either $(x, 1)$ are not mutually consistent under Γ', in which case the right-hand side of (15) is zero, or they are mutually consistent, in which case the right-hand side of (15) is equal to $U^{\Gamma'}(x, 1|x)p^{\Gamma'}(x, 1)$, which is non-negative because $p^{\Gamma'}(x, 1) > 0$ and $U^{\Gamma'}(x, 1|x) \geq 0$.

[50] That is, $\forall z > 0$, $\exists\Delta > 0$ so that $\forall\ 0 < \epsilon < \Delta$, $|J^{\epsilon,\delta}(x) - J(x)| \leq z$, $\forall x \geq \bar{x} + \delta$.

θ_H follows from the assumption that $\{\theta \in (\inf \Theta(\bar{x}), \theta_H) : \pi'(1|\theta) > 0\}$ has strict positive F-measure. Furthermore, $u(\theta_L, 1 - P(\bar{x}|\theta_L)) < 0$.[51] Also observe that $\inf \Theta(\bar{x}) < \theta_L$. This follows from the fact that, as shown above, $\bar{x} < \bar{x}_G$, which, together with Property 1 in Condition M, implies that $\inf \Theta(\bar{x}) < 0$. Because $\theta_L \geq 0$, we thus have that $\inf \Theta(\bar{x}) < \theta_L$.

For any $\gamma > 0$, let $\theta_L^\gamma \equiv \theta_L + \gamma$ and $\theta_H^\gamma \equiv \theta_H - \gamma$. Pick $\gamma, e_L, e_H > 0$ small such that (i) $\pi'(1|\theta_L^\gamma) > 0$ and $\pi'(1|\theta) > 0$ for $F-$almost $\theta \in (\theta_L^\gamma, \theta_L^\gamma + e_L)$, (ii) $\pi'(1|\theta_H^\gamma) < 1$ and $\pi'(1|\theta) < 1$ for $F-$almost all $\theta \in (\theta_H^\gamma - e_H, \theta_H^\gamma)$, and (iii) $\theta_L^\gamma + e_L < \theta_H^\gamma - e_H$.[52] Next, pick $\eta \in (0, \bar{x}_G - \bar{x})$ small such that $U^{\Gamma'}(x, 1|x) > \eta$ for all $x \geq \bar{x} + \eta$. Pick $\epsilon > 0$ also small and let $\delta(\epsilon, \eta)$ be implicitly defined by

$$\begin{aligned} &\textstyle\int_{\theta_L^\gamma}^{\theta_L^\gamma+\epsilon} u(\theta, 1 - P(\bar{x}+\eta|\theta))\pi'(1|\theta)p(\bar{x}+\eta|\theta)\mathrm{d}F(\theta) = \\ &\textstyle\int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma} u(\theta, 1 - P(\bar{x}+\eta|\theta))(1-\pi'(1|\theta))p(\bar{x}+\eta|\theta)\mathrm{d}F(\theta). \end{aligned} \tag{16}$$

For $\epsilon > 0$ small, $\theta_L^\gamma + \epsilon < \theta_H^\gamma - \delta(\varepsilon, \eta)$. Consider the policy $\Gamma^{\epsilon,\gamma,\eta} = (\{0,1\}, \pi^{\epsilon,\gamma,\eta})$ defined by the following properties: (a) $\pi^{\epsilon,\gamma,\eta}(1|\theta) = \pi'(1|\theta)$ for all $\theta \notin \{[\theta_L^\gamma, \theta_L^\gamma + \epsilon] \cup [\theta_H^\gamma - \delta(\epsilon,\eta), \theta_H^\gamma]\}$; (b) $\pi^{\epsilon,\gamma,\eta}(1|\theta) = 0$ for all $\theta \in [\theta_L^\gamma, \theta_L^\gamma + \epsilon]$; and (c) $\pi^{\epsilon,\gamma,\eta}(1|\theta) = 1$ for all $\theta \in [\theta_H^\gamma - \delta(\epsilon,\eta), \theta_H^\gamma]$. Note that Condition (16) implies that

$$U^{\Gamma^{\epsilon,\gamma,\eta}}(\bar{x}+\eta, 1|\bar{x}+\eta)p^{\Gamma^{\epsilon,\gamma,\eta}}(\bar{x}+\eta, 1) = U^{\Gamma'}(\bar{x}+\eta, 1|\bar{x}+\eta)p^{\Gamma'}(\bar{x}+\eta, 1) > 0.$$

We now show that $U^{\Gamma^{\epsilon,\gamma,\eta}}(x, 1|x) \geq 0$ for any x with $(x, 1)$ mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$. Clearly, for any (ϵ, γ, η), and any $x \leq x^*(\theta_L)$ such that $(x, 1)$ are mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$ (alternatively, under Γ') $U^{\Gamma^{\epsilon,\gamma,\eta}}(x, 1|x) > 0$ (alternatively, $U^{\Gamma'}(x, 1|x) > 0$). This is because, for any such x, $\theta_0(x) < \theta_L$ implying that $u(\theta, 1 - P(x|\theta)) > 0$ for all $\theta > \theta_L$. The result then follows from the fact that, under both Γ' and $\Gamma^{\epsilon,\gamma,\eta}$, $\int_{-\infty}^{\theta_L} \pi^{\epsilon,\gamma,\eta}(1|\theta)\,\mathrm{d}F(\theta) = \int_{-\infty}^{\theta_L} \pi'(1|\theta)\,\mathrm{d}F(\theta) = 0$, meaning that all agents assign probability one to the event that $\theta \geq \theta_L$.

Furthermore, the continuity of $\int_{\theta_L}^{+\infty} u(\theta, 1 - P(x|\theta))\,p(x|\theta)\,\pi'(1|\theta)\,\mathrm{d}F(\theta)$ in x, along with the fact that $\int_{\theta_L}^{+\infty} u(\theta, 1 - P(x|\theta))\,p(x|\theta)\,\pi'(1|\theta)\,\mathrm{d}F(\theta) > \eta$ for all $x \geq \bar{x}+\eta$ with $(x, 1)$ mutually consistent under Γ', imply that there exists $\xi > 0$ so that, for any $x \in [x^*(\theta_L), x^*(\theta_L) + \xi] \cup [\bar{x}+\eta, +\infty)$, if $(x, 1)$ are mutually consistent under Γ', then $U^{\Gamma'}(x, 1|x)p^{\Gamma'}(x, 1) > \xi$.

[51] That $u(\theta_L, 1 - P(\bar{x}|\theta_L)) < 0$ follows from the fact that, by definition of $\bar{x}$ and θ_L, $\int_{\theta_L}^{+\infty} u(\theta, 1 - P(\bar{x}|\theta))\pi'(1|\theta)p(\bar{x}|\theta)dF(\theta) = 0$, together with the single-crossing property of $u(\theta, 1 - P(\bar{x}|\theta))$ in θ.

[52] If a single γ satisfying properties (i)-(iii) does not exist, let $\gamma = (\gamma_L, \gamma_H) \in \mathbb{R}^2_{++}$ satisfying properties (i)-(iii). The arguments below then apply verbatim by letting $\theta_L^\gamma = \theta_L + \gamma_L$ and $\theta_H^\gamma = \theta_H + \gamma_H$.

Let $S^{\Gamma^{\epsilon,\gamma,\eta}}(x) \equiv \int_{\theta_L}^{+\infty} u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right) \mathrm{d}F(\theta)$. For any η, the function family $\left(S^{\Gamma^{\epsilon,\gamma,\eta}}(\cdot)\right)_{\epsilon,\gamma}$ is continuous in (γ,ϵ) in the sup-norm, in a neighborhood of $(0,0)$[53] and $x^*\left(\theta\right)$ is continuous in θ. Hence, there exist $\bar{\gamma}, \bar{\epsilon} > 0$ such that, when $\gamma \le \bar{\gamma}$ and $\epsilon \le \bar{\epsilon}$, for any $x \notin (x^*(\theta_L^\gamma + \epsilon), \bar{x}+\eta)$ such that $(x,1)$ are mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$, $U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x) \ge 0$.

Next observe that, for any $x \in (x^*(\theta_L^\gamma + \epsilon), x^*\left(\theta_H^\gamma - \delta\left(\epsilon,\eta\right)\right)]$,

$$\begin{gathered} -\int_{\theta_L^\gamma}^{\theta_L^\gamma+\epsilon} u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) \pi'\left(1|\theta\right) \mathrm{d}F\left(\theta\right) \\ +\int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma} u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) \left(1-\pi'\left(1|\theta\right)\right) \mathrm{d}F\left(\theta\right) \ge 0, \end{gathered}$$

where the inequality follows from the fact that the integrand in the first integral is non-positive, whereas the integrand in the second integral is non-negative. Hence, for any such x, if $(x,1)$ are mutually consistent under Γ', meaning that $p^{\Gamma'}\left(x,1\right) = \int_{\theta_L}^{+\infty} p\left(x|\theta\right) \pi'\left(1|\theta\right) \mathrm{d}F\left(\theta\right) > 0$, and are also mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$, meaning that $p^{\Gamma^{\epsilon,\gamma,\eta}}\left(x,1\right) = \int_{\theta_L}^{+\infty} p\left(x|\theta\right) \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right) \mathrm{d}F\left(\theta\right) > 0$, it must be that $U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x) \ge 0$. Indeed, for any such x,

$$\begin{gathered} U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x) p^{\Gamma^{\epsilon,\gamma,\eta}}\left(x,1\right) = U^{\Gamma'}(x,1|x) p^{\Gamma'}\left(x,1\right) - \int_{\theta_L^\gamma}^{\theta_L^\gamma+\epsilon} u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) \pi'\left(1|\theta\right) \mathrm{d}F\left(\theta\right) \\ +\int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma} u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) \left(1-\pi'\left(1|\theta\right)\right) \mathrm{d}F\left(\theta\right) \end{gathered}$$

and $U^{\Gamma'}(x,1|x) p^{\Gamma'}\left(x,1\right) = \int_{\theta_L}^{+\infty} u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) \pi'\left(1|\theta\right) \mathrm{d}F\left(\theta\right) \ge 0$. If, instead, for any such x, $(x,1)$ are mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$ but not under Γ', then

$$\begin{aligned} U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x) p^{\Gamma^{\epsilon,\gamma,\eta}}\left(x,1\right) &= \int_{\theta_L}^{+\infty} u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right) \mathrm{d}F\left(\theta\right) \\ = \int_{\theta_L}^{+\infty} u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) \pi'\left(1|\theta\right) \mathrm{d}F\left(\theta\right) &- \int_{\theta_L^\gamma}^{\theta_L^\gamma+\epsilon} u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) \pi'\left(1|\theta\right) \mathrm{d}F\left(\theta\right) \\ &+\int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma} u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) \left(1-\pi'\left(1|\theta\right)\right) \mathrm{d}F\left(\theta\right) \\ &= \int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma} u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right) \left(1-\pi'\left(1|\theta\right)\right) \mathrm{d}F\left(\theta\right) \ge 0, \end{aligned}$$

where the first equality follows from the fact $p^{\Gamma^{\epsilon,\gamma,\eta}}\left(x,1\right) > 0$ and the definition of $U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x)$, the second equality is by construction, the third equality follows from the fact that $p^{\Gamma'}\left(x,1\right) = 0$, and the last inequality follows from the fact that, when $x \in (x^*(\theta_L^\gamma + \epsilon), x^*\left(\theta_H^\gamma - \delta\left(\epsilon,\eta\right)\right)]$, the integrand is non-negative. We thus conclude that, for any such x, $U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x) \ge 0$.

Next, consider $x \in \left(x^*\left(\theta_H^\gamma - \delta\left(\epsilon,\eta\right)\right), x^*(\theta_H^\gamma)\right)$. For any (x,θ), let

$$\Delta S(x) \equiv \int_{\theta_L}^{+\infty} u(\tilde{\theta}, 1-P(x|\tilde{\theta})) p(x|\tilde{\theta}) (\pi^{\epsilon,\gamma,\eta}(1|\tilde{\theta}) - \pi'(1|\tilde{\theta})) \mathrm{d}F(\tilde{\theta})$$

[53] This means that, for any $z>0$, there exists $\Delta > 0$ such that, for any (ϵ,γ) with $0<\epsilon<\Delta$ and $0<\gamma<\Delta$, and all x, $|S^{\Gamma^{\epsilon,\gamma,\eta}}(x) - S^{\Gamma^{0,0,\eta}}(x)| \le z$, where $\Gamma^{0,0,\eta} = \Gamma'$.

and $q(\theta,x) \equiv |u(\theta,1-P(x|\theta))|\, p(x|\theta)$. Note that, for any $x \in (x^*(\theta_H^\gamma - \delta(\epsilon,\eta)), x^*(\theta_H^\gamma))$,

$$\begin{aligned}
\Delta S(x) &= \int_{\theta_L^\gamma}^{\theta_H^\gamma-\delta(\epsilon,\eta)} -u(\theta,1-P(x|\theta))\, p(x|\theta)\left(\pi'(1|\theta)-\pi^{\epsilon,\gamma,\eta}(1|\theta)\right)\mathrm{d}F(\theta) \\
&\quad + \int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_0(x)} -u(\theta,1-P(x|\theta))\, p(x|\theta)\left(\pi'(1|\theta)-\pi^{\epsilon,\gamma,\eta}(1|\theta)\right)\mathrm{d}F(\theta) \\
&\quad + \int_{\theta_0(x)}^{\theta_H^\gamma} -u(\theta,1-P(x|\theta))\, p(x|\theta)\left(\pi'(1|\theta)-\pi^{\epsilon,\gamma,\eta}(1|\theta)\right)\mathrm{d}F(\theta) \\
&\geq \int_{\theta_L^\gamma}^{\theta_H^\gamma-\delta(\epsilon,\eta)} \frac{q(\theta,x)}{q(\theta,\bar{x}+\eta)} q(\theta,\bar{x}+\eta)\left(\pi'(1|\theta)-\pi^{\epsilon,\gamma,\eta}(1|\theta)\right)\mathrm{d}F(\theta) \\
&\quad + \int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_0(x)} \frac{q(\theta,x)}{q(\theta,\bar{x}+\eta)} q(\theta,\bar{x}+\eta)\left(\pi'(1|\theta)-\pi^{\epsilon,\gamma,\eta}(1|\theta)\right)\mathrm{d}F(\theta) \\
&\quad + \frac{q(\theta_H^\gamma-\delta(\epsilon,\eta),x)}{q(\theta_H^\gamma-\delta(\epsilon,\eta),\bar{x}+\eta)} \int_{\theta_0(x)}^{\theta_H^\gamma} q(\theta,\bar{x}+\eta)\left(\pi'(1|\theta)-\pi^{\epsilon,\gamma,\eta}(1|\theta)\right)\mathrm{d}F(\theta) \\
&\geq \frac{q(\theta_H^\gamma-\delta(\epsilon,\eta),x)}{q(\theta_H^\gamma-\delta(\epsilon,\eta),\bar{x}+\eta)} \int_{\theta_L^\gamma}^{\theta_H^\gamma} q(\theta,\bar{x}+\eta)\left(\pi'(1|\theta)-\pi^{\epsilon,\gamma,\eta}(1|\theta)\right)\mathrm{d}F(\theta) \\
&= \frac{q(\theta_H^\gamma-\delta(\epsilon,\eta),x)}{q(\theta_H^\gamma-\delta(\epsilon,\eta),\bar{x}+\eta)} \Delta S(\bar{x}+\eta) = 0.
\end{aligned}$$

The first inequality follows from the fact that (i) for any $\theta \leq \theta_0(x)$, $u(\theta,1-P(x|\theta)) < 0$, whereas for any $\theta > \theta_0(x)$, $u(\theta,1-P(x|\theta)) > 0$, and (ii) for $\theta \in [\theta_0(x), \theta_H^\gamma]$, $\pi'(1|\theta) \leq \pi^{\epsilon,\gamma,\eta}(1|\theta)$. Together, these properties imply that

$$\begin{aligned}
&\int_{\theta_0(x)}^{\theta_H^\gamma} -u(\theta,1-P(x|\theta))\, p(x|\theta)\left(\pi'(1|\theta)-\pi^{\epsilon,\gamma,\eta}(1|\theta)\right)\mathrm{d}F(\theta) \\
\geq 0 \geq\ & \frac{q(\theta_H^\gamma-\delta(\epsilon,\eta),x)}{q(\theta_H^\gamma-\delta(\epsilon,\eta),\bar{x}+\eta)} \int_{\theta_0(x)}^{\theta_H^\gamma} q(\theta,\bar{x}+\eta)\left(\pi'(1|\theta)-\pi^{\epsilon,\gamma,\eta}(1|\theta)\right)\mathrm{d}F(\theta).
\end{aligned}$$

The second inequality follows from the fact that, $\pi'(1|\theta) - \pi^{\epsilon,\gamma,\eta}(1|\theta)$ turns from positive to negative at $\theta = \theta_H^\gamma - \delta(\epsilon,\eta) \leq \theta_0(x)$, along with the fact that, for any $\theta \in [\theta_L^\gamma, \theta_0(x)]$, the function $q(\theta,x)/q(\theta,\bar{x}+\eta)$ is non-increasing in θ as implied by the log-supermodularity of $|u(\theta,1-P(x|\theta))|\, p(x|\theta)$ over $\{(\theta,x) \in [0,1] \times \mathbb{R} : u(\theta,1-P(x|\theta)) \leq 0\}$, which in turn follows from Property (2') of Condition M and the assumption that $p(x|\theta)$ is log-supermodular. Finally, the last two equalities follow from the fact that $\theta_0(\bar{x}+\eta) > \theta_0(\bar{x}) > \theta_H \geq \theta_H^\gamma$, which implies that $u(\theta, 1-P(\bar{x}+\eta|\theta)) \leq 0$ for all $\theta \leq \theta_H^\gamma$, and hence that

$$\int_{\theta_L^\gamma}^{\theta_H^\gamma} q(\theta,\bar{x}+\eta)\left(\pi'(1|\theta)-\pi^{\epsilon,\gamma,\eta}(1|\theta)\right)\mathrm{d}F(\theta) = \Delta S(\bar{x}+\eta)$$

along with the fact that, by construction of the policy $\Gamma^{\epsilon,\gamma,\eta}$, $\Delta S(\bar{x}+\eta) = 0$. Hence, for any

$x \in \left(x^*\left(\theta_H^\gamma - \delta\left(\epsilon,\eta\right)\right), x^*(\theta_H^\gamma)\right)$, $\Delta S(x) \geq 0$, which implies that, for any x such that $(x,1)$ are mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$, $U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x) \geq 0$.

Similar arguments imply that, for any $x \in [x^*(\theta_H^\gamma), \overline{x}+\eta]$,

$$\Delta S(x) = \int_{\theta_L^\gamma}^{\theta_H^\gamma} -u\left(\theta, 1-P\left(x|\theta\right)\right) p\left(x|\theta\right)\left(\pi'\left(1|\theta\right) - \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right) \mathrm{d}F\left(\theta\right)$$
$$= \int_{\theta_L^\gamma}^{\theta_H^\gamma} \tfrac{q(\theta,x)}{q(\theta,\bar{x}+\eta)} q\left(\theta,\bar{x}+\eta\right)\left(\pi'\left(1|\theta\right) - \pi^{\epsilon,\gamma,\eta}\left(1|\theta\right)\right) \mathrm{d}F\left(\theta\right) \geq \tfrac{q(\theta_H^\gamma - \delta(\epsilon,\eta),x)}{q(\theta_H^\gamma - \delta(\epsilon,\eta),\bar{x}+\eta)} \Delta S(\bar{x}+\eta) = 0,$$

implying that, for such x too, if $(x,1)$ are mutually consistent under $\Gamma^{\epsilon,\gamma,\eta}$, then $U^{\Gamma^{\epsilon,\gamma,\eta}}(x,1|x) \geq 0$ (this result also follows from Property (2') of Condition M along with the log-supermodularity of $p\left(x|\theta\right)$). Together, the results above thus imply that, when ϵ, γ, η are small, the new policy $\Gamma^{\epsilon,\gamma,\eta} \in \mathbb{G}$.

We now show that, when property (2") in Condition M holds, the new policy yields the policy maker an expected payoff strictly higher than Γ'. To see this, observe that, fixing (γ, η), for any $\epsilon > 0$, the policy maker's payoff under the policy $\Gamma^{\epsilon,\gamma,\eta}$ is equal to

$$\begin{aligned} \mathcal{U}^P[\Gamma^{\epsilon,\gamma,\eta}] &= \int_{-\infty}^{\theta_L^\gamma+\epsilon} U^P(\theta,0)\mathrm{d}F\left(\theta\right) + \int_{\theta_H^\gamma-\delta(\epsilon,\eta)}^{\theta_H^\gamma} U^P(\theta,1)\mathrm{d}F\left(\theta\right) \\ &+ \int_{(\theta_L^\gamma+\epsilon,\theta_H^\gamma-\delta(\epsilon,\eta))\cup(\theta_H^\gamma,+\infty)} \left\{\pi'(1|\theta)U^P(\theta,1) + (1-\pi'(1|\theta))U^P(\theta,0)\right\}\mathrm{d}F\left(\theta\right). \end{aligned}$$

Differentiating $\mathcal{U}^P[\Gamma^{\epsilon,\gamma,\eta}]$ with respect to ϵ, and taking the limit as $\epsilon \to 0^+$, we have that

$$\lim_{\epsilon\to0^+} \tfrac{d\mathcal{U}^P[\Gamma^{\epsilon,\gamma,\eta}]}{d\epsilon} = f(\theta_H^\gamma)(1-\pi'(1|\theta_H^\gamma))\left[U^P(\theta_H^\gamma,1) - U^P(\theta_H^\gamma,0)\right] \times \lim_{\epsilon\to0^+}\tfrac{\partial\delta(\epsilon,\eta)}{\partial\epsilon}$$
$$-f(\theta_L^\gamma)\pi'(1|\theta_L^\gamma)\left[U^P(\theta_L^\gamma,1) - U^P(\theta_L^\gamma,0)\right]$$
$$= f(\theta_L^\gamma)\pi'(1|\theta_L^\gamma)\left(\left[U^P(\theta_H^\gamma,1) - U^P(\theta_H^\gamma,0)\right]\tfrac{p(\bar{x}+\eta|\theta_L^\gamma)u(\theta_L^\gamma,1-P(\bar{x}+\eta|\theta_L^\gamma))}{p(\bar{x}+\eta|\theta_H^\gamma)u(\theta_H^\gamma,1-P(\bar{x}+\eta|\theta_H^\gamma))} - \left[U^P(\theta_L^\gamma,1) - U^P(\theta_L^\gamma,0)\right]\right).$$

Therefore, $\lim\limits_{\epsilon\to0^+}\tfrac{d\mathcal{U}^P[\Gamma^{\epsilon,\gamma,\eta}]}{d\epsilon} > 0$ if and only if

$$\frac{U^P\left(\theta_H^\gamma,1\right) - U^P\left(\theta_H^\gamma,0\right)}{U^P\left(\theta_L^\gamma,1\right) - U^P\left(\theta_L^\gamma,0\right)} > \frac{p\left(\bar{x}+\eta|\theta_H^\gamma\right)u\left(\theta_H^\gamma,1-P\left(\bar{x}+\eta|\theta_H^\gamma\right)\right)}{p\left(\bar{x}+\eta|\theta_L^\gamma\right)u\left(\theta_L^\gamma,1-P\left(\bar{x}+\eta|\theta_L^\gamma\right)\right)}.$$

Property (2") in Condition M, together with the fact that $\bar{x} \leq \bar{x}_G$, guarantee that the last inequality holds. We conclude that the policy $\Gamma^{\epsilon,\gamma,\eta} \in \mathbb{G}$ yields the policy maker a payoff strictly higher than Γ'. This completes the proof of Claim S1-B. □

Claim C. *Suppose that Condition M holds and that $\Gamma' \in \mathbb{G}$ is such that*

$$\left\{\theta \in \left(\inf\Theta\left(\bar{x}\right), \theta_H\right) : \pi'(1|\theta) > 0\right\} \text{ has zero } F\text{-measure.} \tag{17}$$

Then, $\pi'(1|\theta) = 0$ *for* F*-almost all* $\theta \leq \theta^*$ *and* $\pi'(1|\theta) = 1$ *for* F*-almost all* $\theta > \theta^*$*.*

Proof of Claim C. Condition (17), together with the definition of θ_H and the fact that $U^{\Gamma'}(\bar{x}, 1|\bar{x}) = 0$, jointly imply that $\theta_H < \sup \Theta(\bar{x})$ and that $U^{\Gamma'}(\bar{x}, 1|\bar{x}) = U^{\Gamma^{\theta_H}}(\bar{x}, 1|\bar{x})$, where $\Gamma^{\theta_H} = (\{0, 1\}, \pi^{\theta_H})$ is the monotone deterministic policy with cut-off θ_H.[54] In other words, from the perspective of an agent with signal $\bar{x}$, the information learned under Γ', by the announcement that $s = 1$ is the same as the one learnt under Γ^{θ_H}.

Suppose that $\theta_H > \theta^*$. For any deterministic monotone policy $\Gamma^{\hat{\theta}} = (\{0, 1\}, \pi^{\hat{\theta}})$, any $\tilde{\theta} \geq \hat{\theta}$, let $\varphi(\tilde{\theta}; \hat{\theta}) \equiv \int_{\hat{\theta}}^{\sup \Theta(x^*(\tilde{\theta}))} u(\theta, 1 - P(x^*(\tilde{\theta})|\theta))p(x^*(\tilde{\theta})|\theta)\mathrm{d}F(\theta)$ and $\bar{\varphi}(\hat{\theta}) \equiv \inf_{\tilde{\theta}\geq\hat{\theta}} \varphi(\tilde{\theta}; \hat{\theta})$. Note that, for any $\tilde{\theta}$ such that $(x^*(\tilde{\theta}), 1)$ are mutually consistent under the policy $\Gamma^{\hat{\theta}}$, $\varphi(\tilde{\theta}; \hat{\theta}) = U^{\Gamma^{\hat{\theta}}}(x^*(\tilde{\theta}), 1|x^*(\tilde{\theta}))p^{\Gamma^{\hat{\theta}}}(x^*(\tilde{\theta}), 1)$. We claim that, for any $\hat{\theta} > \theta^*$, $\bar{\varphi}(\hat{\theta}) > 0$. To see this, consider first the case where $\hat{\theta} \in \arg\min_{\tilde{\theta}\geq\hat{\theta}} \varphi(\tilde{\theta}; \hat{\theta})$. Observe that, if each agent follows a threshold strategy with cut-off $x^*(\hat{\theta})$, then default occurs only for fundamentals weakly below $\hat{\theta}$. Because $u(\theta, 1 - P(x^*(\hat{\theta})|\theta)) > 0$ for all $\theta > \hat{\theta}$ and because $p(x^*(\hat{\theta})|\theta) > 0$ in a right-neighborhood of $\hat{\theta}$, then necessarily $\bar{\varphi}(\hat{\theta}) = \varphi(\hat{\theta}; \hat{\theta}) > 0$. Next, suppose that $\hat{\theta} \notin \arg\min_{\tilde{\theta}\geq\hat{\theta}} \varphi(\tilde{\theta}; \hat{\theta})$. Then, observe that, for almost any $\hat{\theta} \geq \theta^*$, and any $\tilde{\theta}_m \in \arg\min_{\tilde{\theta}\geq\hat{\theta}} \varphi(\tilde{\theta}; \hat{\theta})$, with $\tilde{\theta}_m > \hat{\theta}$,[55] $\partial\varphi(\tilde{\theta}_m; \hat{\theta})/\partial\hat{\theta} = -u(\hat{\theta}, 1 - P(x^*(\tilde{\theta}_m)|\hat{\theta}))p(x^*(\tilde{\theta}_m)|\hat{\theta})f(\hat{\theta}) \geq 0$, where the inequality follows from the fact that $u(\hat{\theta}, 1 - P(x^*(\tilde{\theta}_m)|\hat{\theta})) < 0$ which, in turn, is a consequence of (i) the definition of $x^*(\cdot)$ and (ii) the fact that $\tilde{\theta}_m > \hat{\theta}$.

By the definition of θ^*, $\bar{\varphi}(\theta^*) = 0$, and $d\bar{\varphi}(\theta^*)/d\hat{\theta} > 0$. The above properties thus imply that, for any $\hat{\theta} > \theta^*$, $\bar{\varphi}(\hat{\theta}) > 0$, as claimed.

By the definition of $\bar{x}$, $U^{\Gamma'}(\bar{x}, 1|\bar{x}) = 0$. Under Condition (17), this implies that, when agents pledge for $x > \bar{x}$ and refrain from pledging for $x < \bar{x}$, the default outcome $\theta_0(\bar{x})$ must necessarily satisfy $\theta_0(\bar{x}) > \theta_H$, for, otherwise, an agent with signal $\bar{x}$ would strictly prefer pledging to not pledging. Because $U^{\Gamma'}(\bar{x}, 1|\bar{x}) = U^{\Gamma^{\theta_H}}(\bar{x}, 1|\bar{x})$, that $\theta_0(\bar{x}) > \theta_H > \theta^*$, along with the fact that $\varphi(\theta_0(\bar{x}); \theta_H) > 0$, however, implies that $U^{\Gamma'}(\bar{x}, 1|\bar{x}) > 0$, a contradiction.

Hence, it must be that $\theta_H \leq \theta^*$. However, by definition of θ^*, if $\theta_H < \theta^*$, then there exists $\theta > \theta_H$ such that $(x^*(\theta), 1)$ are mutually consistent under Γ^{θ_H} and such that

$$U^{\Gamma^{\theta_H}}(x^*(\theta), 1|x^*(\theta))p^{\Gamma^{\theta_H}}(x^*(\theta), 1) = \int_{\theta_H}^{\sup \Theta(x^*(\theta))} u(\tilde{\theta}, 1 - P(x^*(\theta)|\tilde{\theta}))p(x^*(\theta)|\tilde{\theta})\mathrm{d}F(\tilde{\theta}) < 0.$$

[54] If $\theta_H \geq \sup \Theta(\bar{x})$ then $p^{\Gamma'}(\bar{x}, 1) \equiv \int p(\bar{x}|\theta)\pi'(1|\theta)\mathrm{d}F(\theta) = 0$ contradicting the assumption that $U^{\Gamma'}(\bar{x}, 1|\bar{x}) = 0$ which requires that $(x, 1)$ are mutually consistent under Γ'.

[55] Note that $\varphi(\tilde{\theta}; \hat{\theta})$ is absolutely continuous in $\hat{\theta}$, and therefore is differentiable in $\hat{\theta}$ almost everywhere.

Now note that

$$U^{\Gamma'}(x^*(\theta),1|x^*(\theta))\,p^{\Gamma'}(x^*(\theta),1) = \int_{\inf\Theta(x^*(\theta))}^{\theta_H} u(\tilde{\theta},1-P(x^*(\theta)\,|\tilde{\theta}))\pi'(1|\tilde{\theta})p(x^*(\theta)\,|\tilde{\theta})\mathrm{d}F(\tilde{\theta}) + \int_{\theta_H}^{\sup\Theta(x^*(\theta))} u(\tilde{\theta},1-P(x^*(\theta)\,|\tilde{\theta}))p(x^*(\theta)\,|\tilde{\theta})\mathrm{d}F(\tilde{\theta})$$

with $p^{\Gamma'}(x^*(\theta),1) = \int_{\inf\Theta(x^*(\theta))}^{\theta_H}\pi'(1|\tilde{\theta})p(x^*(\theta)|\tilde{\theta})\mathrm{d}F(\tilde{\theta}) + p^{\Gamma^{\theta_H}}(x^*(\theta),1) > 0$. Because, for any $\tilde{\theta} < \theta_H$, $u(\tilde{\theta},1-P(x^*(\theta)\,|\tilde{\theta})) < 0$, we thus have that $U^{\Gamma'}(x^*(\theta),1|x^*(\theta)) < 0$. But this contradict the assumption that $\Gamma' \in \mathbb{G}$. We thus conclude that necessarily $\theta_H = \theta^*$. Furthermore, because $\{\theta \in (\inf\Theta(\bar{x}),\theta_H) : \pi'(1|\theta) > 0\}$ has 0 F-measure, it must be that $U^{\Gamma'}(\bar{x},1|\bar{x}) = U^{\Gamma^{\theta^*}}(\bar{x},1|\bar{x})$. Furthermore, because $\theta_0(\bar{x}) > \theta^*$, we also have that $U^{\Gamma^0}(\bar{x},1|\bar{x}) \leq U^{\Gamma^{\theta^*}}(\bar{x},1|\bar{x}) = 0$. Hence, $\bar{x} \leq \bar{x}_G$, which, by virtue of Property 1 in Condition M, implies that $\inf\Theta(\bar{x}) \leq 0$. Condition (17), along with the fact that $\pi'(1|\theta) = 0$ for all $\theta \leq 0$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \theta_H = \theta^*$, thus imply that Γ' is such that $\pi'(1|\theta) = 0$ for F-almost all $\theta \leq \theta^*$ and $\pi'(1|\theta) = 1$ for F-almost all $\theta > \theta^*$. This completes the proof of Claim C. □

Step 2. Step 1 implies that $\arg\max_{\tilde{\Gamma}\in\mathbb{G}}\{\mathcal{U}^P[\tilde{\Gamma}]\} \neq \emptyset$ and that any $\Gamma \in \arg\max_{\tilde{\Gamma}\in\mathbb{G}}\{\mathcal{U}^P[\tilde{\Gamma}]\}$ is such that $\pi(1|\theta) = 0$ for F-almost all $\theta \leq \theta^*$ and $\pi(1|\theta) = 1$ for F-almost all $\theta > \theta^*$. The result in the theorem then follows from observing that, given any $\Gamma \in \arg\max_{\tilde{\Gamma}\in\mathbb{G}}\{\mathcal{U}^P[\tilde{\Gamma}]\}$, there exists a nearby deterministic monotone policy $\Gamma^{\hat{\theta}} \in \mathbb{G}$ with cut-off $\hat{\theta} = \theta^* + \tilde{\varepsilon}$, for $\tilde{\varepsilon} > 0$ but small, such that $\Gamma^{\hat{\theta}}$ satisfies PCP (i.e., $U^{\Gamma^{\hat{\theta}}}(x,1|x) > 0$ all x such that $(x,1)$ are mutually consistent under $\Gamma^{\hat{\theta}}$) and yields the policy maker a payoff arbitrarily close to that under Γ. Q.E.D.

Proof of Theorem 4. The formal proof follows from arguments similar to those establishing Theorems 1-3 and is omitted for brevity.[56] Here we discuss the novel effects due to the enrichments introduced above and the role played by the conditions in the theorem.

First, consider part (a). When default depends on variables only imperfectly correlated with θ, perfect coordination cannot be induced by announcing to the investors the fate of the

[56] Because, in the generalized model, the default outcome need not be a deterministic function of θ, the definition of $x^*(\theta)$ and $\theta_0(x)$ in the main text must be amended as follows: $x^*(\theta)$ is the critical signal threshold such that, when agents pledge for $x > x^*(\theta)$ and do not pledge for $x < x^*(\theta)$, the agents' expected payoff differential $u(\tilde{\theta},1-P(x^*(\theta)|\tilde{\theta}))$ changes sign at $\tilde{\theta} = \theta$; $\theta_0(x)$ is the critical fundamental threshold such that, when agents pledge of $\tilde{x} > x$ and do not pledge for $\tilde{x} < x$, the agents' expected payoff differential $u(\theta,1-P(x|\theta))$ changes sign at $\theta = \theta_0(x)$. As in the baseline model, we assume that these functions are continuous.

regime under MARP, as in the proof of Theorem 1. Perfect coordination, however, can still be induced by announcing, at any θ, the *sign* of the investors' expected payoff differential under the original policy. Arguments similar to those establishing Theorem 1 then imply that, when the investors learn that their expected payoff differential under the original policy was positive, under the new policy, they all pledge, irrespective of their signals. Likewise, when they hear their payoff was negative, they all refrain from pledging. That the new policy makes the investors better off then follows from the fact that the investors' payoff differentials are non-decreasing in the size of the aggregate pledge. In the special case in which θ is a perfect predictor of the default outcome, because the sign of the investors' expected payoff differential is determined by the default outcome, perfect coordination is obtained by informing the investors of the default outcome, as in the baseline model. In this case, the ability to coordinate perfectly the market while inducing the same default outcome as under the original policy extends to an even richer class of economies. In particular, economies in which (i) agents' prior beliefs need not be consistent with a common prior, nor be generated by signals drawn independently across agents, conditionally on θ, (ii) the number of agents is arbitrary (in particular, finitely many agents), (iii) agents' have a level-K degree of sophistication, (iv) payoffs may be heterogenous across agents, and (v) the designer may disclose different information to different agents (see the document "Additional Material" on the authors' websites for details).

Next, consider part (b). As explained above, when $p(x|\theta)$ is log-supermodular and $u(\theta, 1 - P(x|\theta))$ has the single-crossing property of Condition FB, then, under MARP, the investors' strategies are monotone in their private signals, no matter the structure of the policy Γ and the shape of the induced common posterior, G. Arguments similar to those establishing Theorem 2 in the main text then imply that the new policy that perfectly coordinates the investors does not need to reveal anything more than the sign of the investors' expected payoff differential under the original policy.

Next, consider part (c). The pass/fail policy described above clearly makes all investors weakly better off. In general, it need not make the policy maker better off. However, when Condition PC also holds, possible losses to the policy maker from inducing fewer agents to pledge in states in which the agents' expected payoff differential is negative are compensated by having more agents pledge in those states in which their expected payoff differential is positive. When this is the case, the new policy leads to a Pareto improvement.

Finally, consider part (d). As discussed in the main text, in general, the optimal pass/fail policy need not be monotone in θ. However, it is monotone when, in addition to the conditions in part (c), Condition M in the main text also holds. Q.E.D.

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